Quantification of Data and Production Uncertainties for Tire Design Parameters

Maria Böttcher, Wolfgang Graf, Michael Kaliske
Institute for Structural Analysis, TU Dresden, 01062 Dresden, Germany,
{*maria.boettcher; wolfgang.graf; michael.kaliske*}*@tu-dresden.de*

Abstract. The design process for products such as tires constitutes a multi-objective optimization to enhance the product's performance for a specific usage. However, the selected geometry and material parameters as well as the boundary conditions, which are the basis of the performance prediction resulting from a numerical simulation, cannot be guaranteed to coincide with the real parameters of a final product in use. Most parameters are subjected to variation in production and service, which can lead to significant variation in performance.
This contribution has the objective of describing how to model uncertainties of tire design parameters with regard to production variation as well as to non-precise data (Viertl, 1996), to enable uncertainty analysis for tire performance. Thus, the choice of the product design parameters will not be solely based on the predicted value, but also on the reliability of the performance. Uncertainties of design parameters, divided into the two types aleatory and epistemic with respect to their causation, are quantified for several geometry and material parameters based on the situation of available data (Leichsenring et al., 2018) as well as their sensitivity (Pannier and Graf, 2014; Götz et al., 2018). Using statistical analysis methods for provided data from production plants, different approaches for probabilistic, possibilistic and polymorphic uncertainty modeling, combining different types of uncertainties, are selected, including random variables, fuzzy variables, probability-boxes (p-boxes) and other fuzzy stochastic approaches (Möller et al., 2007).

Keywords: polymorphic uncertainty, non-precise data, tire design, production variation

1. Introduction

The objective of any product design is finding the optimal parameter configuration for its intended usage. This can include a variety of different objectives: style, size, durability, weight, costs, sustainability and more. In engineering tasks, the focus often lies on durability and performance for the intended load exposure. Specifically in tire design, the selection of parameters is based on its intended driving situation, e.g. an increased tread pattern depth for better grip and safety when snow and ice are present on the roads, whereas sleeker, stiffer tires are used for racing scenarios. Relevant tire performances include, but are not limited to, dry and wet braking, noise, rolling resistance, wear and cornering stiffness. A small difference in these performances can have large impact on gas consumption, longevity, driving comfort and safety over the

lifetime of the tire. Therefore, a lot of effort is spent on optimizing these performances. However, viewing the design process as a purely deterministic problem, the performance deviation based on production and service (e.g. tire pressure) fluctuations is neglected. Yet, given the severe impact of minor performance deviations, a consideration of the parameter uncertainty can improve the process of the tire design, by allowing for not only a performance prediction, but for an analysis of the robustness of that performance. The robustness consideration has grown more important in recent years, made possible by surrogate models replacing computationally expensive simulations and, thus, allowing for an uncertainty analysis. The modeling of parametric uncertainty has been applied in numerous examples. (Serafinska et al., 2016) describes a fail-safe design optimization process by computing crack propagation with interval and fuzzy geometry parameters. In (Leichsenring et al., 2018), the fluctuating material parameters of wood are modeled using fuzzy-probability based random variables. A comparison of a probabilistic with an interval modeling approach is given in (Faes et al., 2018) for the variability in the forming process of plate sheets.
This paper describes strategies of modeling parametric uncertainty in tire design, with the focus lying on the performance of the tire's cornering stiffness. A distinction between the two types of uncertainty - aleatory and epistemic - is necessary, as different modeling approaches are commonly used to account for them. Based on both existing data and expert knowledge, the selection of models to quantify the parameters' uncertainties is discussed. Another step in the process of uncertainty modeling is the sensitivity analysis of each design parameter on the cornering stiffness, made possible by a computationally inexpensive performance prediction tool, which allows for the generation of a large data base. The level of detail of the uncertainty quantification can then be selected based on a parameters sensitivity to the performance, i.e. a high level of detail for highly sensitive parameters, whereas non-sensitive parameters can be considered as deterministic.
The structure of the paper is as follows. Section 2 provides a compact overview of uncertainty modeling and gives a brief definition of several existing modeling strategies. In the subsequent Section 3, sensitivity analysis approaches are sketched, both sampling and non-sampling based, with application recommendations and limitations. Section 4 follows with a description of the data situation for different tire design parameters, from the scope of geometry and material, as well as the main results of an exploratory data analysis and the selected uncertainty modeling strategies. A summary and an outlook conclude this contribution in Section 5.

2. Models for Parametric Uncertainty Quantification

For the selection of a model to quantify parameter uncertainties, it is essential to first consider the causes of these uncertainties. There is consensus in the scientific community, that two types can be distinguished: aleatory and epistemic (Möller and Beer, 2008; Der Kiureghian, 2009). The group of aleatory uncertainties considers the inherent variability of parameters, e.g. resulting from production fluctuation or load variation on structures. They can be measured but not reduced. As a modeling strategy, a stochastic approach is commonly used. As for epistemic uncertainties, they originate from a lack of knowledge or information, including incomplete data, e.g. caused by a small sample size, as well

as imprecise data, i.e. samples with a certain measurement error. Given the necessary resources to accumulate additional data as well as to improve the measurement accuracy, the epistemic uncertainty is reducible, yet not completely avoidable. An adequate modeling strategy here is subject of discussion as seen in (Ferson et al., 2004) and (Mullins et al., 2016), with different approaches including evidence theory, possibility theory and interval analysis.
Since the two types of uncertainty are present simultaneously, a joint model is necessary to combine both of them. Also, it is preferable to allow for a distinction of the uncertainty causes, i.e. aleatory or epistemic, as a result of the parameter uncertainty propagation in the uncertainty analysis. Given the results, it is possible to decide if spending additional resources is expedient to reducing the output uncertainty based on the influence of the epistemic uncertainty. Combined modeling strategies include the Bayesian approach (O'Hagan and Oakley, 2004) and imprecise probability, e.g. evidence theory, fuzzy randomness, probability-boxes and fuzzy-probability based randomness. However, since Bayesian probability is a purely stochastic method, it results in a stochastic output variable and, thus, does not allow for an inference of the uncertainty causation, the focus here lies on imprecise probabilities. These modeling strategies can be considered as an extension of a probabilistic model by including possibility theory, in order to add different aspects of epistemic uncertainty.
The subject of model uncertainty is to be viewed separately from that of parametric uncertainty, as discussed in, e.g., (Ferson et al., 2003; Der Kiureghian, 2009). However, it lies outside the scope of this contribution and will not be considered further.

