Numerical simulation of a 3D concrete printing process under polymorphic uncertainty

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Abstract. During the past few years, additive manufacturing techniques for concrete have gained extensive attention. In particular, the extrusion-based 3D concrete printing exhibited a rapid development. Previous investigations are mostly based on experimental studies or even trial-and-error tests. A more profound understanding of the relationships between the process and material parameters and the manufactured structure can be advanced by numerical modeling and simulation. It enables to study a wide range of parameters such that dependencies of properties of the printed product on different influencing factors can be identified. Taking into account the uncertain nature of process and material parameters of the extrusion-based concrete printing, the process can be reliably controlled and finally optimized. The presented study uses pseudo-density for an efficient finite element based modeling approach, where predefined elements are activated layer-by-layer. Material parameters are described as temporally and cross-correlated random processes, while an interval description is used to take the vague information on their correlation parameters into account. First results of a reliability estimation are shown for a 2D modeled additively manufactured wall.

Keywords: Random Process, Polymorphic Uncertainty, Concrete Printing, 3DCP

1. Introduction

In the past decade 3D concrete printing (3DCP) technology is getting recognition in the construction industry. The technology has several advantages such as decreased construction time, design flexibility compared to conventional construction methods, cost reduction by avoiding formwork costs, reduced waste, reduced manpower which decreases injuries and fatalities on construction sites and increased sustainability of the construction industry (Wu et al., 2016; Hager et al., 2016). A growing number of projects can be observed in both private enterprises and research institutes worldwide (Buswell et al., 2018). According to Wangler et al. (Wangler et al., 2016) among the different concrete printing processes like particle-bed 3D printing (Le et al., 2012), the extrusion-based process is the most widely used. Therefore, the presented contribution considers the extrusion-based process which has been demonstrated to be applicable to the construction industry (Wu et al., 2016; Hager et al., 2016). However, there are several challenges to be addressed to fully implement the technology. An improved profound understanding of the relationship between design, material behavior, and process parameters is eminent. The materials' rheology and the process parameters, such as printing speed, time, temperature etc., have an impact on the fresh and hardened state of the printed structure (Panda et al., 2017): Effects like geometric variation of the printed layers (Buswell et al., 2018), stability failure due to local material strength (Wangler et al., 2016) and global buckling (Gosselin et al., 2016) have been reported. Under-filling influenced by printing process parameters (nozzle velocity, pumping rate etc.) and material properties (rheology) are reported (Le et al., 2012). Due to this complex behavior and the variability of material parameters (Buswell et al., 2018) and printing process parameters, adequate process parameters are commonly determined by means of trial-and-error (Suiker, 2018).

By implementing a numerical simulation of the 3DCP process a better understanding of the system parameters on the structure behavior can be achieved. The influence of the parameter uncertainty can be studied. In this contribution a finite element framework is proposed to model the time depending printing process efficiently and is combined with an random process model of fresh concrete material parameters, where auto- and cross-correlation are considered as intervals yielding a polymorphic uncertainty model. A reliability analysis evaluates the dependence on these parameters.

2. Method

2.1. Finite element model of a printing process

According to the layer-wise production process of the concrete structure, the finite element (FE) model "needs to grow". The idea is to work on a previously generated FE mesh, where the elements are activated sequentially in correspondence to the printing process, cf. Fig. 1. This approach enables an efficient simulation avoiding the computational demanding re-meshing procedure. During the

