Free vibration of functionally graded graphene-platelets reinforced composite (FG-GPLRC) plate with interval parameters

Youqin Huang and Donghui Luo
Research Centre for Wind Engineering and Engineering Vibration, Guangzhou University, Guangzhou 510006, China, yqhuang@gzhu.edu.cn

Abstract: Functionally graded graphene-platelets (GPLs) reinforced composite (FG-GPLRC) is a recently proposed and promising nanocomposite, whose mechanical behaviors have been extensively studied in the field of composite structures. Studies on the uncertainty effect of material properties on performances of FG-GPLRC are very few, so an interval analysis is conducted in this work to investigate the influence of uncertain Young's modulus on natural frequencies of an FG-GPLRC plate. The study indicates that the position of the layer with interval parameter play a vital role on the uncertainty degree of first order natural frequency of the plate when only a single layer has an interval parameter. The weight fraction of GPLs in the whole plate should be decreased for controlling the deviation of natural frequency if all layers have interval parameters.

Key words: functionally graded graphene-platelets reinforced composite (FG-GPLRC), free vibration, interval analysis, generalized eigenvalue problem

1. Introduction

The material composition and mechanical properties of functionally graded materials (FGMs)can be tailored to continuously vary over one or more dimension(s) for simultaneously meeting different mechanical requirements (Suresh and Mortensen, 1998). Considering the extraordinary mechanical, thermal and electrical properties of graphene, Yang and his coworkers firstly proposed the functionally graded graphene-platelets (GPLs) reinforced composite (FG-GPLRC) (Yang et al. 2017; Song et al. 2017; Feng et al., 2017). Mechanical performances of FG-GPLRC composite have quickly attracted worldwide attentions in the research field of composite materials (Zhao et al. 2020).

The free vibration of FG-GPLRC polymer nanocomposite plate has been studied extensively. Zhang et al. (2020) investigated the free vibration of FG-GPLRC annular plate resting on elastic substrate and subjected to nonlinear temperature gradient. Quang et al. (2020) presented a three-variable high order shear deformation plate theory (THSDT) for free vibration analysis of FG-GPLRC porous plates. Thai and Phung-Van (2020) carried out free vibration analysis of FG-GPLRC plates with complicated shapes by a meshfree approach using a naturally stabilized nodal integration (NSNI). Chiker et al. (2020) compared the free vibrational behavior of FG-GPLRC plates and multilayer functionally graded carbon nanotube reinforced composite (FG-CNTRC) plates. Noroozi et al. (2020) developed a meshfree radial point interpolation method (RPIM) to investigate the free vibration of FG-GPLRC perforated plates. Majidi-Mozafari et al. (2020) provided an analytical solution for free vibration of an FG-GPLRC sandwich plate enclosed by piezoelectric layers according to Maxwell's equation.

It should be mentioned that all the existing studies on FG-GPLRC composites have not considered the uncertainty of material parameters such as Yong's modulus. However, the presence of uncertainty in material performance is inevitable owing to that the fabrication of FG materials is still immature. Therefore, uncertainty free vibration analysis on an FG-GPLRC plate is conducted in this work by assuming the uncertain material parameter of Young's modulus as an interval variable.

2. Methods based on interval analysis

An FG-GPLRC plate with length *a*, width *b* and thickness *h* is shown in Figure 1. Due to the constraint of current manufacturing technology, the FG-GPLRC plate is simulated by a multilayer structure consisting of sufficient number of layers where GPL weight fraction (w.f. = weight of GPLs / weight of composite) follows a layer-wise change. The plate is composed of N_L layers and each layer has the same thickness $\Delta h = h / N_L$. Neighboring layers are firmly bonded, and there exists no slide among layers. The effective material parameters of the *k*-th layer including Young's modulus $E_C^{(k)}$ can be determined by the Voigt-Reuss mode (Guzmán and Miravete, 2007).

Four types of FG patterns are considered as demonstrated in Figure 2. The w.f. of GPLs varies linearly among layers. Pattern 1 is a special case with uniform GPLs in all layers. For Pattern 2, w.f. of GPLs decreases from midplane to surface of plate, and Pattern 3 is the opposite. In Pattern 4, w.f. of GPLs decreases from one side to the other side of the plate.