2.1. BASIC UNCERTAINTY CONCEPTS

As a probabilistic model, the random variable is used. It is based on the probability space (Ω, Σ, P) with Ω as the set of all elementary events, Σ as a σ-algebra as a set containing all subsets of elements in Ω, and P as a measure of probability, assigning to each event a probability $P : \Sigma \mapsto [0, 1]$. The random variable is a function mapping an event to a real value $X : \Omega \mapsto \mathbb{R}$.
To describe the function of a random variable, the cumulative distribution function (CDF) is defined as

$$F_X = P(X \leq x) = \int_{-\infty}^{x} f_X(t)dt, \tag{1}$$

with f_X denoting the probability density function (PDF), for which $\int_{-\infty}^{\infty} f_X(t)dt = 1$. The PDF can be considered as the relative frequency $h(x)$ resulting from an infinite number of experiments.
In contrast to a random variable, a fuzzy variable follows possibility theory and is defined as a set A of pairs, assigning a possibility to each value, by assigning to each value x a possibility of occurrence using a membership function μ

$$A = \{(x,\, \mu(x)) \,|\, x \in \mathbb{R},\, 0 \leq \mu(x) \leq 1\};\ \exists\, x \,|\, \mu(x) = 1, \tag{2}$$

$$\mu(x) : \mathbb{R} \rightarrow [0,\, 1]. \tag{3}$$

The possibility of a value is to be interpreted linguistically only and does not assign a value of probability.
An alternative way of describing a fuzzy variable is through a family $A = (A^\alpha)_{\alpha \in]0,1]}$ of α-level intervals

$$A^\alpha = \{x \in \mathbb{R} \,|\, \mu(x) \geq \alpha\},\ \alpha \in]0,1], \tag{4}$$
$$A^\alpha = [\underline{x}_\alpha, \overline{x}_\alpha],\ A^\alpha \subseteq \mathbb{R}. \tag{5}$$

Often, fuzzy variables are described by a simple shape, e.g. as a triangle or a trapezoid. Their definition can be simplified by using the sets $x = \langle x_{0,l}, x_1, x_{0,u}\rangle$ and $x = \langle x_{0,l}, x_{1,l}, x_{1,u}, x_{0,u}\rangle$, respectively. Also, an interval variable can be considered as a special case of a fuzzy variable, where the membership function can merely assume the values $\{0, 1\}$, with $\mu(x) = 1\ \forall\, x \in I$. The interval can be described by a lower and an upper bound $I = [\underline{x}, \overline{x}]$, given a fuzzy variable described by the set $x = \langle x_l, x_u\rangle$. A visualization of these fuzzy variables can be found in Figure 1.

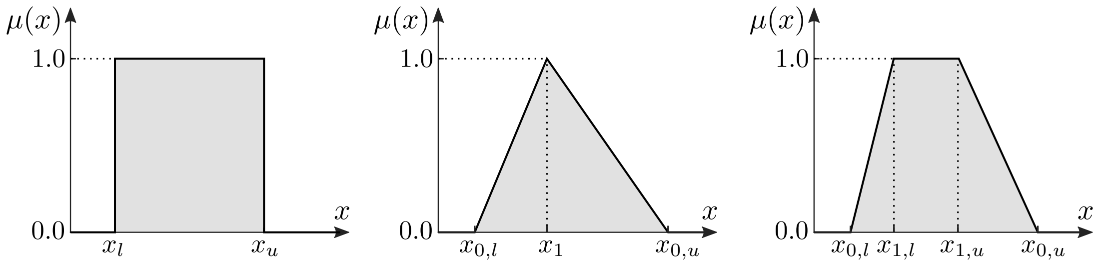

Figure 1. Illustration of an interval (left), a triangular (middle) and a trapezoid (right) fuzzy variable.

2.2. Consideration of parameter uncertainty interactions

As there often exist several parameters for, e.g., one part of the product, the production variation of these can be assumed to show a correlation to some extend. Considering as an example the width of the lower belt in a tire as x_1 and the difference of the lower and upper belt as x_2, then it stands to reason that an increased value in x_1 leads also to an increase in x_2. Accounting for these dependencies is an important part of the uncertainty quantification, since a neglect leads to an overestimation of the predicted performance uncertainty.
If design parameters are modeled as random variables, then interacting parameters can be considered a multivariate random variable, constituted of their individual distribution and joined through a covariance matrix. As for fuzzy variables, (Schietzold et al., 2018) uses a spatial plane dividing a multivariate fuzzy variable in a permissible and a non-permissible region.