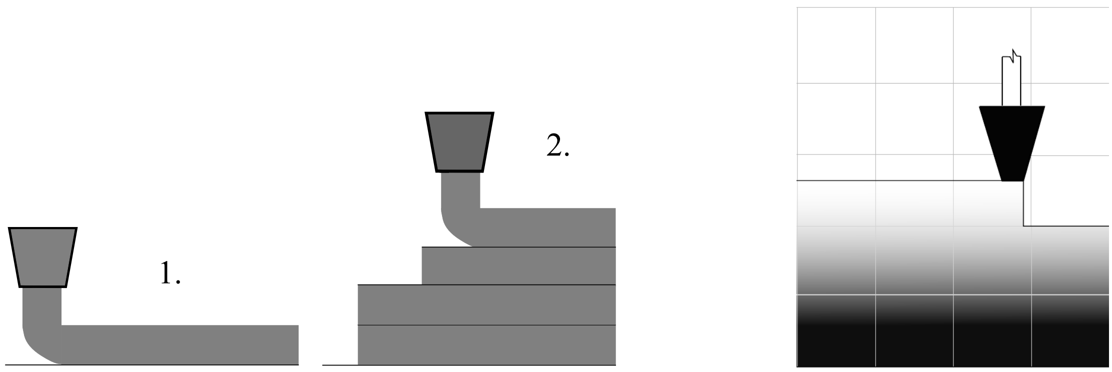

Figure 1. (left) Schematic of a layer-by-layer process; (right) Temporal evolution of a material parameter on a pre-meshed FE model with sequentially activated elements

simulation, i.e. the solution of a finite element system of the form

$$\boldsymbol{K}(\boldsymbol{r},t)\boldsymbol{U}(\boldsymbol{r},t) = \boldsymbol{F}(r,t) \tag{1}$$

is solved, where $\boldsymbol{K}$ is the global stiffness matrix, $\boldsymbol{U}$ is the vector of nodal displacements, the loads due to self-weight are considered in the right-hand-side vector $\boldsymbol{F}$ and $\boldsymbol{r}$ and t denote the space and

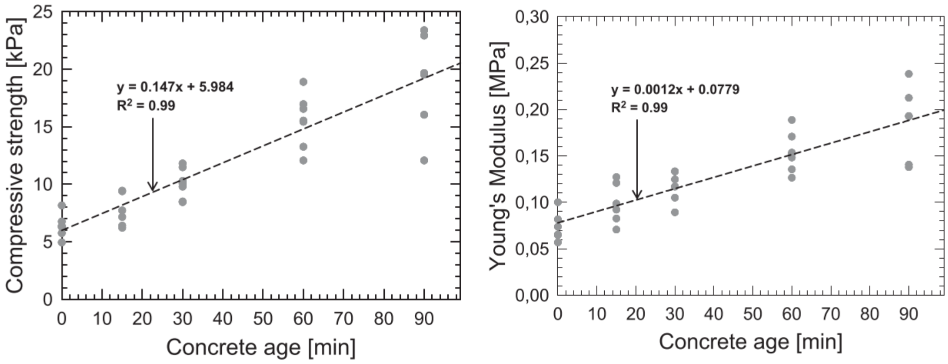

Figure 2. Experimentally determined evolutions of the compressive strength (left) and of the Young's modulus (right), reported in (Wolfs et al., 2018); fit was based on average test results.

time variable, respectively. Additionally, as all parameters do change with time according to the hardening process the material parameters are both spatially and temporarily varying, indicated by the space and time dependency in Eq. 1. An example of the temporal development of the material parameters compressive strength and Young's modulus is shown in Fig. 2 (Wolfs et al., 2018). Obviously, both the compressive strength and the Young's modulus of the printed concrete increase with the passing time. Additionally, the experimental results show a significant scatter of the material properties leading to the necessity of an uncertainty model for the FE model parameters where temporal and spatial correlations are taken into account.

2.2. Polymorphic uncertainty

A principal scheme of the proposed uncertainty description of a 3DCP model is depicted in Fig. 3. The material parameters of the printed structure originate from the time-dependent material parameters $M_1(t)$ and $M_2(t)$ of the pumped concrete, where t denotes the time. While the material properties undergo a natural variation it is reasonable to assume a temporal auto-correlation of the concrete properties since the fresh pumped concrete is taken from the same supply. Hence, a random process description is proposed, described by an auto-correlation function $R(d_t, t_c)$, where d_t denotes the time difference and t_c the auto-correlation time.

