Sensitivity of an hysteretic material model for random vibration analysis of base-isolated rigid-block historical artifacts

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Abstract. We present the computation and the implementation of the sensitivity relevant to a recently formulated phenomenological nonlinear material. Such a constitutive model is capable of reproducing several typologies of hysteretic loops without requiring iterative algorithms to determine its response. Moreover, it is based on a set of parameters having a peculiar physical meaning and provides responses characterized by smooth derivative. Hence, the model is particularly feasible to be implemented within a finite-element based reliability code aiming to perform tail-equivalent linearization and random vibration analysis. Within this context, the implemented model has been tested by analyzing a base-isolated structural system inspired at the Riace Bronze A. Numerical tests confirm the effectiveness of the sensitivity formulation and of its capabilities for tail-equivalent linearization purposes.

Keywords: art objects; nonlinear dynamics; tail-equivalent linearization.

1. Introduction

Nonlinear response of rigid blocks has received a significant attention during the last decade due to the need of assessing the seismic risk of museum artifacts (Roussis et al., 2008). Such a response depends on a complex interaction between the block base and a shaking surface, often presenting nonlinear behavior, and the dynamics of a rigid block.

In such a context, seismic isolation is conveniently adopted in order to reduce the vulnerability of art objects. Among several isolation technologies currently available, elastomeric bearings usually present hysteretic behaviors due to the flexibility and energy dissipation capacity of the rubber, whereas the same effects in sliding bearings rely upon friction and contact between two flat or curved steel surfaces. Moreover, wire ropes devices are typically adopted when the isolated system needs to be centered after a seismic excitation and when the static loads are not significant.

The hysteresis of such bearing typologies strongly influences the maximum displacements and accelerations of the isolated rigid block subjected to a base excitation. Within a seismic context, it is essential to determine the first excursion probability and to assess the properties of the bearing devices in order to maintain it within a safety interval.

To this end, Tail-equivalent Linearization Method (TELM), developed by Fujimura and Der Kiureghian (2007), is an effective and appealing procedure to perform random vibration analysis of such structural typologies since a failure condition of the isolated rigid block can be expressed by means of a limited set of responses.

In particular, the TELM defines the base excitation as a sequence of random pulses. Then, fixed a response of interest and a set of threshold values, the procedure determines a *performance point* as the random process for which the response is equal to the threshold and turns out to be associated with the maximum probability of occurrence. Finally, the performance point permits the definition of a linearized system as a collection of impulse response functions.

The determination of the performance point is carried out by the application of a First-Order Reliability Method (FORM) actually consisting in a constrained-optimization algorithm (Ditlevsen and Madsen, 1996). Specifically, the FORM aims to maximize the probability relevant to the considered random variables (for the TELM case, these are the values in time of the base acceleration random process) by fulfilling a nonlinear limit state function. The latter is defined as difference between the response of interest and a fixed threshold.

An important issue of TELM consists in the fact that all the available algorithms are of iterative nature requiring, at each iteration, the computation of the gradient of the limit state function with respect to the random variables (Broccardo et al., 2017). For what concerns the TELM technique, this consists in computing the derivative of the structural response with respect to a set of acceleration values in which the base excitation process is discretized.

In seismic applications, accurate TELM analyses require to adopt properly discretized random processes lasting at least the time required by the structural system to reach stationarity. For this reason, the base excitation, and the related response gradient, can be made of several hundreds of components. Thus, a finite-difference-based computation of the response gradient would be excessively demanding from the computational point of view.

An essential step of the TELM is represented by the Direct Differentiation Method (DDM) which is able to numerically compute the response gradient with affordable computational burden (Haukaas and Der Kiureghian, 2006). However, its implementation represents the main limitation to a widespread diffusion of TELM since DDM is not common in commercial codes and requires the evaluation of the sensitivity of the response relevant to each object which is part of the analysis procedure.

Specifically, within the context of finite elements, the OpenSees framework includes an effective implementation of the DDM and of the TELM algorithm although this has been developed for a limited set of constitutive models. In particular, the most versatile one is the Bouc-Wen model.