2.3. Polymorphic uncertainty

The concept of polymorphic uncertainty, which was introduced in (Graf et al., 2013) and extended in (Graf et al., 2015; Götz et al., 2015; Götz, 2017), describes different approaches combining probabilistic and possibilistic approaches, including probability-boxes, fuzzy randomness and fuzzy-probability based randomness. A guide for selecting an appropriate uncertainty model for different data situations is given as follows:

– few, imprecise, non-assessed data : interval
– few, imprecise, assessed data : fuzzyness
– very many, deterministic data : randomness

} basic uncertainty models

– many, imprecise, non-assessed data : probability-box
– many, imprecise, assessed data : fuzzy random variable
– some deterministic data : fuzzy-probability based randomness

} polymorphic uncertainty models

In the following, the three mentioned polymorphic uncertainty models will be shortly introduced.
In (Ferson and Ginzburg, 1996) and (Ferson et al., 2003), the uncertainty model probability-box (p-box) is proposed. It deviates from the traditional random variable in the definition of its cumulative density function, which is now not described as a singe function F_X, but by an lower and upper bound $\underline{F}_X$ and $\overline{F}_X$, and, therefore, giving an interval for possible values of F_X

$$\underline{F}_X \leq F_X \leq \overline{F}_X \tag{6}$$

for a random variable X. Thereby, the restriction based on selecting and fitting of the data to a predefined type of function is somewhat reduced. For the lower and upper bounds of the CDF, both predefined and empirical distributions can be used. In the context of a p-box, each realization of the random variable can be considered an interval $X = [\underline{X}, \overline{X}]$, i.e. mapping

$$X : \Omega \mapsto \mathcal{I}(\mathbb{R}), \tag{7}$$

with $\underline{X} : \underline{F}_X(x)$ and $\overline{X} : \overline{F}_X(x)$.
A p-box is described by the tuple $\langle \underline{F}_X, \overline{F}_X, m, v, \mathbf{F}_X \rangle$ in (Götz, 2017), where $\mathbf{F}_X$ includes all possible distribution functions F_X and parameters m and v are the intervals of mean value and variance, for which

$$\int_{-\infty}^{\infty} x \frac{dF_X}{dx} dx \in m, \tag{8}$$

$$\left(\int_{-\infty}^{\infty} x \frac{dF_X}{dx} dx \right) - \left(\int_{-\infty}^{\infty} x \frac{dF_X}{dx} dx \right)^2 \in v. \tag{9}$$

A further extension of the random variable is the model of fuzzy randomness (fr). While for a p-box each realization of the random variable is defined as an interval, here a realization is described by a fuzzy variable

$$X : \Omega \mapsto \mathcal{F}(\mathbb{R}), \tag{10}$$

giving

$$X = ([\underline{X}_\alpha, \overline{X}_\alpha])_{\alpha \in]0,1]}. \tag{11}$$

The definition of the fuzzy cumulative probability function is, similarly to that of a p-box, for each α-level

$$\underline{F}_{X,\alpha} \leq F_{X,\alpha} \leq \overline{F}_{X,\alpha}. \tag{12}$$

Another model consisting of both fuzzy and random variables is fuzzy-probability based randomness (fp-r). Contrary to fuzzy randomness though is the mapping of the random variable to a real value

$$X : \Omega \mapsto \mathbb{R}. \tag{13}$$

To combine epistemic uncertainty quantification with the stochastic model, the parameters θ of the distribution function, from hereon called hyperparameters, are fuzzified

$$F_X = (F_\theta \,|\, \theta \in \theta^\alpha)_{\alpha \in]0,1]}. \tag{14}$$

Therefore, each realization of X has, for each α-level, an interval of probability

$$F_X = ([\underline{F}_{X,\alpha}, \overline{F}_{X,\alpha}])_{\alpha \in]0,1]}. \tag{15}$$

A visualization of these three modeling approaches can be found in Figure 2.

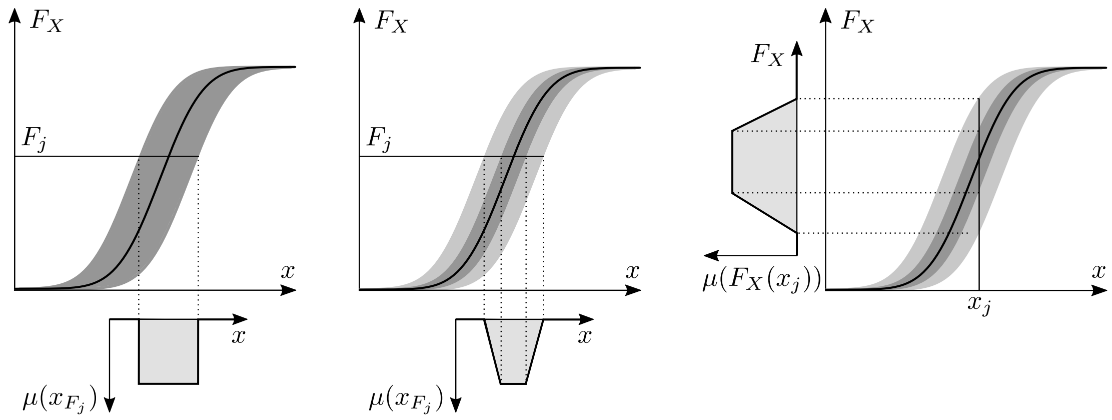

Figure 2. Illustration of the polymorphic uncertainty models p-box (left), fr (middle) and fp-r (right).

3. Sensitivity Analysis

Another important step in quantification of design parameter uncertainties for a specific performance is the investigation of the importance of each parameter, i.e. to what extend the resulting performance is influenced by them. To select adequate analysis methods, the types of the design parameters need to be regarded, given that certain analysis procedures are limited to, e.g., only continuous parameters, and cannot be applied to discrete, categorical or functional ones.

Dependent on the task at hand, the importance of each design parameter can be assigned based on expert knowledge as well as from information of white-box models, if available. Otherwise, or in addition, a sensitivity analysis is performed on data samples $\mathcal{D}$ mapping

input to output $\mathbf{x} \mapsto \mathbf{y}$. Here, a variety of methods can be utilized, including, but not limited to, visualization, measures of correlation, regression coefficients and variance-based indices. A methodology overview with recommendations for selecting a procedure for different tasks can be found in (Pianosi et al., 2014).
When performing a sensitivity analysis, the main question is if the analysis is to be performed as non-sampling, i.e. is based on a set of existing data samples $\mathcal{X}$, or if a simulation model $f(\mathbf{x})$ is available for sampling after a specific design plan (Design of Experiments, DoE) and how computationally expensive a single model run is.

