Reliability Analysis of Randomly Excited Structures with Interval Stiffness Parameters via Sensitivity Analysis

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Abstract: The present study focuses on reliability analysis of linear discretized structures with interval stiffness parameters subjected to stationary Gaussian multi-correlated random excitation. The reliability function for the *extreme value* stress process is evaluated in the framework of the *first-passage* theory. Such a function turns out to have an interval nature due to the uncertainty affecting the stiffness parameters. The aim of the analysis is the evaluation of the bounds of the *interval reliability function* which provides a range of structural performance. The range of stress-related quantities may be significantly overestimated as a consequence of the so-called *dependency phenomenon*. To reduce overestimation, a sensitivity-based procedure is proposed in the context of the *Improved Interval Analysis* via *Extra Unitary Interval*. The main advantage of this procedure is the capability of providing appropriate combinations of the endpoints of the uncertain parameters which yield accurate estimates of the bounds of the *interval reliability function* for the *extreme value* stress process.

A wind-excited steel telecommunication antenna mast with uncertain Young's modulus is analyzed to demonstrate the accuracy and efficiency of the proposed method.

Keywords: Interval reliability function; Interval analysis, Sensitivity analysis.

1. Introduction

The actual values of parameters involved in any engineering design are affected by several sources of uncertainty ensuing from manufacturing inaccuracies, model or measurement errors etc. (see e.g., Ayyub and Klir, 2006; Der Kiureghian, 2008; Corotis, 2015). Such uncertainties have been traditionally taken into account in structural reliability analysis by applying well-established probabilistic approaches. Furthermore, failure probabilities are highly sensitive to the assumed probabilistic distribution of input parameters (Ben-Haim, 1994; Elishakoff, 1995). This entails that the outcomes of the classical probabilistic reliability analysis may be considered accurate only when sufficient information is available to define the probability density function of the uncertain parameters. Nowadays, it is largely recognized that, if only vague, incomplete or fragmentary data are available, the non-probabilistic approaches (see e.g., Moens and Vandepitte, 2005; Elishakoff and Ohsaki, 2010) are more appropriate to obtain reliable predictions of the safety level. This issue has been first addressed by Ben-Haim (Ben-Haim, 1994) who presented a non-probabilistic concept of reliability in the framework of the convex model of uncertainty. Besides uncertainty affecting the structural parameters, the inherent random nature of environmental loads, such as earthquake ground motion, sea waves or gusty winds (see e.g., Lutes and Sarkani, 1997; Roberts and Spanos, 2003; Li and Chen, 2009) has to be taken into account in the context of structural safety assessment. Studies relying on the traditional probabilistic uncertainty model have shown that reliability analysis becomes quite challenging when uncertainties affecting geometrical and/or mechanical properties and the random nature of excitations are simultaneously taken into account (see e.g., Gupta and Manohar, 2006; Goller et al., 2013).

To the best of the authors' knowledge, in literature only a few studies have focused on reliability analysis of structures subjected to random excitation with uncertain parameters described using non-probabilistic models. Among these, the most popular approach is the interval model which describes the uncertain parameters as interval variables with given lower bound and upper bound (Moore, 1966; Moore et al., 2009). Under this assumption, the statistics of structural response in the presence of random excitation have an interval nature and the measure of structural performance is provided by lower and upper values of the *interval reliability function*. In this framework, recently, Muscolino et al. (Muscolino et al., 2015; 2016a; 2016b) addressed the reliability analysis of linear discretized structures with interval stiffness properties subjected to stationary Gaussian random excitation by interval extension of the formulation of the *first-passage problem* (see e.g., Lutes and Sarkani, 1997). A selected displacement component has been assumed as critical response quantity and the uncertainties have been described by means of the *Improved Interval Analysis* via *Extra Unitary Interval* (*IIA* via *EUI*) (Muscolino and Sofi, 2012). The *IIA* via *EUI* has been introduced to limit the conservatism affecting computations based on the *Classical Interval Analysis* (*CIA*) (Moore, 1966; Moore et al., 2009). Such conservatism is caused by the so-called *dependency phenomenon* which is related to the inability of the *CIA* to treat multiple occurrences of the same interval variables in a mathematical expression as dependent ones.

In this paper, it is assumed that the structure fails as soon as a selected stress process at a critical location firstly exceeds a prescribed safe domain. In this case, the evaluation of the bounds of the *interval reliability function* is much more challenging due to the high overestimation of the range of stress-related interval functions which typically affects structural analysis based on the rules of the *CIA*. Indeed, stresses are more sensitive to the *dependency phenomenon* than displacements since their expression involves multiple occurrences of the same interval variables. Recently, this issue has been successfully addressed by Sofi et al. (2020) who proposed a sensitivity-based procedure for estimating the bounds of the *interval reliability function* for structures with interval axial stiffness. The main purpose of the present study is the extension of such sensitivity-based procedure to general finite element models involving uncertain stiffness properties. The formulation is developed in the context of the *first-passage* theory, under the Poisson assumption of independent up-crossings of a prescribed threshold (Lutes and Sarkani, 1997; Li and Chen, 2009). The issue of overestimation is tackled by describing interval uncertainties affecting the stiffness matrix of the structure through the *IIA* via *EUI* (Muscolino and Sofi, 2012). In order to ensure a conservative design, the worst case scenario corresponding to the lower bound of the *interval reliability function* has to be considered. The proposed method is able to identify through sensitivity analysis the set of uncertain parameters which lead to this scenario. Then, the lower bound of the *interval reliability function* is evaluated by performing a stochastic analysis of the randomly excited structure.