Besides of its capability of reproducing several typologies of smooth hysteresis loops, such a model is affected by two significant drawbacks. First, the model is defined by means of an implicit differential equation governing the derivative of a non-linear generalized strain. This implies the necessity of determining the response by iterative approaches such as the Newton's or the Backward Euler algorithms. As a second, and more important consideration, the Bouc Wen material is a purely numerical model defined by means of a set of coefficients which lacks of physical meaning and can be identified, by matching experimental data, with significant difficulty.

To overcome such drawbacks, a new class of phenomenological constitutive models has been recently proposed by Vaiana et al. (2018) including, in particular, the *algebraic material* formulated in (Vaiana et al., 2019).

The latter model was developed for reproducing the hysteresis loops of several typologies of seismic devices including elastomeric bearings. Its main benefit consists in the fact that the response is computed in closed form so that no iterative algorithms are required. Moreover, the model is defined by a set of physically meaningful parameters directly identifiable by experimental data so that their calibration is straightforward (Sessa et al., 2020).

Within this context, the paper illustrates the development of the response sensitivity of the algebraic material proposed in (Vaiana et al., 2019) to be implemented within a DDM in order to perform the TELM analysis of a base-isolated rocking rigid block simulating the Riace Bronze A.

Specifically, a brief review of the TELM is reported in Section 2 while the computation of the response gradient relevant to the algebraic material is formulated in Section 3. A numerical application consisting in a vulnerability analysis of the Riace Bronze A is illustrated in Section 4 and, finally, conclusions are discussed in Section 5.

2. Review of the Tail-Equivalent Linearization Method

The Tail-Equivalent Linearization Method, developed by Fujimura and Der Kiureghian (2007), defines a linearized model of a nonlinear structural system by means of a collection of Impulse Response Functions (IRFs) associated with different threshold values of a response of interest.

Such a strategy adopts a discretized description of a zero-mean, second-order Gaussian base excitation which is expressed as an array of random pulses:

$$F\left(t\right)=\sum_{i=1}^{n}s_{i}\left(t\right)\upsilon_{i}=\mathbf{s}^{T}\left(t\right)\boldsymbol{\upsilon} \tag{1}$$

where n is the total number of pulses, $\boldsymbol{\upsilon}$ is a vector of standard normal variables and $\mathbf{s}\left(t\right)$ is a deterministic vector depending on the covariance of the base excitation process.

We also denote by $X_{NL}\left(t_n,\, \boldsymbol{\upsilon}\right)$ a response of interest of the nonlinear structural system at time t_n associated with a realization of the input random pulses $\boldsymbol{\upsilon}$. Fixed a threshold x of such a response, a limit state function is defined as:

$$\Theta\left(x,\, t_n,\, \boldsymbol{\upsilon}\right)=x-X_{NL}\left(t_n,\, \boldsymbol{\upsilon}\right) \tag{2}$$

Denoting by *tail probability* the quantity $\Pr\left[\Theta\left(x,\, t_n,\, \boldsymbol{\upsilon}\right)\leq 0\right]$, i.e., the probability that the response of interest is greater than the fixed threshold x, its first-order approximation is computed by applying the FORM algorithm (Ditlevsen and Madsen, 1996) provided that the limit state function has continuous gradient.

The FORM algorithm determines a *performance point* $\boldsymbol{\upsilon}^{\star}\left(x,\, t_n\right)$, associated with threshold x at time t_n, as the solution of the following constrained optimization problem:

$$\boldsymbol{\upsilon}^{\star}\left(x,\, t_n\right)=\arg\,\min\left\{\|\boldsymbol{\upsilon}\|\;|\Theta\left(x,\, t_n,\, \boldsymbol{\upsilon}\right)=0\right\} \tag{3}$$

In particular, the first-order approximation of the tail probability turns out to be:

$$\Pr\left[\Theta\left(x,\, t_n,\, \boldsymbol{\upsilon}\right)\leq 0\right]\propto\Phi\left[-\beta\left(x,\, t_n\right)\right] \tag{4}$$

where Φ denotes the standard-normal cumulative probability function and β is the reliability index:

$$\beta\left(x,\, t_n\right)=\boldsymbol{\alpha}\left(x,\, t_n\right)\boldsymbol{\upsilon}^{\star}\left(x,\, t_n\right) \tag{5}$$

where:

$$\boldsymbol{\alpha}\left(x,\,t_n\right)=-\frac{\nabla_{\boldsymbol{\upsilon}}\Theta\left(x,\,t_n,\,\boldsymbol{\upsilon}^{\star}\right)}{\left\|\nabla_{\boldsymbol{\upsilon}}\Theta\left(x,\,t_n,\,\boldsymbol{\upsilon}^{\star}\right)\right\|} \tag{6}$$

is the negative unit vector parallel to the gradient of the limit state function at the performance point $\boldsymbol{\upsilon}^{\star}$.