3.1. Non-sampling-based approaches

For a non-sampling sensitivity analysis, the method selection as well as the accuracy of the results are somewhat limited. However, they can allow for a good approximation and validation of expert knowledge. A commonly used approach is data visualization, including scatter- and swarmplots, parallel coordinate plots, Andrew plots and self-organizing maps (SOM). A variety of visualization methods can be found in (Schnell, 1994). However, visual analysis is subjective and difficult to quantify. Also, for high-dimensional input and/or output domains, as is the case in most engineering tasks, the disregard of higher order sensitivities can lead to false interpretation of the results. As a quantification of spatial dependencies, coefficients of correlation can be employed. For linear correlation, the Pearson coefficient

$$\rho_P(x,y) = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y} \in [-1,1] \tag{16}$$

is commonly used. Alternatively, a rank correlation coefficient, e.g. Spearman

$$\rho_S(x,y) = \frac{\sum_i (r(x_i)-\overline{r}_x)(r(y_i)-\overline{r}_y)}{\sqrt{\sum\limits_i (r(x_i)-\overline{r}_x)^2}\sqrt{\sum\limits_i (r(y_i)-\overline{r}_y)^2}} \in [-1,1] \tag{17}$$

with $r(y_i) = j \in \mathbb{N}$ yielding the rank, i.e. the position of the sample y_i within the sample set $\mathcal{Y}$, sorted in ascending order, also shows non-linear dependencies, while reacting less sensitive to outliers. The benefit of a correlation coefficient is the information about positive and negative correlation, which leads to an improved insight of the output dependencies.
With a standardized regression coefficient (SRC), a regression model, e.g. $\hat{y} = b_0 + \sum_{j=1}^{n_x} b_j x_j$ as a linear model, is fitted. To compute the constants b_j, an optimization is performed to minimize the squared error $\sum_{i=1}^{N}(\hat{y}_i - y_i)^2$. The sensitivity of a parameter x_j is proportional to its associated factor b_j. In addition, using the coefficient of determination (COD)

$$R^2 = \frac{SS_{reg}}{SS_{tot}} = \frac{\sum_{i=1}^{N}(\hat{y}_i - \bar{y})}{\sum_{i=1}^{N}(y_i - \bar{y})} \in [0,1], \tag{18}$$

the fitness of the regression model can be evaluated. If $R^2 \approx 1$, the selected model gives a good approximation of the data samples, whereas a low value for R^2 shows that the

model and, therefore, the sensitivity measure is insufficient.
The analysis of variance (ANOVA) is based on the variance decomposition of the output into

$$y = f_0 + \sum_{i=1}^{k} f_i(\mathbf{x}_i) + \sum_{i<1}^{k} f_{i,j}(\mathbf{x}_i, \mathbf{x}_j) + \ldots + f_{1,2,\ldots,k}(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k) \tag{19}$$

with

$$\begin{aligned} f_0 &= E(y), \\ f_i(\mathbf{x}_i) &= E(y|\mathbf{x}_i) - f_0, \\ f_{i,j}(\mathbf{x}_i, \mathbf{x}_j) &= E(y|\mathbf{x}_i, \mathbf{x}_j) - f_0 - f_i - f_j, \\ &\ldots. \end{aligned} \tag{20}$$

Based on this decomposition, the SOBOL indices S_i were introduced by their namesake as a parameter x_i sensitivity by defining S_i as the variance of the conditional expected value of the output, normalized by the unconditional variance of the output

$$S_{I,i} = \frac{V[E(y|x_i)]}{V(y)}, \quad \sum_i S_{I,i} \leq 1. \tag{21}$$

While Eq. (21) describes the first order SOBOL index, i.e. the main effect of a parameter x_i on the output y, thereby neglecting any interaction effects with one or more other parameters x_j, the total SOBOL index can be computed as

$$S_{tot,i} = 1 - \frac{V[E(y|\mathbf{x}_{\sim i})]}{V(y)}, \quad \sum_i S_{tot,i} \geq 1, \tag{22}$$

with $E(y|\mathbf{x}_{\sim i})$ as the expected value of the output by varying all parameters except x_i. A sum of first order indices $\sum_i S_{I,i}$ close to 1 indicates that almost only main effects of the parameters exist, while a lower value shows that interaction effects are not to be ignored.
For a approximation of the first order SOBOL indices for existing data samples (KVest), (Götz, 2017) proposes a separation of the input domain $\mathcal{X}_i$ in k equidistant intervals and a computation of $\hat{S}_{I,i}$ using

$$\hat{S}_{I,i} = \frac{V(\mathbf{E})}{V(\mathbf{y})}, \quad E_j = E(\mathbf{y}|\mathbf{x_i})_j \; \forall \; j \in k \tag{23}$$

given a sufficient and somewhat equally distributed number of samples.
Another variance-based measure is achieved by performing the fast FOURIER sensitivity test (FAST) method (Cukier et al., 1978), which, while also being perform-able as an approximation for existing data samples using the effective algorithm for computing sensitivity indices (EASI), is limited to continuous input and output parameters.

3.2. Sampling-based approaches

Is a simulation model for the generation of (additional) data samples available, a DoE can be constructed. For a visualization or a regression-based analysis, it is most desirable to create samples that cover the whole design space. To that purpose, possible experiments can follow full-factorial or fractional-factorial designs for categorical or discrete variables with a small number of possible realizations (mostly ≤ 3). Otherwise, random sampling (RS) or latin hypercube sampling (LHS) is utilized, whereas LHS shows faster convergence (Aistleitner et al., 2013) than RS and is often preferred. With the goal of approximating first order and total Sobol indices, (Saltelli et al., 2010) suggests specific sampling designs (radial sampling, winding stairs sampling) for a more accurate indices approximation.
In addition to the described data based computation of sensitivity measures, some surrogate models allow for an analytical result. In (Sobol and Kucherenko, 2009) a derivative-based approach is described. (Montano and Palmer, 2003) establishes a method for weight-based sensitivity measures based on feedforward neural network models.

