Serviceability assessment of footbridges via interval analysis

Roberta Santoro[1)], Alba Sofi[2)], and Federica Tubino[3)]
[1)] Department of Engineering, University of Messina, Messina, 98166, Italy, roberta.santoro@unime.it
[2)] Department of Architecture and Territory, University "Mediterranea" of Reggio Calabria, Reggio Calabria, 89124, Italy, alba.sofi@unirc.it
[3)] Department of Civil, Chemical and Environmental Engineering, University of Genoa, Genova, 16145, Italy, federica.tubino@unige.it.

Abstract: This paper studies serviceability assessment of footbridges through a non-deterministic approach. Both the parameters defining pedestrian-induced loading and the structural dynamic properties are uncertain quantities, and they can be characterized through possible ranges of variation. Thus, a suitable tool for taking uncertainties into account is interval analysis. In this paper, the improved interval analysis is applied together with classical optimization that allows obtaining the bounds of the spectral moments of the response, of the cumulative distribution function of the maximum footbridge acceleration, and of the expected value of the maximum acceleration. Based on this approach, an interval of probability of reaching a suitable comfort level can be estimated.

Keywords: Footbridges; Human-induced vibration; Interval Analysis; Serviceability.

1. Introduction

The increasing strength of new materials and aesthetic requirements lead to the design of lightweight, slender and, consequently, vibration-sensitive structures. As regards footbridges, this trend is further stimulated by the relatively small service loads. For these structures, the vibration serviceability under human-induced loading has become a key design criterion.

In order to estimate human-induced response of footbridges, pedestrian traffic should be modelled probabilistically, considering several sources of randomness among which pedestrian arrivals, step frequencies and velocities, force amplitudes and pedestrian weights. The method generally used to statistically characterize the maximum dynamic response of the footbridge accounting for the probabilistic nature of pedestrian excitation is Monte Carlo simulation. However, due to the computational burden of Monte Carlo simulations, which makes them not practical in view of engineering applications, current guidelines (e.g., SETRA, 2006; HIVOSS, 2009) provide simplified procedures to assess footbridges' serviceability. An interesting alternative to simplified procedures is the introduction of spectral models of pedestrian excitation (e.g., Brownjohn et al., 2004; Piccardo and Tubino, 2012), that allow dealing with the problem of serviceability assessment in the frequency domain through the methods of random dynamics. The Equivalent Spectral Model (ESM) introduced by Piccardo and Tubino (2012) has been derived analytically, based on suitable probabilistic models of all the parameters involved. It allows deriving closed-form expressions for the spectral moments of the dynamic response, as functions of the loading and structural parameters.

Serviceability assessment of footbridges is commonly carried out, based on current guidelines, by comparing a deterministic value of the expected maximum acceleration and suitable thresholds defining different comfort classes. Actually, both pedestrian-induced loading parameters (i.e. dynamic loading factor, pedestrian weight and step frequency) and the structural dynamic properties (i.e. modal damping ratios, natural frequencies and modal masses) are affected by remarkable uncertainties. Thus, serviceability assessment should be tackled through a non-deterministic approach. A first attempt accounting only for the errors connected with the estimate of structural parameters and based on simplified loading models has been introduced through a fuzzy analysis (Lievens et al., 2016). In principle, serviceability assessment could be carried out based on Monte Carlo simulations of pedestrian excitation, taking into account the uncertainties of the involved parameters, resulting in very time consuming analyses. However, the ESM constitutes a very efficient tool to perform uncertainty propagation analyses, thanks to the availability of closed-form solutions for the spectral moments of the response as explicit functions of the uncertain parameters.

Tubino et al. (2020) carried out a literature analysis in order to characterize probabilistically the uncertainties in the loading and structural parameters. The mean values and coefficient of variation of the involved uncertain parameters have been estimated, and the propagation of uncertainties on the serviceability assessment of footbridges has been studied adopting the Taylor series expansion technique. Furthermore, Monte Carlo simulations have been performed assuming probability distributions of the uncertain parameters, and the probability distribution of the mean value of the maximum acceleration has been estimated numerically (Tubino et al., 2020).