The *tail-equivalent linear oscillator* associated with time t_n and threshold x is therefore defined as a linear, single degree-of-freedom system having a tail-probability equal to the first-order tail probability of the nonlinear structure. In this sense, the linear response $X\left(t\right)$ can be expressed by means of its Impulse Response Function (IRF) $h\left(t\right)$ by a convolution integral:

$$X\left(t\right)=\int_0^t h\left(t-\tau\right)\sum_{i=1}^{n}s_i\left(\tau\right)\upsilon_i\,d\tau=\mathbf{a}\left(t\right)^T\boldsymbol{\upsilon} \tag{7}$$

where the deterministic vector $\mathbf{a}\left(t\right)$ is defined by the components:

$$a_i\left(t\right)=\int_0^t h\left(t-\tau\right)s_i\left(\tau\right)\,d\tau \tag{8}$$

Referring to the original paper (Fujimura and Der Kiureghian, 2007) for further details, we emphasize that enforcing equality between the response of the linear and of the nonlinear systems, and exploiting geometrical considerations, it is possible to relate such a deterministic vector to the performance point:

$$\mathbf{a}\left(t_n\right)=\frac{x}{\left\|\boldsymbol{\upsilon}^{\star}\left(x,\,t_n\right)\right\|}\frac{\boldsymbol{\upsilon}^{\star}\left(x,\,t_n\right)}{\left\|\boldsymbol{\upsilon}^{\star}\left(x,\,t_n\right)\right\|} \tag{9}$$

Hence, computed the performance point associated with time t_n and threshold x, it is possible to determine in turn the vector $\mathbf{a}\left(t_n\right)$ and an impulse response function $h^{\star}\left(x,\,t_n,\,t_i\right)$. Such a function, referred to the linearized system, is time discretized in terms of the set of instants t_i and associated with x and t_n.

The *Tail-Equivalent Linearized System* (TELS) it determined by adopting a time t_n sufficiently large to make the nonlinear system reach stationarity and an exhaustive set of thresholds x; in this sense, the TELS is defined by means of a collection of impulse response functions.

3. Sensitivity of the algebraic material

A key issue of the TELM concerns the determination of the performance point by the FORM algorithm applied to the optimization problem in Eq. (3) as well as the computation of the tail probability. Both tasks need the gradient $\nabla_{\boldsymbol{\upsilon}}\Theta\left(x,\,t_n,\,\boldsymbol{\upsilon}^{\star}\right)$ of the limit state function and, subsequently, of the response $X_{NL}\left(t_n,\,\boldsymbol{\upsilon}\right)$.

It is convenient to determine such a gradient by a DDM procedure in order to limit the computational burden. To this end, the constitutive models adopted in the analysis have to be equipped with their sensitivity with respect to all variables depending on the base excitation.

For this reason we present a brief review of the algebraic material formulated in (Vaiana et al., 2019) and report the detailed computation of its sensitivity required by the TELM procedure.

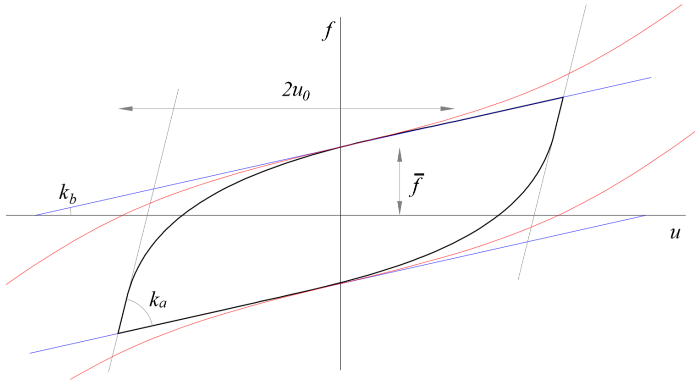

Figure 1. Riace Bronze A resting on a seismic-isolated basement.