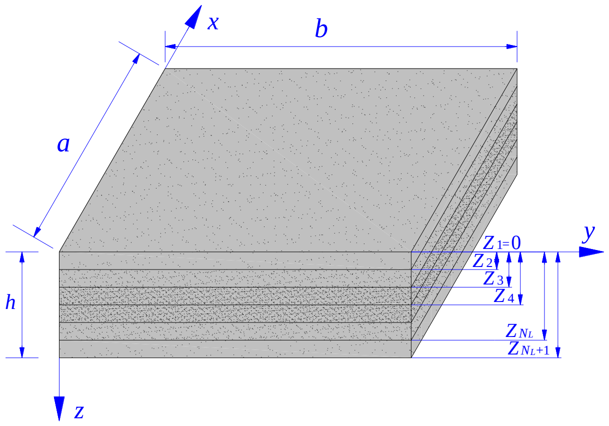


Figure 1. Multi-layer FG-GPLRC plate.

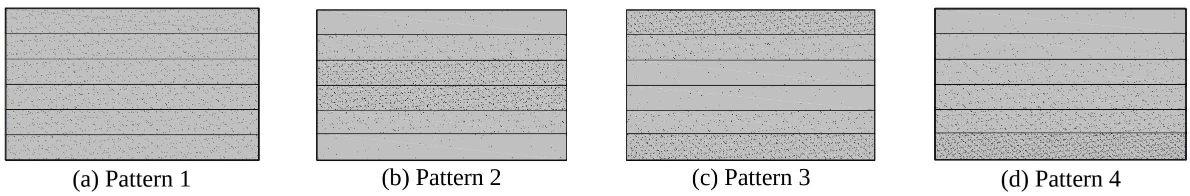


Figure 2. Different GPL distribution patterns.

Free vibration of FG-GPLRC plate with interval parameters

According to the work of Song et al. (2017), free vibration of an FG-GPLRC plate corresponds to a generalized eigenvalue problem:

$$\mathbf{K}\boldsymbol{\varphi} = \lambda \mathbf{M}\boldsymbol{\varphi} \tag{1}$$

where **M** and **K** are matrices of mass and stiffness; λ and $\boldsymbol{\varphi}$ are eigenvalue and eigenvector respectively, and λ is square of natural frequency, i.e., $\lambda = \omega^2$.

Expressions of elements in **M** and **K** can be found in Song et al. (2017), indicating that ω is related with Young's modulus $E_C^{(k)}$. Considering that $E_C^{(k)}$ could be uncertain owing to difficult manufacturing processes of FG materials and probabilistic information of such an uncertain parameter is not easy to be obtained, *Ec* is to be modeled as non-probabilistic interval variable in this study, and the interval analysis is carried out to evaluate the effect of uncertain Young's modulus of each layer on the natural frequencies of the FG-GPLRC plate.

An interval variable α^I denotes a variable closed by a lower bound $\underline{\alpha}$ and an upper bound $\overline{\alpha}$, i.e.

$$\alpha^I = [\underline{\alpha}, \overline{\alpha}] = \{\alpha \in R : \underline{\alpha} \le \alpha \le \overline{\alpha}\} \tag{2}$$

In engineering interval analysis, interval variable is generally represented by its midpoint and uncertainty degree:

$$\alpha^C = \frac{\overline{\alpha} + \underline{\alpha}}{2} \tag{3}$$

$$\Delta\alpha^I = \frac{\overline{\alpha} - \underline{\alpha}}{\overline{\alpha} + \underline{\alpha}} \tag{4}$$

For a system with r interval variables, the interval variables can be integrated as an interval vector, $\boldsymbol{\alpha}^I = \{\alpha_1^I, \alpha_2^I, \cdots, \alpha_r^I\}$, and free vibration of the system is governed by:

$$\mathbf{K}(\boldsymbol{\alpha})\boldsymbol{\varphi} = \lambda \mathbf{M}(\boldsymbol{\alpha})\boldsymbol{\varphi}, \ \boldsymbol{\alpha} \in \boldsymbol{\alpha}^I = [\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}] \tag{5}$$

For the studied FG-GPLRC plate, N_L interval parameters are considered, i.e.