Actually, probability distributions of the loading and structural parameters are not available, but the literature analysis in Tubino et al. (2020) provides possible ranges of variation of these parameters. Thus, a suitable tool for taking into account uncertainties in serviceability assessment of footbridges is interval analysis (Moore et al., 2009; Muscolino and Sofi, 2012). Indeed, the key idea of the interval model of uncertainty is to describe the uncertain parameters as interval variables with given lower bound and upper bound without requiring a full probabilistic characterization. Since the mid-1990s, several studies have been devoted to the development of efficient procedures for the propagation of interval uncertainties in engineering problems (see e.g., Faes and Moens, 2020). The main challenge is the evaluation of tight bounds of system’s response, which is described by an interval rather than assuming a single value. Both static (see e.g., Sofi et al. 2019) and dynamic (see e.g., Liu et al., 2013) behaviors of structures with interval parameters have been thoroughly investigated in literature. Over the last decade, the analysis of randomly excited systems in the presence of interval uncertainties has also been addressed (see e.g., Muscolino and Sofi, 2012; Muscolino et al., 2016).

In this paper, starting from the analytical expression for the spectral moments of the structural response (Piccardo and Tubino, 2012), the *improved interval analysis* (Muscolino and Sofi, 2012) is applied together with classical optimization that allows obtaining the bounds of the standard deviation, of the cumulative distribution function (Muscolino et al., 2016) and of the expected value of the maximum footbridge acceleration. Based on this approach, serviceability assessment is carried out within a non-probabilistic framework, and an interval of probability of reaching a suitable comfort level can be estimated.

2. Analytical formulation

2.1. EQUATION OF MOTION

Let us consider a footbridge, modelled as a linear mono-dimensional classically damped dynamical system. Its equation of motion can be written as follows:

$$m_s(x)\frac{\partial^2 q(x,t)}{\partial^2 t}+\mathcal{C}\left[\frac{\partial q(x,t)}{\partial t}\right]+\mathcal{L}\left[q(x,t)\right]=f(x,t) \tag{1}$$

where q is the vertical displacement of the footbridge; x is the abscissa along the structure; t is the time; $m_s(x)$ is the structural mass; $\mathcal{C}$ is the damping operator; $\mathcal{L}$ is the stiffness operator; $f(x,t)$ is the external force.
Focusing attention only on the first walking harmonic for each pedestrian, the force induced by N_p pedestrians can be expressed as:

$$f(x,t)=\sum_{i=1}^{N_p}\alpha_i G_i \sin\left(\Omega_i(t-\tau_i)+\Psi_i\right)\delta\left[x-c_i(t-\tau_i)\right]\left[H(t-\tau_i)-H\left(t-\tau_i-\frac{L}{c_i}\right)\right] \tag{2}$$

where $\delta(\bullet)$ and $H(\bullet)$ are the Dirac function and the Heaviside function, respectively; L is the span length of the structure; furthermore, F_i $(=\alpha_i G_i)$, ω_i, Ψ_i, c_i and τ_i are, respectively, the force amplitude, the step circular frequency, the phase-angle, the walking velocity and the arrival time of the i-th pedestrian, while α_i and G_i are the Dynamic Loading Factor (DLF) and the weight of the i-th pedestrian. All these quantities can be considered as random variables (Racic et al. 2009; Tubino and Piccardo, 2016).

Under the hypothesis of classical damping, Eq. (1) is usually solved applying the principal transformation. Considering the contribution of one vibration mode (namely the j-th), the structural displacement can be expressed as:

$$q(x,t)=\varphi_j(x)\,p_j(t) \tag{3}$$

where $\varphi_j(x)$ is the j-th mode of vibration, $p_j(t)$ is the corresponding principal coordinate.

