Fusion of Probabilistic Knowledge as Foundation for Sliced-Normal Approach

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Abstract

In many practical applications, it turns out to be efficient to use Sliced-Normal multi-D distributions, i.e., distributions for which the logarithm of the probability density function (pdf) is a polynomial – – to be more precise, it is a sum of squares of several polynomials; see, e.g., Colbert, Crespo, and Peet (2019); Crespo (2019); Crespo, Colbert, Kenny, and Giesy (2019). This class is a natural extension of normal distributions, i.e., distributions for which the logarithm of the pdf is a quadratic polynomial.

In this paper, we provide a possible theoretical explanation for this empirical success. In this explanation, we look for families F of distributions which continuously depend on finitely many parameters. Our first reasonable requirement is that if we have two independent estimates for the same random vector, and both pdfs are in F, then the result of fusing this knowledge – i.e., a normalized product of the pdfs – should also belong to the same family F. From this, we conclude that the class L of logarithms of pdfs is closed under linear combination (modulo an additive constant).

Our second reasonable requirement is that the family F should not change if we simply change the starting point and/or the measuring unit of one of the quantities. It turns out that, from these two requirements, we can imply that the family L contains only polynomials – which explains the appearance of Sliced-Normal distributions.

References

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