3.3. Design parameter sensitivities for cornering stiffness

As for the tire design performance cornering stiffness, an inexpensive performance prediction tool is available, allowing for generation of data samples for sensitivity analysis. The structural model is a black-box and, therefore, does not give any information about parameter sensitivities based on the underlying functions/equations. The $n_x = 31$ input parameters consist of 24 continuous and 4 discrete, as well as 3 categorical variables. Since the objective of this sensitivity analysis is to contribute to the uncertainty quantification of the examined input parameters, categorical parameters are not of interest, since they are considered as deterministic. Table I gives an overview of the continuous and discrete input parameters, separated into geometry, compound and service. Each parameter index provides information about its corresponding tire part (grouped into A-F), or indicates an association to the categories of general tire dimensions (e.g. nominal width, rim diameter) or of service during usage. The model output consists of two continuous cornering stiffness parameter $\mathbf{y} = \{C_{\alpha,1}, C_{\alpha,2}\}$, for two different loads F_z.
A visual analysis proves to be only somewhat useful, since the input dimension is rather large, and will not be further discussed for the sake of brevity. The results of coefficient of correlation after Spearman (Eq. (17)) are depicted in Figure 3. The results for the

Table I. Tire design parameter for cornering stiffness prediction, sorted by tire part, with (g) for geometry, (c) for compound and (m) for miscellaneous parameter.

	general tire dimensions	tire part A	tire part B	tire part C	tire part D	tire part E	tire part F	service
g	$\{x^g_{0,i}, i \in 5\}$	$\{x^g_{a,i}, i \in 4\}$	$\{x^g_{b,i}, i \in 2\}$	$\{x^g_{c,i}, i \in 3\}$	$\{x^g_{d,i}, i \in 2\}$	$\{x^g_{e,i}, i \in 2\}$		
c		$\{x^c_{a,i}, i \in 2\}$	$\{x^c_{b,i}, i \in 2\}$	$\{x^c_{c,i}, i \in 1\}$	$\{x^c_{d,i}, i \in 1\}$	$\{x^c_{e,i}, i \in 2\}$	$\{x^c_{f,i}, i \in 1\}$	
m								$\{x_s\}$

Pearson correlation coefficient (Eq. (16)) have been found to coincide, with a maximum deviation of $\Delta(\rho_S, \rho_P) = 0.02$. The same result was acquired as the result of a regression analysis when using a linear model. However, the coefficients ot determination $R^2 = 0.71$ for $C_{\alpha,1}$ and $R^2 = 0.52$ for $C_{\alpha,2}$ indicate an insufficient representation of the target function by a linear model. An improvement was achieved by separating the general tire dimension parameter nominal tire width, resulting in $R^2 > 0.8$ for each category and still leading to similar results for the standardized regression coefficients. In addition, the radial sampling scheme described in (Saltelli et al., 2010) has been performed in order to approximate the total Sobol indices $S_{tot,i}$. Here, each nominal width was investigated separately, and the maximum of the resulting indices was selected for each parameter. For that reason, the sum of all total Sobol indices is much larger than 1, without necessarily indicating the presence of higher order interaction effects. The resulting sensitivities are depicted in Figure 3.

With the focus on the total Sobol indices, the design parameters are separated based on their sensitivity as shown in Table II. The total Sobol index was selected instead of the first order one, because the total effect of a parameter is of more relevance for the description of parameter importance in the context of uncertainty quantification. Also, a faster convergence during the numerical approximation of the total indices compared to the first order was observed.

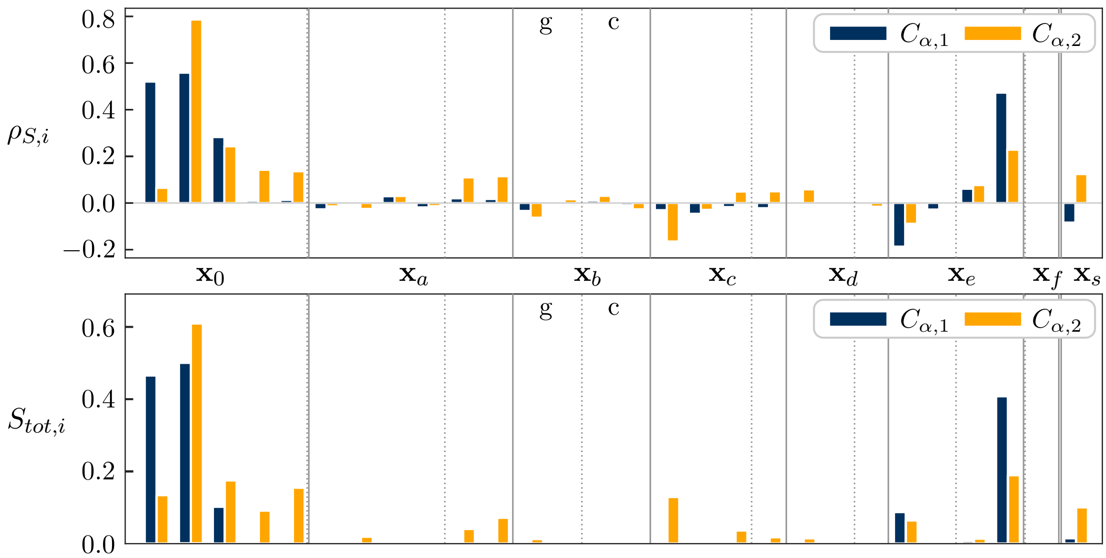

Figure 3. Coefficient of Correlation (Spearman) $\rho_{S,i}$ (top) and total Sobol indices $S_{tot,i}$ (bottom) for tire design parameter, with distinction of tire parts, grouped into (g) geometry and (c) compound (if applicable).