Additionally, a cross-correlation parameter c_{12} is introduced to take into account that most of these material parameters are not independent from each other. The same holds for process parameters (e.g. printing velocity, pumping rate), which are fixed here for the sake of clarity.

The stochastic characterization of the material parameters is based on distribution parameters and the auto- and cross-correlation parameters t_c and c_{12}, respectively. While the distribution parameters can usually be determined from experimental data, information on auto- and cross-correlation is typically vague, which constitutes an epistemic uncertainty. Therefore, in this contribution auto- and cross-correlation are considered as interval variables t_c^{i} and c_{12}^{i} to model this kind of uncertainty, where the first one yields a interval valued auto-correlation function $R^{\mathrm{i}}(d_{\mathrm{t}}, t_c^{\mathrm{i}})$.

The uncertainty in the temporal evolution of the material parameters of the printed structure could also be described in an analog manner but is assumed as deterministic for this study. As a result system responses (for example the maximum displacement u_{mx}) are described in a interval-stochastic space. Hence, a reliability analysis yields an interval failure probability $\hat{p}^{\mathrm{i}}$.

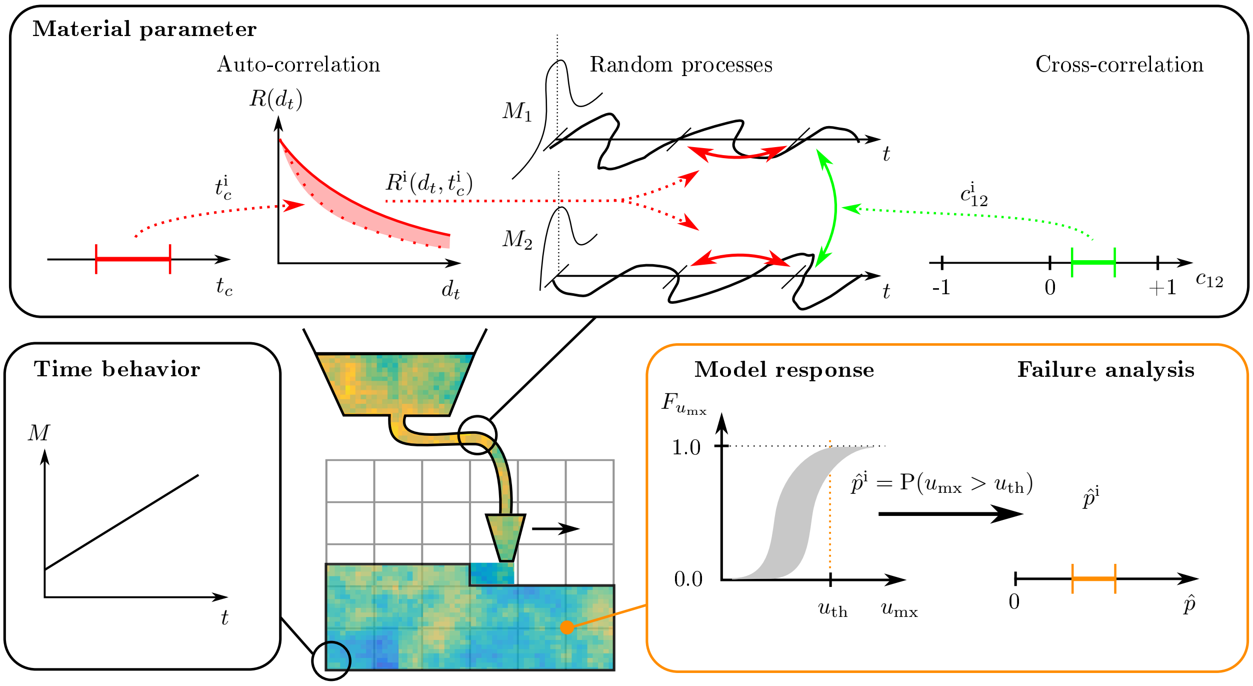

Figure 3. Principle scheme of a 3DCP model with time dependent, uncertain material parameters, which are described as interval probability based random processes