The equation of motion of the j-th principal coordinate is expressed as:

$$\ddot{p}_j(t)+2\xi_j\omega_j\dot{p}_j(t)+\omega_j^2 p_j(t)=\frac{1}{M_j}F_j(t) \tag{4}$$

where ξ_j, ω_j and M_j are, respectively, the j-th modal damping ratio, natural circular frequency and modal mass; $F_j(t)$ is the j-th modal force:

$$F_j(t)=\int_0^L f(x,t)\varphi_j(x)\,\mathrm{d}x \tag{5}$$

2.2. EQUIVALENT SPECTRAL MODEL OF THE LOADING

According to the ESM, multi-pedestrian loading can be modelled as an equivalent Gaussian stationary random process. Considering the first modal shape of a simply-supported beam $\varphi_j(x)=\sin(\pi x/L)$, the power spectral density function (PSDF) of the modal force F_j (Eq. (5)) is given by:

$$S_{F_j}(\omega) = \frac{N_p}{4}(\alpha G)^2 p_\Omega(\omega) \tag{6}$$

where N_p is the number of pedestrians; α is the mean value of the DLF; G is the mean value of the weight of pedestrians; $p_\Omega(\omega)$ is the probability density function (PDF) of the circular step frequency of pedestrians. The circular step frequency of pedestrians can be assumed as normally distributed, and its PDF is given by:

$$p_\Omega(\omega) = \frac{1}{\sigma_{\omega p}\sqrt{2\pi}} \exp\left[-\frac{(\omega - \mu_{\omega p})^2}{2\sigma_{\omega p}^2} \right] \tag{7}$$

being $\mu_{\omega p}$ and $\sigma_{\omega p}$ the mean-value and standard deviation of the circular step frequency of pedestrians, respectively.

The PSDF of the acceleration of the j-th principal coordinate is then given by:

$$S_{\ddot{p}_j}(\omega) = \omega^4 \left| H_j(\omega) \right|^2 S_{F_j}(\omega) \tag{8}$$

where the PSDF of the modal force $S_{Fj}(\omega)$ is provided by Eq. (6), and $H_j(\omega)$ is the frequency response function of the j-th principal coordinate:

$$H_j(\omega) = \frac{1}{M_j} \frac{1}{\omega_j^2 - \omega^2 + 2\mathrm{i}\xi_j \omega_j \omega} \tag{9}$$

where $\mathrm{i} = \sqrt{-1}$ is the imaginary unit. Taking into account that the PSDF of the modal force $S_{Fj}(\omega)$ is nearly constant around the j-th natural circular frequency ω_j, the spectral moments of zero- and second-order of the acceleration of the j-th principal coordinate can be approximated as follows:

$$\lambda_{0,\ddot{p}_j} = \sigma_{\ddot{p}_j}^2 = \int_0^{+\infty} S_{\ddot{p}_j}(\omega)\mathrm{d}\omega = \int_0^{+\infty} \omega^4 \left| H_j(\omega) \right|^2 S_{F_j}(\omega)\mathrm{d}\omega \approx \frac{\pi\omega_j S_{F_j}(\omega_j)}{4\xi_j M_j^2} \tag{10}$$

$$\lambda_{2,\ddot{p}_j} = \int_0^{+\infty} \omega^2 S_{\ddot{p}_j}(\omega)\mathrm{d}\omega = \int_0^{+\infty} \omega^6 \left| H_j(\omega) \right|^2 S_{F_j}(\omega)\mathrm{d}\omega \approx \frac{\pi\omega_j^3 S_{F_j}(\omega_j)}{4\xi_j M_j^2} \tag{11}$$

2.3. EXTREME VALUE OF THE ACCELERATION

Modelling the loading through the ESM, the footbridge dynamic response is a stationary Gaussian random process. The maximum acceleration of the j-th principal coordinate in the time interval T is then defined as the following random process:

$$\ddot{p}_{j,\max}(T) = \max_{0 \le t \le T} \left| \ddot{p}_j(T) \right| \tag{12}$$

where the time interval T can be conventionally defined as a multiple of the average crossing time of pedestrians, $T=NL/c_m$, being $c_m=0.9\mu_{\omega p}/2\pi$ (m/s) the mean walking velocity and assuming N=10 (Piccardo and Tubino, 2012).