Table II. Tire design parameter for cornering stiffness prediction, separated by sensitivity.

parameters with medium to high sensitivity	parameters with low to no sensitivity
$\{x^g_{0,i}, i \in 5\}$	
$\{x^c_{a,i}, i \in 2\}$	$\{x^g_{a,i}, i \in 4\}$
	$\{x^g_{b,i}, i \in 2\}$, $\{x^c_{b,i}, i \in 2\}$
$\{x^g_{c,i}, i \in 2\}$	$\{x^g_{c,3}\}$ $\{x^c_{c,i}, i \in 1\}$
	$\{x^g_{d,i}, i \in 2\}$, $\{x^c_{d,i}, i \in 1\}$
$\{x^g_{e,1}, x^c_{e,2}\}$	$\{x^g_{e,2}, x^c_{e,1}\}$
	$\{x^c_{f,i}, i \in 1\}$
$\{x_s\}$	

4. Uncertainty quantification for tire design parameters

4.1. Data situation

The performance and, therefore, the design of a tire is dependent on a variety of parameters, including different types of rubber compounds and layers of textile and steel, the geometry and the molding process. For $n^g = 7$ geometry parameters $x^g_i, i \in n^g$ and $n^c = 47$ compound parameters $x^c_i, i \in n^c$, measurements taken in the process of tire production were available.
Each data sample for a geometry parameter includes information about the target value, the actual value and the general tire geometry as well as randomized information about the product specification, the production plant and date

$$x^g_i = \langle \Delta x, x_{Target}, Prod, Plant, Date \rangle. \tag{24}$$

As for the compound data, each sample consists of three measurements of the materials storage shear moduli G' for the strains $\varepsilon = \{1, 10, 100\}\%$, with values statistically normalized to $\overline{x}_i = \frac{x_i - \mu_X}{\sigma_X}$, as well as the date the test was performed. The description of the compound is randomized, yet allows for a distinction between body (B) and tread (T) compounds

$$x^c_i = \langle G'_1, G'_{10}, G'_{100}, Date \rangle. \tag{25}$$

No performance measure is associated with these data samples, so no sensitivity analysis based on these experimental values is possible.
In context of the data preparation, extreme outliers are discarded, i.e. data samples with $\Delta x \notin [Q_1 - 3(Q_3 - Q_1), Q_3 + 3(Q_3 - Q_1)]$ with Q_1 and Q_3 being the 25% and 75% quartiles of all samples Δx_j.

4.2. Uncertainties present in data samples

The data samples for the geometric and material parameters show, that a variability is present for each one of the parameters. This type of uncertainty, denoted as the aleatory type, is commonly modeled using a stochastic model. Figure 4 depicts the deviation from

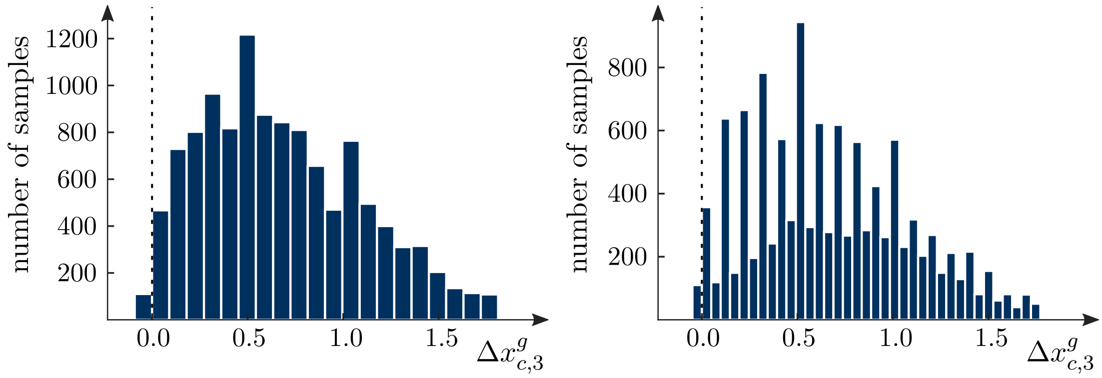

Figure 4. Histogram deviation of the tire geometry parameter $\Delta x^g_{c,3}$ with different number of bins.

the target value of the geometry parameter $x^g_{c,3}$ in form of a histogram with a positively skewed distribution. In addition, by adjusting the number of bins, an alternation of bin heights, i.e. in the number of contained samples, becomes apparent, indicating the limitations of measurement accuracy for the parameter. This lack of accuracy is classified as epistemic uncertainty and can not be disregarded when establishing a detailed uncertainty model. Given that a number of 11 706 measurements (after discarding of outliers) for the parameter $x^g_{c,3}$ can be considered a large sample size, epistemic uncertainty caused by incompleteness will be disregarded.
Between the geometry parameters of the same tire part, a correlation of the variation can be noted. Also, given the information about the target value in addition to the variation, some parameters display a dependency of these values. However, since these parameters show low to no sensitivity and will therefore be set as deterministic, these dependencies will be neglected. Otherwise, separate models for each target level would be advisable.

4.3. Selected uncertainty modeling strategies

As described before, the uncertainty quantification for the tire design parameters is based on their sensitivity for the performance cornering stiffness as well as the available data and expert knowledge. Table III gives an overview of the selected uncertainty modeling strategies.
For sensitive parameters with no production variation data, a fuzzy model is employed, to avoid false assumptions by assigning a specific type of probability distribution. Low sensitivity parameters are considered deterministic, for the sake of reducing the dimensionality of the later following fuzzy analysis in context of an uncertainty analysis and thereby minimizing the numerical effort. For parameters with considerable sensitivity as well as a data base, the measurement accuracy bounds constitute each data sample as an interval and are therefore used to construct p-boxes. Figure 5 shows the p-box modeling on the example of parameter $x^g_{c,3}$. A stochastic modeling of the body compound uncertainty can be seen in Figure 6 for two different strains ε. The fitting of different probability distributions was performed using the maximum-likelihood-estimate, and evaluated with the χ^2-test. As the p-value was higher than the conservatively selected α value of 0.10, the distribution can be considered an accurate fit in both cases. As

the sensitivity of body compounds is comparatively low, a more detailed uncertainty quantification has been refrained from.

