Distribution-free uncertainty propagation

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Abstract

Elementary formulas for propagating information about means and variances through mathematical expressions have long been used by analysts (Lauritzen, 1992). Yet the precise implications of such information are rarely articulated. This paper explores distribution-free techniques of uncertainty propagation that do not require simulation, sampling or approximation of any kind. We describe best-possible bounds on exceedance risks (probabilities of extreme events) that can be inferred given only information about the range, mean and variance of a random variable. These bounds generalize the classical Chebyshev inequality in an obvious way, yet apparently have not been described elsewhere. We also collect in convenient tables several formulas for propagating range and moment information through calculations involving 7 binary convolutions (addition, subtraction, multiplication, division, powers, minimum, and maximum) and 9 unary transformations (scalar multiplication, scalar translation, exponentiation, natural and common logarithms, reciprocal, square, square root and absolute value) commonly encountered in risk expressions (Rowe, N.C., 1988). These formulas are rigorous rather than approximate, and in most cases are either exact or mathematically best possible. The formulas can be used effectively even when only interval estimates of the moments are available (Ferson, 1999). Although most discussions of moment propagation assume stochastic independence among variables, this paper shows the assumption to be unnecessary and generalizes formulas for the case when no assumptions are made about dependence.

References

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