3. Numerical Example

3.1. Description

3.1.1. *Finite Element Model*

We consider a 2D finite element (FE) model of a printed wall with a size of $1\,\mathrm{m} \times 0.2\,\mathrm{m}$. The process parameters are constant: a printing velocity of $v_{\mathrm{p}} = 1\,\mathrm{m/min}$ is chosen while the layer height is set to 10 mm. The total printing time is 1200 s. The wall is modeled with a regular rectangular mesh using 40x20 Q8 elements. A linear-elastic plane-stress material model is assumed. The time-dependent Young's modulus E and compressive strength σ_y are given as

$$E(\boldsymbol{r}, t) = E_0(\boldsymbol{r}) + E_1(t) \tag{2}$$

and

$$\sigma_y(\boldsymbol{r}, t) = \sigma_{y,0}(\boldsymbol{r}) + \sigma_{y,1}(t), \tag{3}$$

where $E_0(\boldsymbol{r})$ and $\sigma_{y,0}(\boldsymbol{r})$ are the starting value as a function of position $\boldsymbol{r}$, E_1 and $\sigma_{y,1}$ are the slopes of the temporal increment, respectively, and t the time. The loading consists of the increasing self-weight, considering a constant mass density of $2020\,\mathrm{kg/m^3}$ (Wolfs et al., 2018). The modeled wall is fixed vertically at the whole bottom edge and horizontally at the middle of the bottom edge.

In order to take the temporal development of the material parameters into account, the total displacement $\boldsymbol{U}$ of the whole system is calculated by activating the elements in layer-wise fashion for all layers $l = 1 \dots n_\mathrm{l}$ (bottom to top), with $n_\mathrm{l} = 20$ in this example, solving

$$\boldsymbol{K}_l \boldsymbol{u}_l = \boldsymbol{F}_l, \tag{4}$$

where the stiffness matrix $\boldsymbol{K}_l$ and the displacement vector $\boldsymbol{u}_l$ take only the elements of the layers 1 to l into account and $\boldsymbol{F}_l$ consists only of the self-weight of layer l. Therefor, the total displacement can be determined as

$$\boldsymbol{U} = \sum_{l=1}^{n_\mathrm{l}} \boldsymbol{u}_l \tag{5}$$

The temporal evolution of the compressive strength can be seen in Fig. 4. An increase of $E_1 = 1.2\,\mathrm{kPa/min}$ and $\sigma_{y,1} = 0.147\,\mathrm{kPa/min}$ are chosen in accordance to (Wolfs et al., 2018), while E_0 and $\sigma_{y,0}$ are modeled via a cross-correlated random process, which is described in the next section.

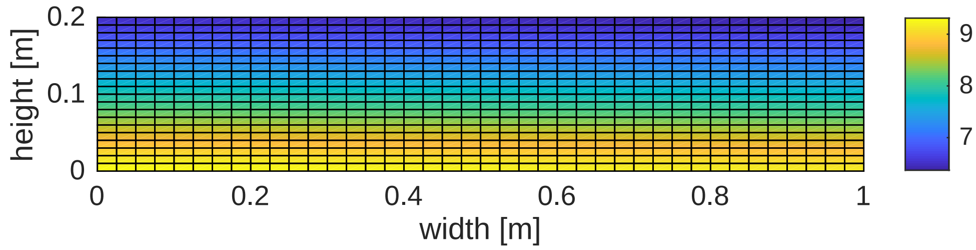

Figure 4. Compressive strength $\sigma_y(\boldsymbol{r}, t)$ in [kPa] for $t = 1200\,\mathrm{s}$ and a constant $\sigma_{y,0} = 6.37\,\mathrm{kPa}$

3.1.2. *Polymorphic Uncertainty Model*

In this contribution the proposed uncertainty scheme of Fig. 3 is applied to the material parameters of the fresh concrete, namely Young's modulus and compressive strength, which are than mapped to E_0 and $\sigma_{y,0}$, respectively.