Under the Poisson's assumption of independent up-crossings of a prescribed threshold, the probability that the extreme value acceleration process is less than or equal to the critical level b>0 within the time interval [0 T] is defined by the following cumulative distribution function (CDF):

$$L_{\ddot{p}_{j,\max}}(b,T) = \Pr\left[\ddot{p}_{j,\max}(T) \le b \right] \approx \exp\left[-2T\, \nu_{\ddot{p}_j} \exp\left(-\frac{b^2}{2\lambda_{0,\ddot{p}_j}} \right) \right] \tag{13}$$

where $\nu_{\ddot{p}_j}$ is the expected frequency of the acceleration of the j-th principal coordinate, given by:

$$\nu_{\ddot{p}_j} = \frac{1}{2\pi}\sqrt{\frac{\lambda_{2,\ddot{p}_j}}{\lambda_{0,\ddot{p}_j}}} \approx \frac{\omega_j}{2\pi}. \tag{14}$$

Considering Eq. (3), and the first modal shape of a simply-supported beam $\varphi_j(x) = \sin(\pi x/L)$, the maximum mid-span acceleration is coincident with the maximum acceleration of the j-th principal coordinate. According to Davenport's formulation (Davenport, 1964), it is given by:

$$\ddot{q}_{\max} = \mathrm{E}\left[\ddot{p}_{j_{\max}}\right] = g_{\ddot{p}_j}\sigma_{\ddot{p}_j} \tag{15}$$

being the peak factor $g_{\ddot{p}_j}$ defined as:

$$g_{\ddot{p}_j} = \sqrt{2\ln(2\nu_{\ddot{p}_j}T)} + \frac{0.5772}{\sqrt{2\ln(2\nu_{\ddot{p}_j}T)}} \tag{16}$$

2.4. COMFORT REQUIREMENTS

Serviceability assessment of footbridges is commonly carried out, based on current guidelines, by comparing a deterministic value of the expected maximum acceleration and suitable thresholds defining different comfort classes. Table I reports the threshold values as defined by SETRA (2006).

Table I. Comfort classes (SETRA)

Comfort Class		*Min*	*Max*
Maximum	[m/s^2]	0	0.5
Medium	[m/s^2]	0.5	1
Minimum	[m/s^2]	1	2.5
Unacceptable	[m/s^2]	2.5	

Assuming deterministic values for all the parameters involved, the serviceability assessment can be carried out estimating the mean-value of the maximum acceleration through Eq. (15) and checking the comfort class through Table I. As an alternative, the probability of falling within each comfort class can be estimated through Eq. (13).

3. Interval serviceability assessment

Interval serviceability assessment is based on the assumption of uncertain loading and structural parameters, represented as interval variables according to the interval model of uncertainty:

$$\mathbf{x}^I = \left\{\mu^I_{\omega p} \quad \sigma^I_{\omega p} \quad \alpha^I \quad G^I \quad \omega^I_j \quad \xi^I_j \quad M^I_j\right\}^{\mathrm{T}}. \tag{17}$$

Each component of the interval vector $\mathbf{x}^I$, i.e. the k-th interval variable x^I_k, is defined as:

$$x^I_k = x_{0,k}\left(1 + \Delta x_k \hat{e}^I_k\right) \tag{18}$$

where $\hat{e}_k^I = [-1, 1]$ is the *extra unitary interval* (Muscolino and Sofi, 2012), $x_{0,k}$ and Δx_k are the midpoint and normalized deviation amplitude, defined as follows:

$$x_{0,k} = \frac{\overline{x}_k + \underline{x}_k}{2}; \tag{19}$$

$$\Delta x_k = \frac{\overline{x}_k - \underline{x}_k}{2x_{0,k}}. \tag{20}$$

In Eqs. (19) and (20), $\underline{x}_k$ and $\bar{x}_k$ represent, respectively, the lower bound (LB) and upper bound (UB) of the interval variable x_k^I.