The algebraic material is defined by means of five constitutive parameters. Namely, k_a is the initial stiffness, k_b is the asymptotic stiffness and α is a transition parameter between the initial elastic phase and the inelastic one. Parameters β_1 and β_2 influence the shape of the hysteretic loop by introducing local maxima and pinching.

Moreover, the model uses the following auxiliary parameters:

$$u_0 = \frac{1}{2}\left[\left(\frac{k_a - k_b}{\delta}\right)^{\frac{1}{\alpha}} - 1\right]; \quad \bar{f} = \frac{k_a - k_b}{2}\left[\frac{(1+2u_0)^{1-\alpha} - 1}{1-\alpha}\right] \tag{10}$$

where δ denotes a geometric tolerance which is usually set equal to 10^{-20}.

A physical interpretation of the parameters is schematized in Figure 1. Specifically, the hysteresis loop presents an initial stiffness k_a and asymptotically tends to a boundary line or curve with vertical intercept at $\bar{f}$. If β_1 and β_2 are both zero, the loop tends to the asymptotic lines with tangent k_b represented in blue in Figure 1. Parameters β_1 and β_2 introduce a curvature of the asymptotic boundary by means of a 5th degree polynomial function (see, e.g., the red curves in Figure 1).

The auxiliary variable u_0 is defined so that if displacement equal to $2u_0$ is applied to a load-displacement state belonging to one of the boundary curves, the distance between the new load-displacement state and the boundary curve is equal to the tolerance δ.

To determine the closed-form expression of the load, a history variable u_j is necessary:

$$u_j = u_c + s_t\,(1+2u_0) - s_t\left\{\frac{s_t\,(1-\alpha)}{k_a - k_b}\left[f_c - \beta_1 u_c^3 - \beta_2 u_c^5 - k_b u_c - s_t\bar{f} + (k_a - k_b)\,\frac{(1+2u_0)^{1-\alpha}}{s_t\,(1-\alpha)}\right]\right\}^{\frac{1}{1-\alpha}} \tag{11}$$

where u_c denotes the current displacement and s_t the algebraic sign of the velocity.

Denoting by f_c the force associated with the displacement u_c and by u_t a *trial displacement*, the trial force is given by:

$$f\,(u_t, u_c,\, s_t) = \beta_1 u_t^3 + \beta_2 u_t^5 + k_b u_t + (k_a - k_b)\left[\frac{(1+s_t u_t - s_t u_j + 2u_0)^{1-\alpha}}{s_t\,(1-\alpha)} - \frac{(1+2u_0)^{1-\alpha} - 1}{1-\alpha}\right] + s_t\bar{f} \tag{12}$$

Figure 2. Riace Bronze A resting on a seismic-isolated basement.

Within the TELM framework, the sensitivity of the trial force must be computed with respect to quantities related to the sequence of acceleration pulses. In particular, the derivatives of the displacements u_c and u_t are computed by the DDM procedure disposing of the derivative of $f\left(u_t, u_c, s_t\right)$ with respect to such displacements as well as with respect to f_c.

Derivatives of the auxiliary parameters are zero while those of the history variable turn out to be:

$$\frac{\partial u_j}{\partial f_c} = 1 - \frac{s_t^2}{k_a - k_b}\left\{\frac{s_t\left(1-\alpha\right)}{k_a - k_b}\left[f_c - \beta_1 u_c^3 - \beta_2 u_c^5 - k_b u_c - s_t \bar{f} + \left(k_a - k_b\right)\frac{\left(1+2u_0\right)^{1-\alpha}}{s_t\left(1-\alpha\right)}\right]\right\}^{\frac{\alpha}{1-\alpha}} \tag{13}$$

$$\frac{\partial u_j}{\partial u_c} = \frac{\partial u_j}{\partial f_c}\left[-3\beta_1 u_c^2 - 5\beta_2 u_c^4 - k_b\right] \tag{14}$$

while the derivatives of the trial force are:

$$\frac{\partial f\left(u_t, u_c, s_t\right)}{\partial f_c} = -\frac{\partial u_j}{\partial f_c}\left(k_a - k_b\right)\left[1 + s_t u_t - s_t u_j + 2u_0\right]^{-\alpha} \tag{15}$$

$$\frac{\partial f\left(u_t, u_c, s_t\right)}{\partial u_c} = -\frac{\partial u_j}{\partial u_c}\left(k_a - k_b\right)\left[1 + s_t u_t - s_t u_j + 2u_0\right]^{-\alpha} \tag{16}$$

and

$$\frac{\partial f\left(u_t, u_c, s_t\right)}{\partial u_t} = 3\beta_1 u_t^2 + 5\beta_2 u_t^4 + k_b + \left(k_a - k_b\right)\left[1 + s_t u_t - s t u_j + 2u_0\right]^{-\alpha} \tag{17}$$