$$\boldsymbol{\alpha}^I = \{E_C^{(1)I}, \cdots E_C^{(k)I}, \cdots, E_C^{(N_L)I}\} \tag{6}$$

The lower and upper bounds of natural frequency of any order can be obtained by solving two deterministic eigenvalue problem (Sofi et al., 2015):

$$\mathbf{K}(\boldsymbol{\alpha}^{(\mathrm{LB})})\boldsymbol{\varphi} = \underline{\lambda}\mathbf{M}(\boldsymbol{\alpha}^{(\mathrm{LB})})\boldsymbol{\varphi} \tag{7}$$

$$\mathbf{K}(\boldsymbol{\alpha}^{(\mathrm{UB})})\boldsymbol{\varphi} = \overline{\lambda}\mathbf{M}(\boldsymbol{\alpha}^{(\mathrm{UB})})\boldsymbol{\varphi} \tag{8}$$

The elements of $\boldsymbol{\alpha}^{(\mathrm{LB})}$ and $\boldsymbol{\alpha}^{(\mathrm{UB})}$ are determined by:

$$\alpha_i^{(\mathrm{UB})} = \overline{\alpha}_i, \alpha_i^{(\mathrm{LB})} = \underline{\alpha}_i \text{ when } S_{\alpha_i} > 0 \tag{9}$$

$$\alpha_i^{(\mathrm{LB})} = \overline{\alpha}_i, \alpha_i^{(\mathrm{UB})} = \underline{\alpha}_i \text{ when } S_{\alpha_i} < 0 \tag{10}$$

with the sensitivity function:

$$S_{\alpha_i} = \left.\frac{\partial \lambda(\boldsymbol{\alpha})}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}^{\mathrm{C}}} = \phi_0^{\mathrm{T}} \left.\frac{\partial \mathbf{K}(\boldsymbol{\alpha})}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}^{\mathrm{C}}} \phi_0 - \lambda_0 \phi_0^{\mathrm{T}} \left.\frac{\partial \mathbf{M}(\boldsymbol{\alpha})}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}^{\mathrm{C}}} \phi_0 \tag{11}$$

Hence, after all sensitivity functions are computed for all interval parameters, the bounds of natural frequency of any order of the FG-GPLRC plate can be obtained by solving Eqs. (7) and (8).

3. Results and discussions

An FG-GPLRC plate simply supported at all edges with $a \times b \times h$ = 0.45m×0.45m×0.045m is studied, with the total number of layers as N_L = 10. The w.f. of GPLs in the whole plate is g_{GPL} = 1%. The average length, width and thickness of GPLs is $l_{GPL} \times w_{GPL} \times h_{GPL}$ = 2.5μm×1.5μm×1.5nm, and the mass density, Young's modulus and Poisson's ratio of GPL or polymer matrix are ρ_{GPL} = 1.06g / cm3, ρ_M = 1.2g / cm3, E_{GPL} = 1.01TPa, E_M = 3GPa, v_{GPL} = 0.186 and v_M = 0.34, respectively, based on which the midpoints of Young's modulus of each layer $E_C^{(k)C}$ can be obtained for any pattern of GPLs according to modified Halpin-Tsai model. The uncertainty degree of Young's modulus is set as $\Delta\alpha_E = 0.1$ for any layer.

In order to verify the results of eigenvalue analysis in this work, the code written based on the above theory is used to compute the first order dimensionless natural frequency ($\varpi = \omega h\sqrt{\rho_M / E_M}$) of the plate with deterministic material parameters and compare the results with those from Song et al. (2017), as demonstrated in Table I. It can be seen that this study produces identical results with the literature.

Table I. First order dimensionless natural frequency

	Pattern 1	Pattern 2	Pattern 3	Pattern 4
This work	0.1216	0.1020	0.1378	0.1118
Song et al. (2017)	0.1216	0.1020	0.1378	0.1118

The signs of sensitivities of first five orders of eigenvalues with respect to the interval Young's modulus of each layer are shown in Table II. It is indicated that the sensitivities of eigenvalue to Young's moduli of all layers are positive, so the eigenvalue becomes larger when Yong's modulus of any layer increases.