Based on the interval representation of the involved variables, all the quantities in Eqs. (6)-(16) are interval functions/ variables. The impact of each uncertain parameter on serviceability assessment of the footbridge can be predicted performing sensitivity analysis. Sensitivities of the standard deviation of the acceleration, of the CDF of the extreme value acceleration and of the expected value of the maximum acceleration can be evaluated analytically by direct differentiation of Eqs. (10), (13) and (15) with respect to the parameters $x_k \in x_k^I$.

The LB and UB of the standard deviation of the modal acceleration are defined as:

$$\underline{\sigma}_{\ddot{p}_j} = \min_{\mathbf{x}\in\mathbf{x}^I}\left\{\sqrt{\frac{\pi\omega_j S_{F_j}(\omega_j)}{4\xi_j M_j^2}}\right\}; \quad \bar{\sigma}_{\ddot{p}_j} = \max_{\mathbf{x}\in\mathbf{x}^I}\left\{\sqrt{\frac{\pi\omega_j S_{F_j}(\omega_j)}{4\xi_j M_j^2}}\right\}. \tag{21}$$

Analogously, the LB and UB of the interval CDF of the extreme value acceleration process are defined as:

$$\begin{aligned}\underline{L}_{\ddot{p}_{j,\max}}(\mathbf{x};b,T) &= \min_{\mathbf{x}\in\mathbf{x}^I}\left\{\exp\left[-2T\,\nu_{\ddot{p}_j}\exp\left(-\frac{b^2}{2\lambda_{0,\ddot{p}_j}}\right)\right]\right\}\\ \bar{L}_{\ddot{p}_{j,\max}}(\mathbf{x};b,T) &= \max_{\mathbf{x}\in\mathbf{x}^I}\left\{\exp\left[-2T\,\nu_{\ddot{p}_j}\exp\left(-\frac{b^2}{2\lambda_{0,\ddot{p}_j}}\right)\right]\right\}\end{aligned} \tag{22}$$

Finally, the LB and UB of the maximum mid-span acceleration are given by:

$$\begin{aligned}\underline{\ddot{q}}_{\max} &= \min_{\mathbf{x}\in\mathbf{x}^I}\left\{\left[\sqrt{2\ln(2\nu_{\ddot{p}_j}T\ )} + \frac{0.5772}{\sqrt{2\ln(2\nu_{\ddot{p}_j}T\ )}}\right]\sigma_{\ddot{p}_j}\right\}\\ \bar{\ddot{q}}_{\max} &= \max_{\mathbf{x}\in\mathbf{x}^I}\left\{\left[\sqrt{2\ln(2\nu_{\ddot{p}_j}T\ )} + \frac{0.5772}{\sqrt{2\ln(2\nu_{\ddot{p}_j}T\ )}}\right]\sigma_{\ddot{p}_j}\right\}\end{aligned} \tag{23}$$

Since analytical expressions of the functions of interest are available, the global minimum and maximum under the constraint that the optimization parameters range within prescribed intervals can be evaluated using the built-in functions “Minimize” and “Maximize” of Wolfram Mathematica (Wolfram Research 2020).

4. Numerical application

In this Section, serviceability assessment of an ideal footbridge is carried out. Results of a conventional evaluation are compared with those deriving from interval analysis. In particular, at first interval analysis is carried out assuming fixed normalized deviation amplitudes of the parameters. Then, an increasing degree of uncertainty is considered according to the following definition of the interval variables:

$$x_k^I(\beta) = x_{0,k}\left(1+\beta\Delta x_k \hat{e}_k^I\right), \qquad 0 \le \beta \le 1. \tag{24}$$

Furthermore, three different analyses are carried out: considering all the parameters as uncertain, i.e. full uncertainty analysis (FU), taking into account only the uncertainties in the structural parameters (SU), and taking into account only uncertainties in the loading parameters (LU).