The variability of the material parameter of such a printing process can be described as lognormally distributed random processes $E^{\mathrm{rp}}(t)$ and $\sigma_y^{\mathrm{rp}}(t)$, respectively. Experimental studies suggest a strong positive correlation of the chosen material parameters (Wolfs et al., 2018) which is covered by introducing a cross-correlation c between the processes while auto-correlation is described using a exponentially decaying function with the correlation time t_c. Considering the vague information on cross- and auto-correlation, both parameters are modeled as intervals with $t_\mathrm{c}^\mathrm{i} = [7.5; 75]$s and $c^\mathrm{i} = [0.7; 0.9]$, respectively. A mathematically more profound definition of interval probability based random processes can be found in Sec. 3.1 of (Schietzold et al., 2019).

The mean and the standard deviation of the E^{rp} and σ_y^{rp} are taken from (Wolfs et al., 2018) as 74 kPa with 14.5 kPa and 6.37 kPa with 0.95 kPa, respectively. In Fig. 5 realizations of the random processes are plotted for auto-correlation time $t_{\mathrm{c}} = 25\,\mathrm{s}$ and cross-correlation $c = 0.9$.

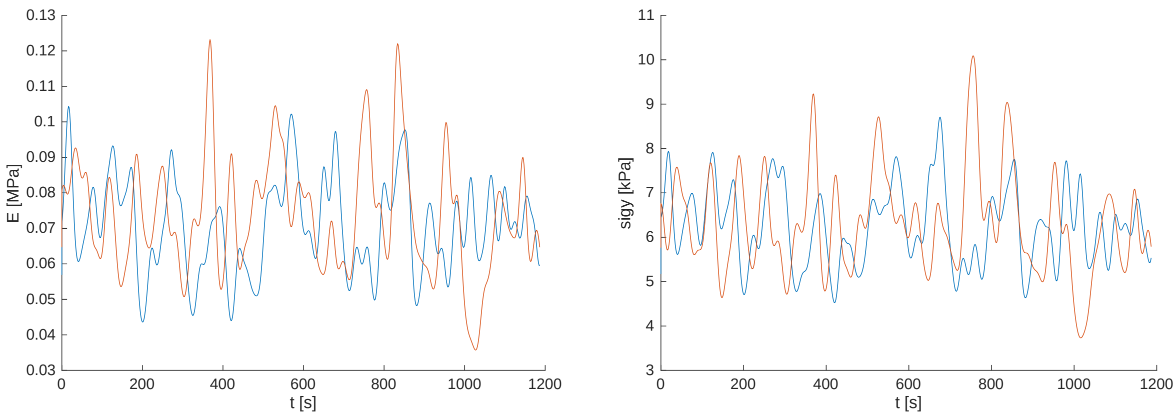

Figure 5. Samples of random processes $E^{\mathrm{rp}}(t)$ (left) and $\sigma_y^{\mathrm{rp}}(t)$ (right) for $t_{\mathrm{c}} = 25\,\mathrm{s}$ and $c = 0.9$

The behavior of the printing process is studied within the interval-stochastic space. A modified Karhunen-Loeve expansion is used to generated the cross-correlated random processes (Vořechovský, 2008). While the interval space $t_{\mathrm{c}}^{\mathrm{i}} \times c^{\mathrm{i}}$ is sampled using a latin hypercube algorithm (50 points), a crude Monte Carlo approach is used in the stochastic space (1000 samples), resulting in 50 000 finite element simulations.

3.2. Results

In Fig. 6 one realization of the Young's modulus at the end of the printing process ($t = 1200\,\mathrm{s}$) is depicted. Both the random nature of the fresh concrete property (E_0) and the increase of stiffness over time can be observed.