Table III. Uncertainty quantification models for different parameters relevant for the performance cornering stiffness in a tire design process

	parameter	sensitivity	data base	modeling strategy
geometry	$x^g_{0,i}, i \in 5$	high	–	individual trapezoid fuzzy variables, based on expert knowledge
	$x^g_{c,3}$	medium	+	p-box with measurement accuracy as bounds
	$x^g_{c,1}, x^g_{c,3}$ $x^g_{e,1}$		–	individual fuzzy interval variables, based on expert knowledge
	$x^g_{a,i}, i \in 4$ $x^g_{b,i}, i \in 2$ $x^g_{c,2}$ $x^g_{d,i}, i \in 2$ $x^g_{e,2}$	low/none	– – – +/– +	deterministic
compound	tread $x^c_{T,i}$	high	+	p-box with measurement accuracy as bounds
	body x^c_B	low/none	+	multivariate (for different strains ε) random variable $X^c_B \sim t$ with covariance matrix
service	x_s	medium	–	individual trapezoid fuzzy variables, based on expert knowledge

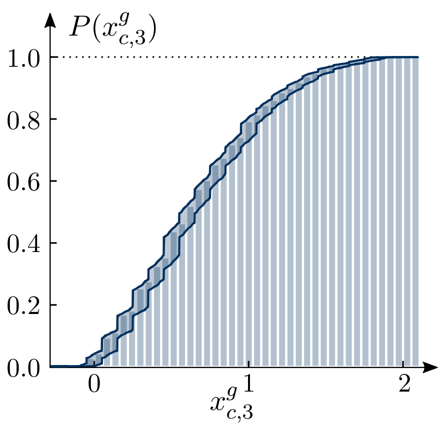

Figure 5.: p-box of parameter $x^g_{c,3}$.

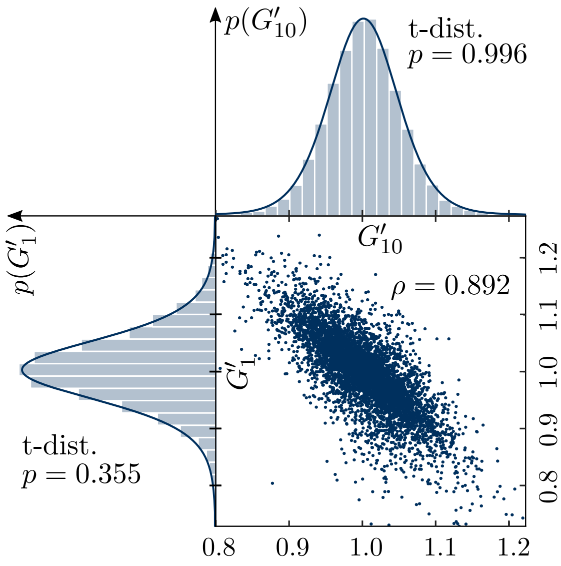

Figure 6.: Histogram & Scatterplot for measurements of body compound G' at $\varepsilon = \{1, 10\}\%$.

5. Conclusion and outlook

The quantification of parametric uncertainty is an important step in many design tasks, as it allows for a robustness analysis of the model output in addition to a deterministic prediction. As has been shown by the data collected from several production plants over a considerable amount of time, the fluctuation of different design parameters is considerable, as there are tolerance limits during any production process. Therefore, an intensive study of modeling these uncertainties is necessary.
The focus of this paper lies on the uncertainty quantification of tire design parameters for the performance cornering stiffness. An overview of basic as well as polymorphic uncertainty modeling approaches has been described. Since a specific product performance was investigated, a sensitivity analysis was performed to distinguish those design parameters which are most important, i.e. who's deviation has the biggest impact on the performances variance, and fixing not important ones as deterministic. For some of the tire design parameters, a data base of the production variation was available and analyzed as a basis for an uncertainty quantification. The resulting selected modeling concepts have been illustrated by examples based on the data situation, the parameters' sensitivities and expert knowledge.
It has been shown that an extensive data base is necessary for a detailed description of the aleatory parameter uncertainty, and, therefore, if not available, an assumption like, e.g., a normal distribution model, is not sufficient, as it may falsify results and under- or overestimates the robustness. In such a case, a quantification of epistemic uncertainty is necessary as well as the aleatory one, therefore a combined modeling is required.
Future work will include an uncertainty analysis for the tire performance run on an inexpensive simulation model, as well as a comparison of different modeling approaches and their influence on the performance variation. A model for robustness evaluation will be developed as a guideline for selecting a tire design which is considered as a multi-criteria optimization for both, the performance value and its robustness.

Acknowledgements

The provision of tire production data and expert knowledge on the subject of tire design as well as the financial support of Continental Reifen Deutschland GmbH is highly appreciated. Special thanks goes to the departments of Mechanics and Simulation Development under R. Nojek and Virtual Development and Computational Mechanics under J. Dehnert as well as the project team members C. Gerstenberger, C. Marschler and C. von Zobeltitz for the continuous and helpful cooperation in all tire-related questions.

References

Aistleitner, C., Hofer, M. and Tichy, R A central limit theorem for Latin hypercube sampling with dependence and application to exotic basket option pricing. *Journal of Theoretical and Applied Finance*, 15:1–20, 2013.

Cukier, R. I., Levine, H. B. and Shuler, K. E. Nonlinear sensitivity analysis of multiparameter model systems. *Journal of Computational Physics*, 26:1–42, 1978.