An ideal, steel, simply-supported beam with span length L=50 m, a deck width b=2 m is analyzed (Tubino et al. 2020). Considering load category III, sparse crowd (SETRA, 2006), a pedestrian density ρ=0.5 pedestrians/m^2 is assumed, i.e. N_p=50, and the predicted maximum acceleration applying SETRA procedure is $\ddot{q}_{\max}$ =2.15 m/s^2, falling into a minimum comfort class according to Table I.

Table II reports the midpoint and the normalized deviation amplitude of the uncertain parameters assumed in numerical simulations, derived from the literature analysis in Tubino et al. (2020). It should be remarked that the degree of uncertainty of the damping is very large. However, the assumed range of variation is in accordance with what proposed by HIVOSS Guidelines for steel footbridges, suggesting a minimum damping 0.2% and a mean value of 0.4%.

Table II. Midpoint and normalized deviation amplitude

Parameter		$x_{0,k}$	Δx_k
$\mu_{\omega p}$	[rad/s]	2 π 1.88	0.03
$\sigma_{\omega p}$	[rad/s]	2 π 0.17	0.17
α	[ ]	0.35	0.20
G	[N]	700	0.10
ω_j	[rad/s]	2 π 1.88	0.10
ξ_j	[%]	0.42	0.560
M_j	[kg]	50000	0.10

The nominal solutions for the standard deviation and the mean-value of the maximum acceleration are $\sigma_{\ddot{p}_j} = 0.50$ m/s^2 (Eq. (10)), $\ddot{q}_{\max} = 1.94$ m/s^2 (Eq. (15)). Table III reports the LB and UB of $\sigma^I_{\ddot{p}_j}$ (see Eq. (21)) and $\ddot{q}^I_{\max}$ (see Eq. (23)) obtained in the three analyses. The significant deviation amplitudes of the uncertain parameters (Table II) lead to very large interval of variation of the response, especially if all the parameters (structural and loading) are considered as uncertain (FU). However, the uncertainty in the dynamic response is mainly governed by the uncertainties in the structural parameters. When only loading parameters are assumed as uncertain (LU), the interval of variation of the response is significantly reduced. It should be remarked that the maximum acceleration obtained applying SETRA procedure is higher than the nominal value and it is close to the UB provided by interval analysis if only loading uncertainties are accounted for (LU). In all the other cases (FU and SU), SETRA prediction is well below the UB provided by interval analysis, and it can be non-conservative.

	Table III. LB and UB of the standard deviation of the response and of the maximum acceleration			
		FU	*SU*	*LU*
$\underline{\sigma}_{\ddot{p}_j}$	[m/s^2]	0.13	0.25	0.32
$\bar{\sigma}_{\ddot{p}_j}$	[m/s^2]	1.29	0.88	0.72
$\underline{\ddot{q}}_{\max}$	[m/s^2]	0.49	0.97	1.26
$\bar{\ddot{q}}_{\max}$	[m/s^2]	5.02	3.42	2.81

Figure 1 provides the UB and LB of $\sigma^I_{\ddot{p}_j}$ versus the coefficient β which measures the degree of uncertainty according to Eq. (24): results obtained for the three different analyses (FU, SU, LU) are reported. It can be deduced that, while the LB of the standard deviation tends to decrease almost linearly with the degree of uncertainty β, the increase of the UB is not linear with β. As a consequence, the LB and UB are not symmetric with respect to the nominal value and the amplitude of the interval of variation is not increasing linearly with β. The comparison among the different analyses (FU, SU, LU) confirms that the uncertainty in the standard deviation is mainly governed by the structural uncertainties. In order to have a narrow interval of variation of the response, the degree of uncertainty should be reduced (i.e. β should be significantly smaller than 1).