Table II. Sign of sensitivities of eigenvalues

λ_j	$E_C^{(1)}$	$E_C^{(2)}$	$E_C^{(3)}$	$E_C^{(4)}$	$E_C^{(5)}$	$E_C^{(6)}$	$E_C^{(7)}$	$E_C^{(8)}$	$E_C^{(9)}$	$E_C^{(10)}$
j = 1	+	+	+	+	+	+	+	+	+	+
j = 2	+	+	+	+	+	+	+	+	+	+
j = 3	+	+	+	+	+	+	+	+	+	+
j = 4	+	+	+	+	+	+	+	+	+	+
j = 5	+	+	+	+	+	+	+	+	+	+

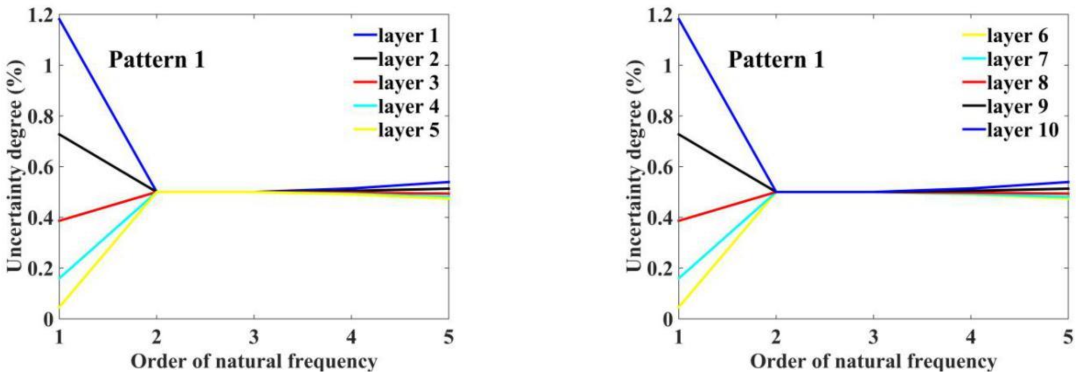
Based on the sensitivities, bounds of natural frequencies of any order can be calculated according to Eqs. (7)-(10). The obtained bounds of first five orders of natural frequencies and the results from Monte Carlo Simulation (MCS) for Pattern 3 are given in Table III. It is shown that the proposed method generates very close results to MCS for all orders of eigenvalue.

Table III. Bounds of natural frequencies

Mode	This study		MCS	
	Upper bound	Lower bound	Upper bound	Lower bound
1	0.1431	0.1324	0.1431	0.1328
2	0.5848	0.5465	0.5846	0.5469
3	1.0164	0.9498	1.0162	0.9498
4	4.2581	3.9114	4.2580	3.9116
5	4.3966	4.0377	4.3966	4.0377

Under various distribution patterns, the effects of uncertain Young's modulus at any layer on natural frequencies are shown in Figure 3. It is clear that the number of layer (i.e. the position) where uncertain Young's modulus is located obviously affects the uncertainty degree of natural frequencies.

When the weight fraction (w.f.) of GPLs is uniform among layers (Pattern 1), the position mainly affects the first order of natural frequency while generates few effects on natural frequencies of higher orders. However, when GPLs are distributed ununiformly among layers (Patterns 2-4), the position also influences higher orders of natural frequencies. Therefore, for higher orders of natural frequencies, the position of interval Young's modulus is not important but the amount of w.f. of GPLs in the uncertain layer is important. This can be proved by comparing the uncertainty degree of natural frequencies of higher orders of Pattern 2 and Pattern 3, which indicates that the uncertainty degree of natural frequency is similar when the layer with highest w.f. of GPLs is uncertain no matter which layer it is located at.