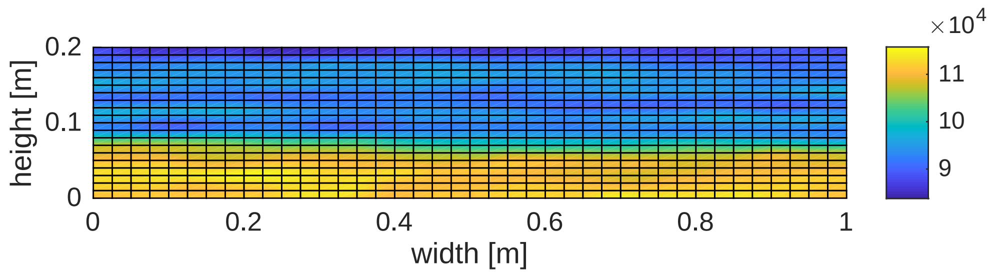

Figure 6. Realization of Young's modulus $E(\boldsymbol{r})$ in [Pa] at $t = 1200\,\mathrm{s}$ for E_0 mapped from the random process $E^{\mathrm{rp}}(t)$

Exemplary results of the structural analysis are shown in Fig. 7. The reasonable deformation shape displays horizontal displacements up to 1.5 cm.

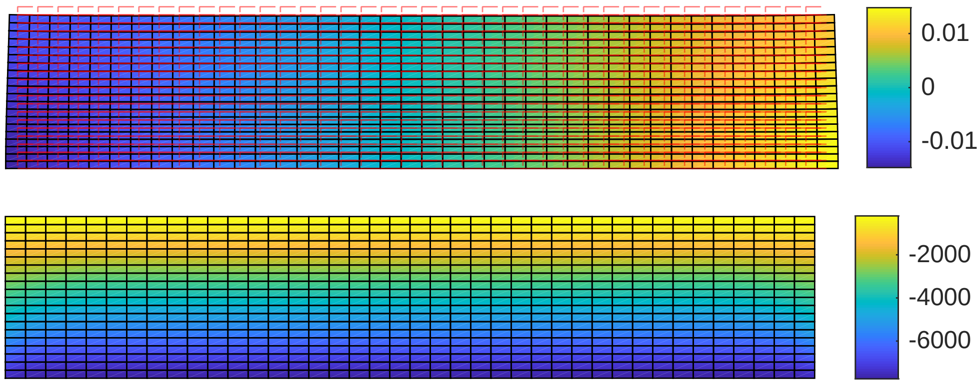

Figure 7. Exemplary results at $t = 1200\,\mathrm{s}$ under self weight: (top) horizontal displacement u_x in [m]; (bottom) normal stress σ_{yy} in [Pa]

For each sample the resulting stresses and strains have been calculated from the simulated nodal displacements. The histogram of the maximum shear strain per sample is displayed in Fig. 8 for two different points in the interval space of t_c and c, displaying a significant difference.