Der Kiureghian, A. Aleatory or epistemic? Does it matter?. *Structural Safety*, 31:105–112, 2009.

Faes, M., Van Doninck, B., Imholz, M. and Moens, D. Product reliability optimization under plate sheet forming process variability. *8th International Workshop on Reliable Engineering Computing, "Computing with Confidence"*, Liverpool, 2018.

Ferson, S. and Ginzburg, L. R. Different methods are needed to propagate ignorance and variability. *Reliability Engineering & System Safety*, 54:133–144, 1996.

Ferson, S., Kreinovich, V., Ginzburg, L. R. and Myers, D. S. *Constructing Probability Boxes and Dempster-Shafer Structures.* Sandia National Laboratories, Livermore, 2003.

Ferson, S. and Hajagos, J. G. Arithmetic with uncertain numbers: rigorous and (often) best possible answers. *Reliability Engineering & System Safety*, 85:135–152, 2004.

Ferson, S., Joslyn, C. A. Helton, J. C., Oberkampf, W. L. and Sentz, K. A Summary from the epistemic uncertainty workshop: consensus amid diversity. *Reliability Engineering & System Safety*, 85: 355–369, 2004.

Graf, W., Götz, M. and Kaliske, M. Analysis of dynamical processes under consideration of polymorphic uncertainty. In: G. Deodatis, B. R. Ellingwood, and D. M. Frangopol (Eds.), *Safety, Reliability, Risk and Life-Cycle Performance of Structures & Infrastructures: Proceedings of the 11th International Conference on Structural Safety and Reliability (ICOSSAR 2013)*, CRC Press, Taylor and Francis Group, pp. 453–460, 2013.

Graf, W., Götz, M. and Kaliske, M. Analysis of dynamical processes under consideration of polymorphic uncertainty. *Structural Safety*, 52(B):194–201, 2015.

Götz, M., Graf, W. and Kaliske, M. Structural design with polymorphic uncertainty models. *International Journal of Reliability and Safety*, 9:112–131, 2015.

Götz, M., Leichsenring, F., Kropp, M., Müller, P., Falk, T., Graf, W., Kaliske, M. and Drossel, W. Data mining and machine learning methods applied to a numerical clinching model. *Computer Modeling in Engineering & Sciences*, 117(3):387–423, 2018.

Götz, M. *Numerische Entwurfsmethoden unter Berücksichtigung polymorpher Unschärfe*. Dissertation, Institute for Structural Analysis, Technische Universität Dresden 2017. Vol. 32.

Jansen, M. J. W. Analysis of variance designs for model output. *Computer Physics Communications*, 117:35–43, 1999.

Leichsenring, F., Jenkel, C., Graf, W. and Kaliske, M. Numerical simulation of wooden structures with polymorphic uncertainty in material properties. *International Journal of Reliability and Safety*, 12:24–45, 2018.

Möller, B. and Beer, M. Engineering computation under uncertainty – Capabilities of non-traditional models. *Computers & Structures*, 86:1024–1041, 2008.

Möller, B., Graf, W., Sickert, J. U. and Reuter, U. Numerical simulation based on fuzzy stochastic analysis. *Mathematical and Computer Modelling of Dynamical Systems*, 13:349–364, 2007.

Montaño, J. J. and Palmer, A. Numeric sensitivity analysis applied to feedforward neural networks. *Neural Computation & Application*, 12:119–125, 2003.

Mullins, J., Ling, Y., Mahadevan, S., Sun, L. and Strachan, A. Separation of aleatory and epistemic uncertainty in probabilistic model validation. *Reliability Engineering & System Safety*, 147:49–59, 2016.

Pannier, S. and Graf, W. Sectional global sensitivity measures.. *Reliability Engineering & System Safety*, 134:110–117, 2014.

Pianosi, F., Beven, K., Freer, J., Hall, J. W., Rougier, J., Stephenson, D. B. and Wagener, T. Sensitivity analysis of environmental models: A systematic review with practical workflow. *Environmental Modelling & Software*, 79:214–232, 2014.

Plischke, E. An effective algorithm for computing global sensitivity indices (EASI). *Reliability Engineering & System Safety*, 95:354–360, 2010.

O'Hagan, A. and Oakley, J. E. Probability is perfect, but we can't elicit it perfectly. *Reliability Engineering & System Safety*, 85:239–248, 2004.

Saltelli, A., Annoni, P., Azzini, I., Campolongo, F., Ratto, M., and Tarantola, S. Variance based sensitivity analysis of model output. Design and estimator for the total sensitivity index. *Computer Physics Communications*, 181:259–270, 2010.

Saltelli, A., Tarantola, S. and Chan, K. P.-S. A Quantitative Model-Independent Method for Global Sensitivity Analysis of Model Output. *Technometrics*, 41:39–56, 1999.

Schietzold, F. N., Graf, W. and Kaliska, M. Polymorphic uncertainty modeling for optimization of timber structures. *8th International Workshop on Reliable Engineering Computing, "Computing with Confidence"*, Liverpool, 2018.

Schnell, R. *Grafisch gestützte Datenanalyse.* Verlag Oldenbourg, München, 1994.

Serafinska, A., Özenc, K., Kaliske, M. and Graf, W. A Fail-Safe Design Approach Based on the Fracture Mechanical Analysis and Epistemic Uncertainty Quantification. *Proceedings of the 7th International Workshop on Reliable Engineering Computing (REC) 2016, Bochum*, 143–158, 2016.

Sobol, I. M. Global sensitivity indices for the investigation of nonlinear mathematical models. *Matematicheskoe Modelirovanie*, 19:23–24, 2007.

Sobol, I. M. and Kucherenko, S. Derivative based global sensitivity measures and their link with global sensitivity indices. *Mathematics and Computers in Simulation*, 79:3009–3017, 2009.

Viertl, R.. *Statistical Methods for Non-Precise Data.* CRC Press, Boca Raton, 1996.