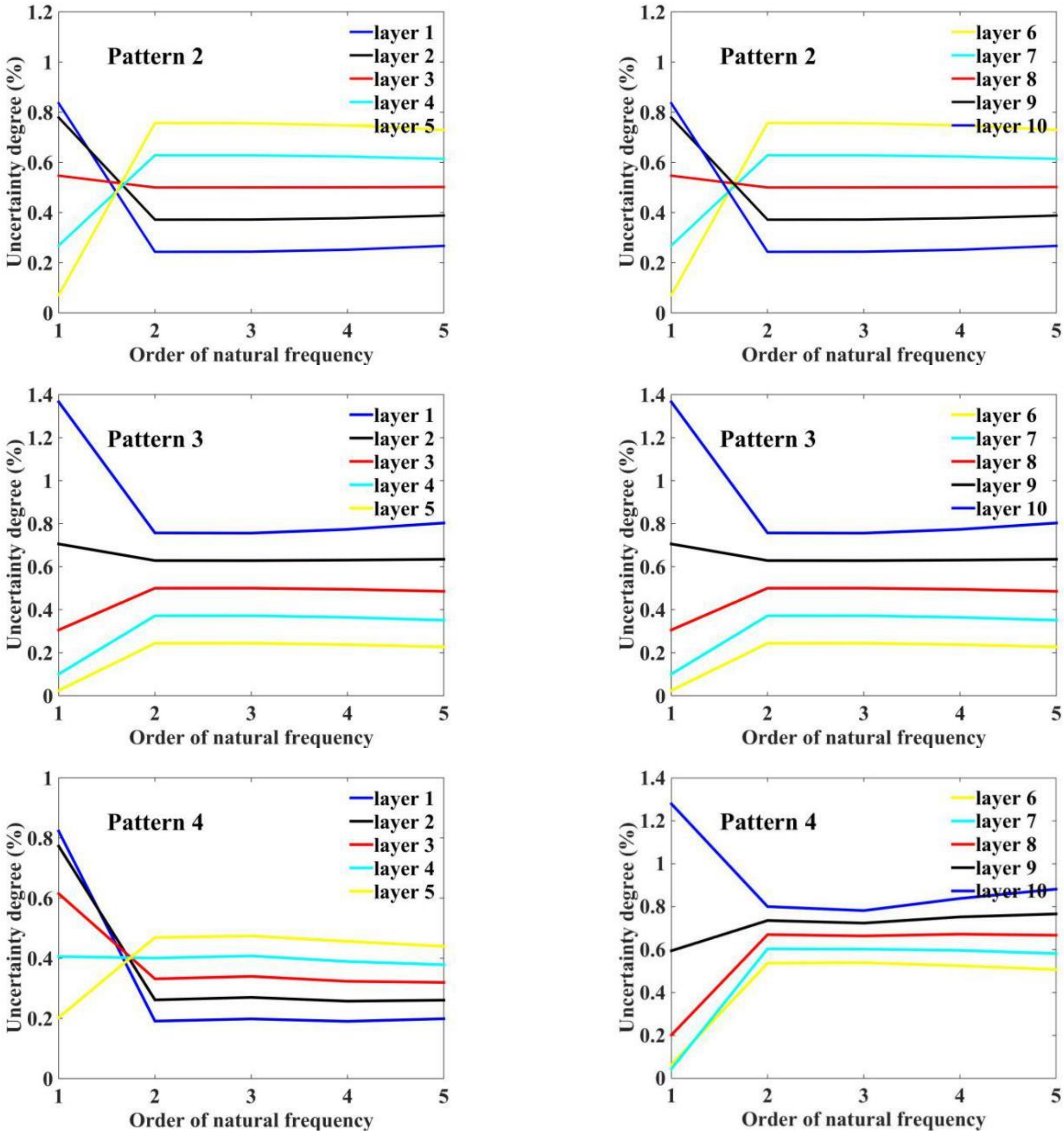


Figure 3. Uncertainty degree of natural frequency for a single uncertain layer.

Assuming that two layers of symmetrical positions in the plate have interval parameters, the uncertainty degree of natural frequency is shown in Figure 4. It can be observed that the corresponding uncertainty degree doubles for all orders of natural frequencies comparing with the case with single interval layer, so the effect of w.f. of GPLs on the uncertainty degree of natural frequency can be linearly superimposed for the layer with the same weight fraction of GPLs. For Pattern 4, although the curves seem different from those with single uncertain layer, they are also the superimposition of results of individual layers.

When more layers are uncertain simultaneously, the curve becomes flat gradually owing to the superimposition effect (Figure 5 and Figure 6). That is, the effects of material uncertainty on natural frequency uncertainty accumulate even though the uncertain layers have different w.f. of GPLs. Finally, all orders of natural frequencies have the same uncertainty degree. Therefore, for controlling the uncertainty degree of natural frequencies, the w.f. of GPLs in the whole plate should be decreased.