The algebraic material has been implemented in OpenSees, an object oriented and open-source framework for finite element analysis in which the DDM and the TELM algorithm have been implemented within a structural reliability environment (Haukaas and Der Kiureghian, 2006).

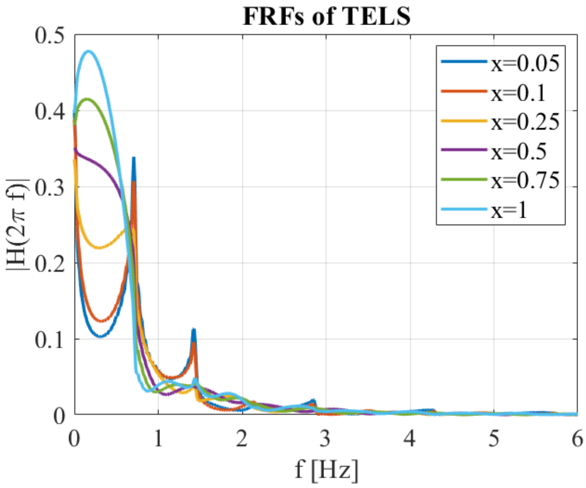

Figure 3. Frequency response functions of the TELS.

4. TELM analysis of the Riace Bronze A

One of the most appealing applications of TELM concerns seismic isolation since the safety of isolated structures can be easily analyzed by computing a single response of interest. Moreover, significant applications of base isolation have been recently developed for archaeological artifacts, e.g. slender statues, because of their vulnerability with respect to seismic actions (Calió and Marletta, 2004; Zuccaro et al., 2019).

In particular, base isolation has recently implemented on the Riace Bronzes (De Canio, 2012) in order to avoid vibrations induced by earthquakes. Bronze A, shown in Figure 2, has been chosen as benchmark for testing the implementation of the algebraic material in TELM.

The bronze, whose inertial properties are reported in (Pellecchia et al., 2020), is constrained to a marble basement equipped with elastomeric bearings. The basement-statue system has a mass of 4235 kg while the isolation device has been modeled by a truss element, having unit length and area of $0.1\,m^2$, made of the algebraic material with parameters reported in Table I.

Table I. Parameter set used in the numerical application.

k_a, $[kN/m]$	k_b, $[kN/m]$	α	β_1, $[kN/m^3]$	β_2, $[kN/m^5]$
1714.95	85.74	25	0	0

Base excitation has been modeled by an unfiltered Gaussian white noise lasting $t_n = 25\,s$ and discretized with time step of $0.01\,s$ corresponding to a cutoff frequency of 50 Hz. The horizontal displacement of the basement has been adopted as response of interest of the TELM procedure for which a set of 20 increasing values of the threshold have been adopted.

The Tail-Equivalent Linearized System (TELS) has been defined by means of a set of Impulse Response Functions. For convenience, it is possible to represent the TELS functions in the frequency

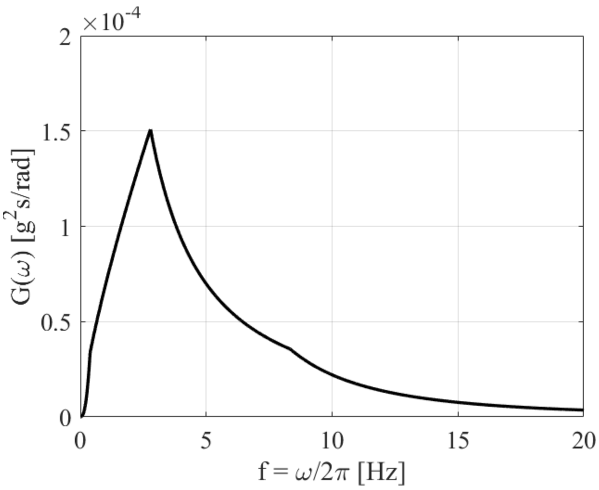

Figure 4. Single-sided Power Spectral Density adopted in the computations (Barone et al., 2015)

domain by computing their Fourier transform:

$$H^{\star}\left(x,\, t_n,\, \omega\right) = \int_0^{\infty} h^{\star}\left(x,\, t_n,\, \tau\right) \exp\left(-i\omega\tau\right)\, d\tau \tag{18}$$