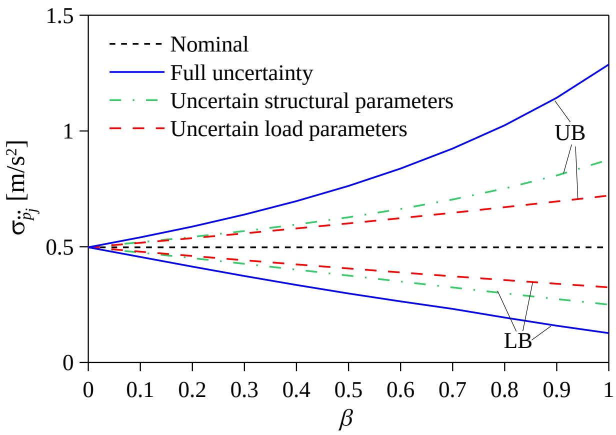

Figure 1. UB and LB of the standard deviation for increasing degree of uncertainty.

A probabilistic assessment of the footbridge serviceability can be obtained through the analysis of the CDF of the extreme value of the acceleration. Figure 2 provides the LB and UB of the CDF of the maximum acceleration for two degrees of uncertainty (β=0.5, β=1), for the full uncertainty analysis (FU, Fig. 2(a)), taking into account only the uncertainties in the structural parameters (SU, Fig. 2(b)), and taking into account only uncertainties in the loading parameters (LU, Fig. 2(c)). From Fig. 2, the LB and UB of the probability of falling within each comfort class can be estimated. It can be deduced that if the nominal values of the parameters are assumed, the footbridge will surely fall within a minimum comfort class. When all the parameters are considered as uncertain (Fig. 2(a)) with the maximum degree of uncertainty, the UB and LB of the CDF are extremely far from each other: considering the LB of the CDF, the maximum footbridge acceleration is surely higher than 4 m/s^2, and thus the comfort level is unacceptable, while considering the UB of the CDF, the maximum acceleration is almost surely lower than 0.5 m/s^2 and the maximum comfort level is assured. When only the structural or the loading parameters are considered as uncertain (Fig. 2(b) and 2(c)), the interval of variation of the CDF becomes narrower. For example, if only loading parameters are considered as uncertain (Fig. 2(c)) and β=0.5, both LB and UB of the CDF provide a minimum comfort class.

As an alternative to the analysis of the CDF, an interval-based serviceability assessment of the footbridge can be carried out focusing on the mean value of the maximum acceleration $\ddot{q}^I_{\max}$. Figure 3 provides the UB and LB of $\ddot{q}^I_{\max}$, compared with the comfort limits provided by SETRA, versus the coefficient β which measures the degree of uncertainty according to Eq. (24). Results obtained for the three different analyses (FU, SU, LU) are reported. It can be deduced that, if all the parameters are considered as uncertain and the maximum degree of uncertainty is assumed, then the interval of variation of the expected value of the maximum response is so wide that the footbridge could fall in any comfort class (from unacceptable to maximum comfort). If only the uncertainties of the loading are considered, the footbridge is almost surely in a minimum comfort class, as predicted by SETRA guideline. As regards structural uncertainties, if the degree of uncertainty is small then interval analysis provides a minimum comfort class, while for large degree of uncertainty a comfort level ranging from minimum to unacceptable can be obtained.