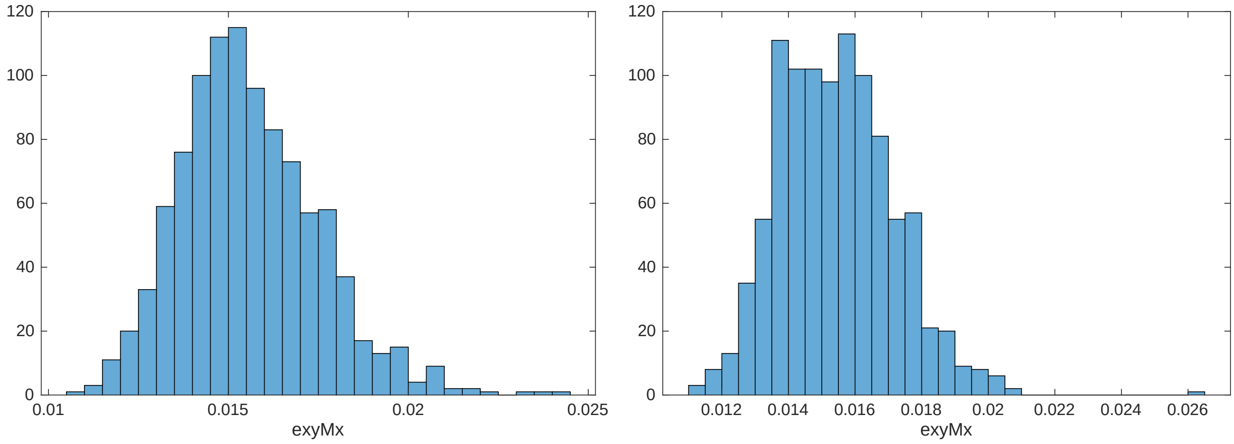

Figure 8. Shear strain histograms for (left) $t_\mathrm{c} = 72.6\,\mathrm{s}$, $c = 0.70$; (right) $t_\mathrm{c} = 40.1\,\mathrm{s}$, $c = 0.89$

Considering a simple limit state of a maximum tolerable deformation of 10% the failure probability for all 50 combinations of auto- and cross-correlation values was estimated. While the failure probability is strongly influenced by the correlation time t_c the cross-correlation c has no effect on the failure probability, cf. Fig. 9

Further, a polynomial-based regression is used to predict the failure probability estimates at the boundaries of the interval, especially for t_c^i. Finally, following the idea of the scheme in Fig. 3 a failure probability interval can be determined as $p_F^\mathrm{i} = [0.23; 0.34]$.

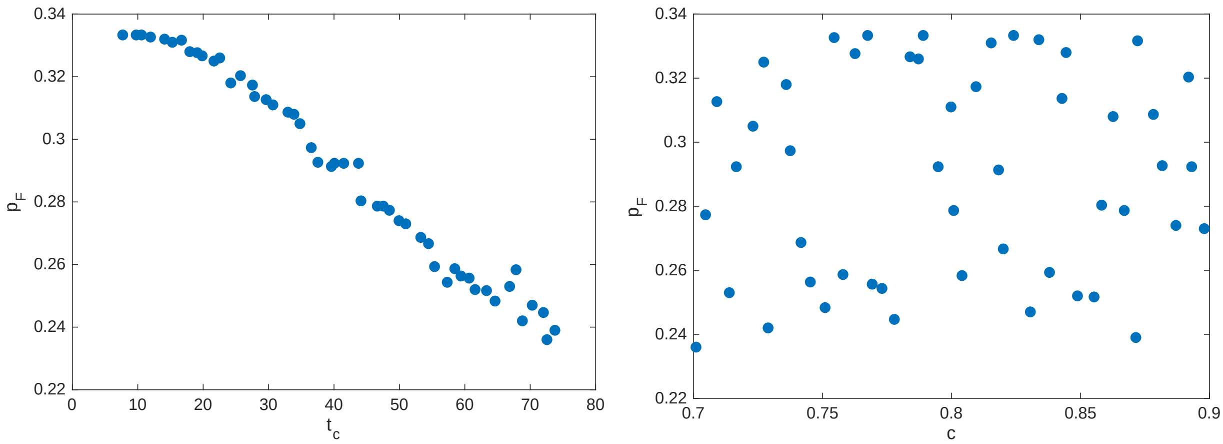

Figure 9. Failure probability p_F as a function of (left) auto-correlation time t_c and (right) cross-correlation c

4. Conclusion

The proposed approach considers an interval probability based random process description to take the uncertain nature of 3D concrete printing material parameters into account. A finite element model utilizing a pre-defined mesh is used to efficiently simulate the time-dependent printing process in a layer-by-layer fashion. For a simple 2D example of a $1\,\mathrm{m} \times 0.2\,\mathrm{m}$ wall, lognormally distributed random processes modelling the Young's modulus and the compressive strength of the fresh, pumped concrete is mapped to a the initial values of the time-dependent stiffness and compressive strength. The statistical properties of the system responses show a strong dependence on the auto-correlation time, which is described by an interval. A reliability analysis limited by the maximal tolerable deformation emphasis the dependence of the failure probability on such vaguely know parameters, which suggests that a realistic, polymorphic uncertainty model is appropriate. Further studies will include additional material and process parameters in the uncertainty model. The effect of the cross-correlation parameter on an stress-based failure mode will be investigated.

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