Such Frequency Response Functions (FRFs) represent the amplitude of the steady-state response induced by an external excitation $f\left(t\right) = \exp\left(i\omega t\right)$. A limited set of FRFs belonging to the TELS is shown in Figure 3.

The main properties of the FRFs are consistent with what is usually expected by the application of TELM. Specifically, as the threshold increases, the peaks of the FRFs, representing the main frequencies of the system, tend to translate leftwards and the amplitude of each peak turns out to be inversely proportional to the threshold.

Such a behavior mainly depends on two aspects: the shifting to the lower frequencies depends on the smaller stiffness of the hysteretic material as long as the displacements increase; the smaller value of the FRF depends on the higher dissipation of elastic energy during the hysteresis as long as force and displacement become larger making the cycles wider.

To perform random vibration analysis, the base excitation power spectral density proposed by Barone et al. (2015) has been adopted. The main benefit of such a function consists in the fact that it is related to the design response spectrum functions provided by the Italian Structural Code so that its parameters can be calibrated consistently with the code hazard coefficients.

The single-sided power spectral density is defined by:

$$G_{FF}\left(\omega\right) = \begin{cases} G_0 \left(\frac{\omega_D}{\omega_C}\right)^{e_2} \left(\frac{\omega}{\omega_D}\right)^{e_1} & 0 \leq \omega \leq \omega_D \\ G_0 \left(\frac{\omega}{\omega_C}\right)^{e_2} & \omega_D \leq \omega \leq \omega_C \\ G_0 \left(\frac{\omega}{\omega_C}\right)^{e_3} & \omega_C \leq \omega \leq \omega_B \\ G_0 \left(\frac{\omega_B}{\omega_C}\right)^{e_3} \left(\frac{\omega}{\omega_B}\right)^{e_4} & \omega \geq \omega_B \end{cases} \tag{19}$$

where $\omega = 2\pi/T$ is the pulsation, G_0 is the intensity, ω_B, ω_C and ω_D are the pulsations associated with standard code periods T_B, T_C and T_D, respectively, and $e_1 \dots e_4$ are exponents calibrated with respect to the design response spectrum.

Coefficients of the power spectral density reported in Table II have been chosen according to (Barone et al., 2015) and are relevant to the Messina Strait area. The function is represended in Figure 4.

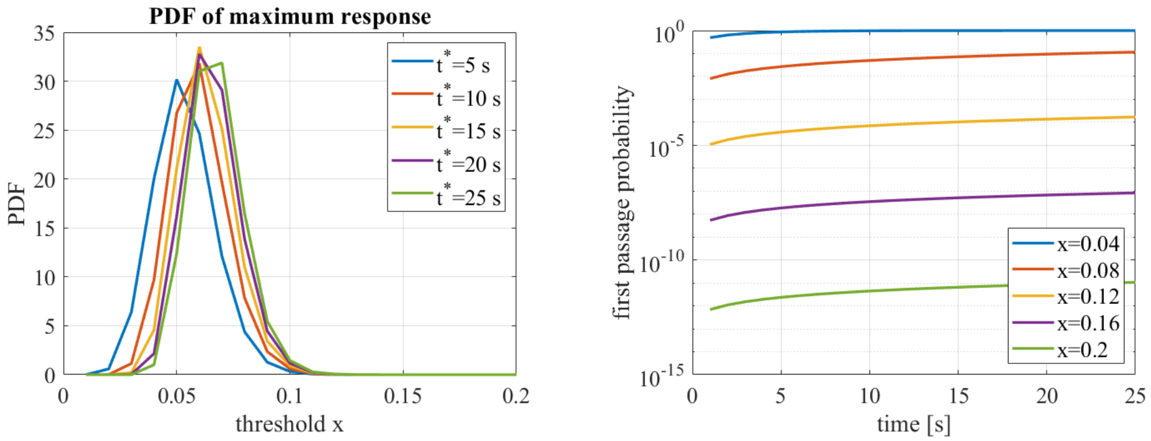

Figure 5. Probability density function of the maximum response and First Passage Probability of the TELS.