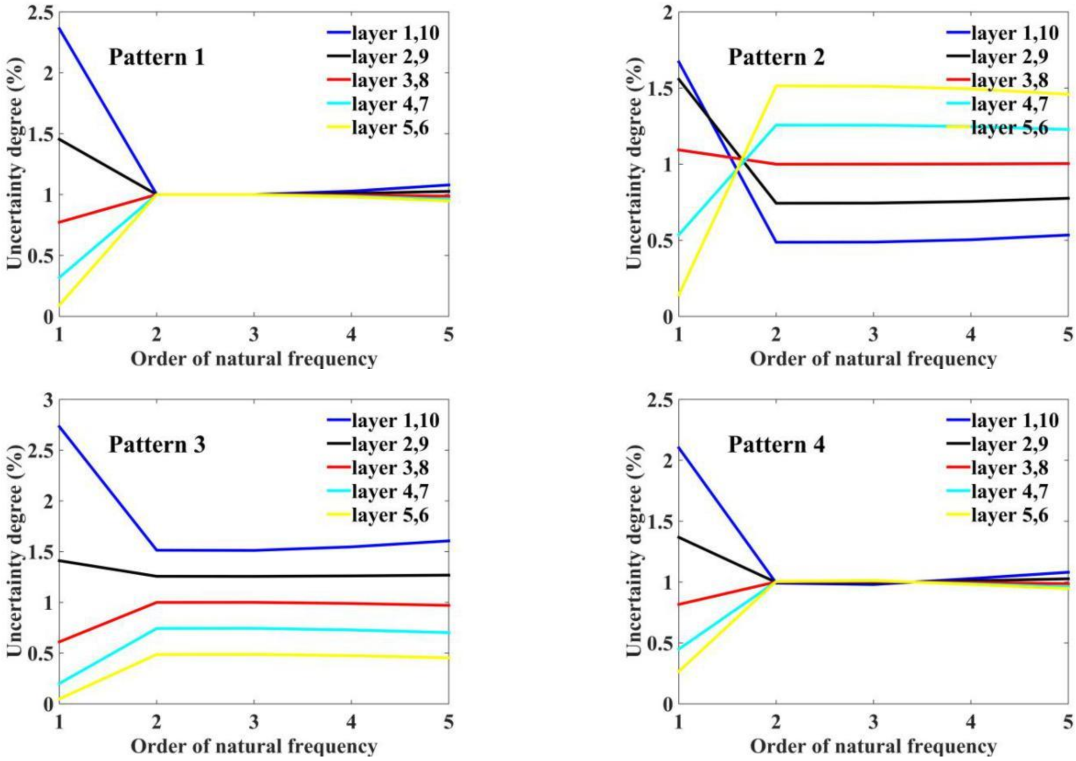
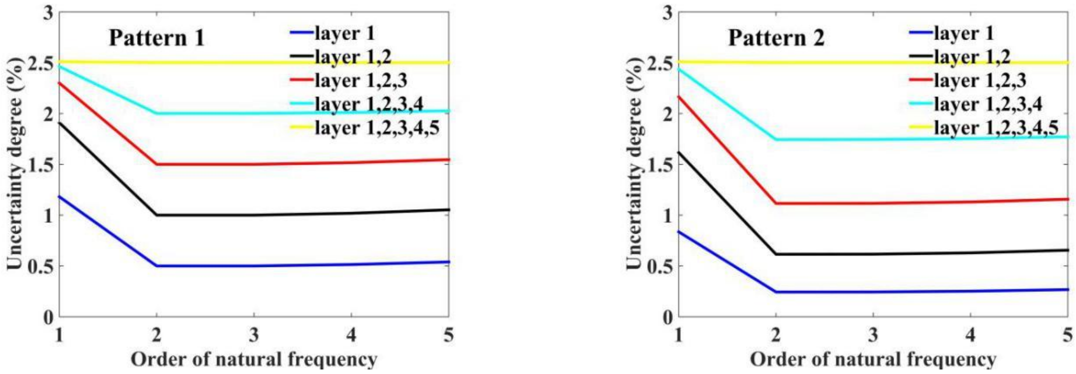


Figure 4. Uncertainty degree of natural frequency under two symmetrical interval layers.