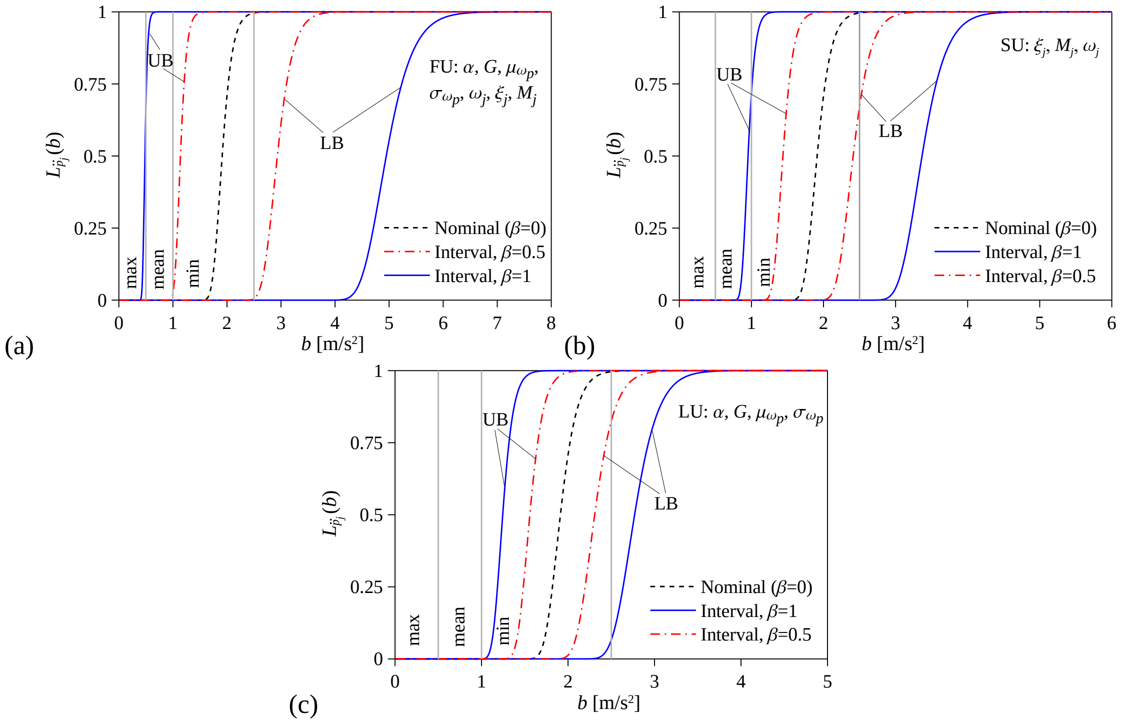

Figure 2. UB and LB of the CDF: full uncertainty (a), structural uncertainty (b), loading uncertainty(c).

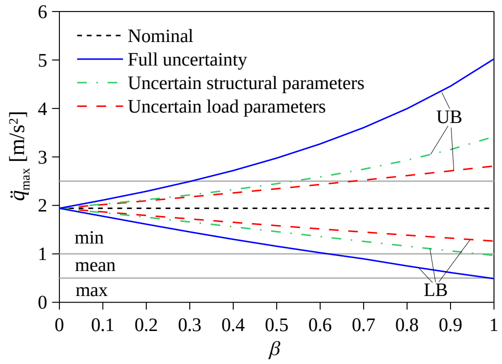

Figure 3. UB and LB of the mean value of the maximum acceleration for increasing degree of uncertainty.

5. Conclusions

Serviceability assessment of footbridges is studied through a non-deterministic approach, modelling structural and loading parameters as interval variables. Based on this approach, the intervals of variation of the standard deviation of the footbridge acceleration, of the cumulative distribution function of the maximum dynamic response and of the expected value of the maximum acceleration are defined. The analysis of the cumulative distribution function of the extreme value of the acceleration allows carrying out a probabilistic assessment of the footbridge serviceability.

The application to an ideal case study has shown that the high degree of uncertainty in the loading and structural parameters leads to very large intervals of variation of the response. The separate analysis of the role of the structural and loading parameters has shown that uncertainty in the structural parameters provides a determinant contribution to the variation of the response. In order to obtain narrow intervals of variation of the response, a reliable characterization of the loading and of the structural properties is mandatory. If all the parameters are considered as uncertain and a relatively high degree of uncertainty is assumed, then the interval of variation of the response is so wide that the footbridge could fall in any comfort class (from unacceptable to maximum comfort).

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