A series-system analysis (Der Kiureghian and Fujimura, 2009) permits to compute the statistics of the maximum response with respect to different time intervals $t^\star$ and to the First-Passage Probability of the structural system, both reported in Figure 5.

The probability density functions of the maximum response show that the procedure is capable of determining non-Gaussian distributions. Moreover, as larger time intervals are adopted, the modal value of the PDF increases as well as the corresponding maximum of the probability density while the standard deviation decreases.

In order to check the effectiveness of the isolation technique for seismic protection, it is useful to compute the statistics of the acceleration of the isolated block. In fact, we remind that the Riace Bronzes were isolated in order to reduce vibrations induced by seismic motions.

To this purpose, the peak ground acceleration of the base motion, equal to 0.248g, can be compared with the square-rooted mean square $\sigma_{\ddot{X}\ddot{X}}$ of the second derivative of the response of interest. Referring to (Lutes and Sarkani, 2004) for further details, we recall that the mean-square of the (acceleration) process representing the second derivative of the response can be computed

Table II. Parameters of the adopted Power Spectral Density.

G_0	T_B	T_C	T_D	e_1	e_2	e_3	e_4
$[g^2 s/rad]$	$[s]$	$[s]$	$[s]$	–	–	–	–
$1.51 \cdot 10^{-4}$	0.12	0.359	2.592	2.5094	0.7594	-1.3177	-2.6209

as:

$$\sigma^2_{\ddot{X}\ddot{X}}(\omega,\, x,\, t_n) = \int_0^{\infty} G_{FF}(\omega)\, |H^{\star}(x,\, t_n,\, \omega)|^2\, \omega^4\, d\omega \tag{20}$$

where dependency on x and t_n is due to the fact that such a parameter is relevant to the linearized system.

Figure 6 shows the values of such acceleration's root-mean-square as function of the threshold. Actually, because of the isolation system, the acceleration acting at the base of the statue is at least 30% smaller than the ground acceleration, this confirming the convenience in adopting such a technique. However, we remark that such a comparison is purely qualitative since the constitutive parameters of the seismic isolating devices have not been determined by a proper design procedure; actually the purpose of the present application is to check the effectiveness of the the algebraic material sensitivity within the TELM context.

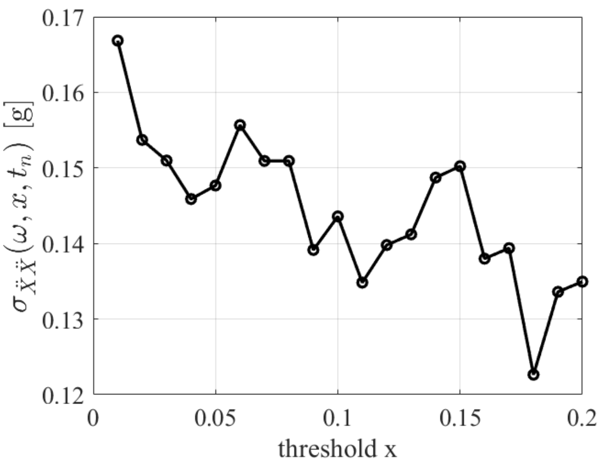

Figure 6. Square root of the mean-square of the acceleration of the isolated block as function of the threshold.

5. Conclusions and future work

The implementation of the sensitivity of a recently formulated phenomenological constitutive model has been presented. Such a constitutive model is capable of reproducing several typologies of hysteretic loops by means of a closed-form analytical expression and does not require iterative procedures to determine the response.

The sensitivity of the constitutive model, intended as the derivative of the response with respect to all quantities depending by the base acceleration, namely, the displacement and the initial force, has been analytically computed and implemented in OpenSees, an open source framework for finite element analysis. Within the context of a Direct Differentiation Method, the implemented material can be used in Tail Equivalent Linearization

Numerical application concerning the random vibration analysis of a base-isolated archaeological artifact shows the effectiveness of the implementation and of the whole procedure.

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