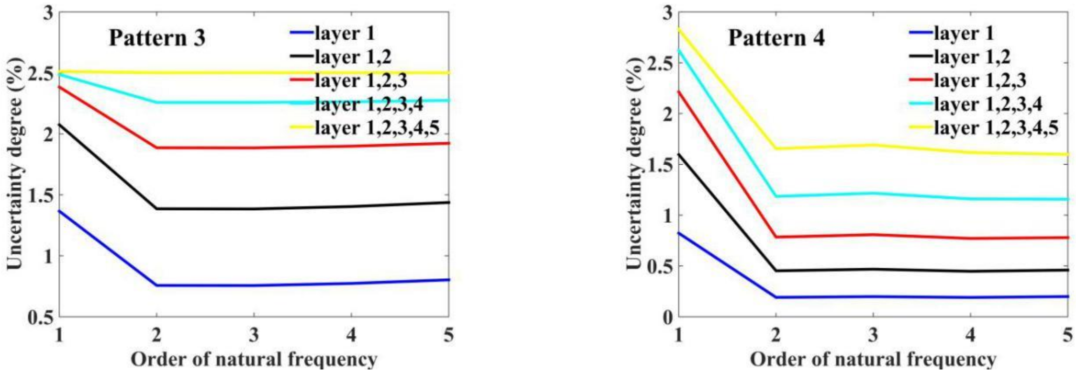


Figure 5. Uncertainty degree of natural frequency under increasing number of interval layers.

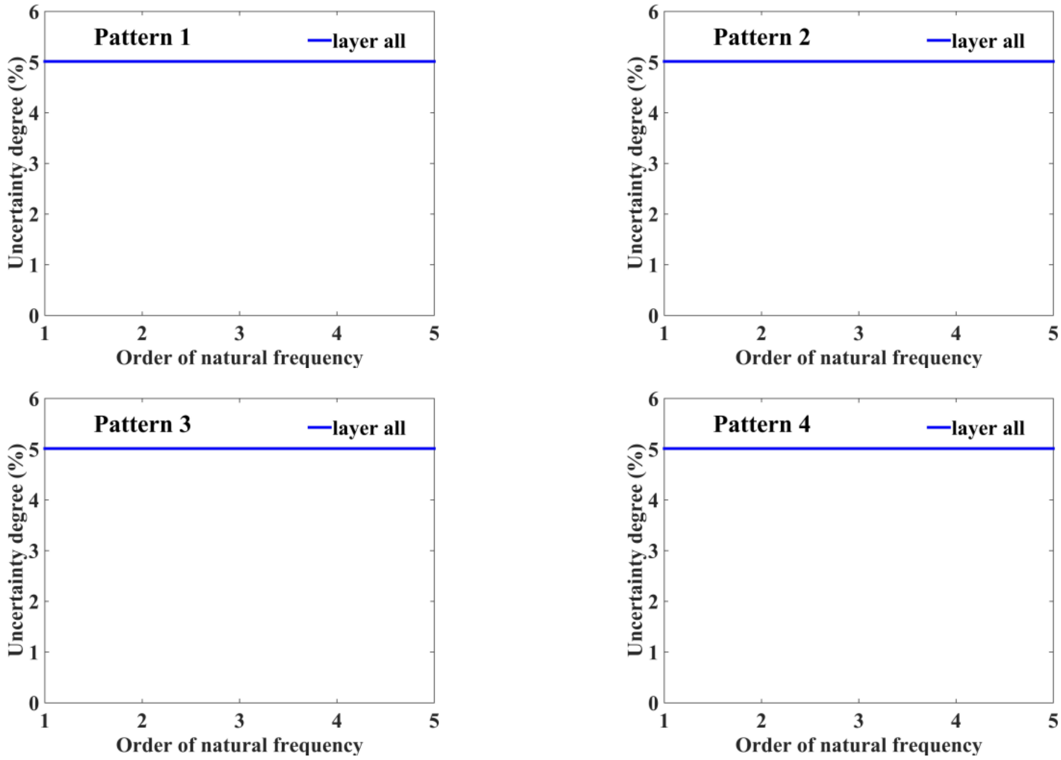


Figure 6. Uncertainty degree of natural frequency when all layers are uncertain.

4. Conclusions

Owing to the difficult fabrication process of functionally graded (FG) materials, uncertainty of material properties exists inevitably. This study deals with the free vibration of an functionally graded graphene-platelets (GPLs) reinforced composite (FG-GPLRC) plate with interval uncertain Young's modulus at each layer of the FG plate. The investigations show that the position of uncertain layer apparently affects the uncertainty degree of natural frequencies. When the weight fraction (w.f.) of GPLs is uniform among layers, the position mainly affects the first order of natural frequency while has few effects on natural frequencies of higher orders. When GPLs are distributed ununiformly among layers, the position also influences higher orders of natural frequencies. The effect of w.f. of GPLs on the uncertainty degree of natural frequency can be superimposed. From the respect of reducing the deviation of natural frequencies, the w.f. of GPLs in the whole plate should be decreased.

Acknowledgements

The research reported was financially supported by the Technology Planning Project of Guangzhou City in China (No. 201904010307).

References

Chiker, Y., M. Bachene, M. Guemana, B. Attaf, and S. Rechak. Free vibration analysis of multilayer functionally graded polymer nanocomposite plates reinforced with nonlinearly distributed carbon-based nanofillers using a layer-wise formulation model. *Aerospace Science and Technology*, 104: 105913, 2020.

Feng, C., S. Kitipornchai, and J. Yang. Nonlinear bending of polymer nanocomposite beams reinforced with non-uniformly distributed graphene platelets (GPLs). *Composites Part B-Engineering*, 110: 132–140, 2017.

Guzmán de Villoria, R. and A. Miravete. Mechanical model to evaluate the effect of the dispersion in nanocomposites. *Acta Materialia*, 55(9): 3025–3031, 2007.

Majidi-Mozafari, K., R. Bahaadini, and A. R. Saidi. An analytical solution for vibration analysis of sandwich plates reinforced with graphene nanoplatelets. *Engineering with Computers*, DOI: 10.1007/s00366-020-01183-9, 2020.

Nguyen, Q. H., L. B. Nguyen, H. B. Nguyen, and H. Nguyen-Xuan. A three-variable high order shear deformation theory for isogeometric free vibration, buckling and instability analysis of FG porous plates reinforced by graphene platelets. *Composite Structures*, 245: 112321, 2020.

Noroozi, A. R., P. Malekzadeh, R. Dimitri, and F. Tornabene. Meshfree radial point interpolation method for the vibration and buckling analysis of FG-GPLRC perforated plates under an in-plane loading. *Engineering Structures*, 221: 111000, 2020.

Sofi, A., G. Muscolino, and I. Elishakoff. Natural frequencies of structures with interval parameters. *Journal of Sound and Vibration*, 347: 79–95, 2015.

Song, M., S. Kitipornchai, and J. Yang. Free and forced vibrations of functionally graded polymer composite plates reinforced with graphene nanoplatelets. *Composite Structures*, 59: 579–588, 2017.

Suresh, S. and A. Mortensen. Fundamentals of functionally graded materials. Institute of Materials Communications Ltd., London, 1998.

Thai, C. H. and P. Phung-Van. A meshfree approach using naturally stabilized nodal integration for multilayer FG GPLRC complicated plate structures. *Engineering Analysis with Boundary Elements*, 117: 346–358, 2020.

Yang, J., H. Wu, and S. Kitipornchai. Buckling and postbuckling of functionally graded multilayer graphene platelet-reinforced composite beams. *Composite Structures*, 161: 111–118, 2017.

Zhang, G., C. Xiao, A. Rahimi, and M. Safarpour. Thermal and mechanical buckling and vibration analysis of FG-GPLRC annular plate using higher order shear deformation theory and generalized differential quadrature method. *International Journal of Applied Mechanics*, 12(2): 2050019, 2020.

Zhao, S., Z. Zhao, Z. Yang, L. Ke, and J. Yang. Functionally graded graphene reinforced composite structures: a review. *Engineering Structures*, 210: 110339, 2020.