For validation purpose, a steel telecommunication antenna mast is selected as case-study. The accuracy of the proposed sensitivity-based procedure is demonstrated through appropriate comparisons with a combinatorial procedure, known as *vertex method* (Dong and Shah, 1987), which yields the exact bounds of the solution in the case of monotonic problems.

2. Problem Formulation

2.1. EQUATIONS OF MOTION

Let us consider a linear-elastic structure subjected to a stationary multi-correlated Gaussian stochastic process $\mathbf{F}(t)$. The structure is discretized into $N^{(e)}$ *finite elements* (*FE*s) resulting into a n-DOFs model. Young's modulus of the i-th *FE* is assumed uncertain and is described as an interval variable by means of the *Improved Interval Analysis* (*IIA*) via *Extra Unitary Interval* (*EUI*) (Muscolino and Sofi, 2012), i.e.

$$E^{(i)}(\alpha_i^I) = E_0^{(i)}\left(1+\alpha_i^I\right) = E_0^{(i)}\left(1+\Delta\alpha_i \hat{e}_i^I\right), \quad i=1,2,\ldots,r \tag{1}$$

where $r \le N^{(e)}$ denotes the number of *FE*s with uncertain stiffness; $\alpha_i^I = \Delta\alpha_i \hat{e}_i^I$ is a symmetric interval variable denoting the dimensionless fluctuation of Young's modulus around the nominal value $E_0^{(i)}$, with $\Delta\alpha_i$ the associated deviation amplitude, and $\hat{e}_i^I \triangleq [-1,+1]$ a particular unitary interval, called *EUI* (Muscolino and Sofi, 2012), which does not obey the rules of the *Classical Interval Analysis* (*CIA*) (Moore, 1966; Moore et al., 2009).

Taking into account Eq.(1), the elastic matrix of the i-th *FE* can be expressed as:

$$\mathbf{E}^{(i)}(\alpha_i^I) = \left(1+\Delta\alpha_i \hat{e}_i^I\right)\mathbf{E}_0^{(i)} \tag{2}$$

where $\mathbf{E}_0^{(i)}$ is the element elastic matrix with nominal Young's modulus $E_0^{(i)}$.

Let $\mathbf{x} = [x_1 \quad x_2 \quad x_3]^{\mathrm{T}}$ indicate the position vector of a generic point referred to a Cartesian coordinate system $O(x_1, x_2, x_3)$. Following the standard displacement-based *FE* formulation, the interval displacement field within the i-th *FE* can be approximated as follows (Sofi and Romeo, 2016; Sofi et al., 2019):

$$\mathbf{u}^{(i)}(\boldsymbol{\alpha}^I, \mathbf{x}, t) = \mathbf{N}^{(i)}(\mathbf{x})\, \mathbf{L}^{(i)}\, \mathbf{U}(\boldsymbol{\alpha}^I, t) \tag{3}$$

where $\mathbf{N}^{(i)}(\mathbf{x})$ denotes the shape-function matrix; and $\mathbf{L}^{(i)}$ a Boolean matrix defined so as to take into account the boundary conditions. In the previous equation, $\mathbf{U}(\boldsymbol{\alpha}^I, t)$ is the vector of global nodal displacements, which is an interval vector, depending both on time t and on the interval fluctuations collected into the vector $\boldsymbol{\alpha}^I = \left[\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}\right] \in \mathbb{IR}^r$, where the symbols $\underline{\boldsymbol{\alpha}}$ and $\overline{\boldsymbol{\alpha}}$ denote the vectors gathering the *lower bound* (*LB*) and *upper bound* (*UB*) of the symmetric interval parameters α_i^I, respectively. Due to the symmetry of α_i^I, the midpoint value vector of $\boldsymbol{\alpha}^I$ is null, while $\underline{\boldsymbol{\alpha}} \equiv -\Delta\boldsymbol{\alpha}$ and $\overline{\boldsymbol{\alpha}} \equiv \Delta\boldsymbol{\alpha}$, where the vector $\Delta\boldsymbol{\alpha}$ collects the deviation amplitudes $\Delta\alpha_i$ of α_i^I.

The strain-displacement equations and the linear-elastic constitutive equations yield the following expressions of the interval strain and stress fields within the i-th *FE*:

$$\begin{aligned}
&\boldsymbol{\varepsilon}^{(i)}(\boldsymbol{\alpha}^I, \mathbf{x}, t) = \mathbf{B}^{(i)}(\mathbf{x})\mathbf{L}^{(i)}\, \mathbf{U}(\boldsymbol{\alpha}^I, t);\\
&\boldsymbol{\sigma}^{(i)}(\boldsymbol{\alpha}^I, \mathbf{x}, t) = \mathbf{E}^{(i)}(\alpha_i^I)\boldsymbol{\varepsilon}^{(i)}(\boldsymbol{\alpha}^I, \mathbf{x}, t) = \left(1+\Delta\alpha_i \hat{e}_i^I\right)\mathbf{E}_0^{(i)}\, \mathbf{B}^{(i)}(\mathbf{x})\mathbf{L}^{(i)}\, \mathbf{U}(\boldsymbol{\alpha}^I, t)
\end{aligned} \tag{4a,b}$$

where $\mathbf{B}^{(i)}(\mathbf{x})$ is the strain-displacement matrix and the definition of the interval elastic matrix $\mathbf{E}^{(i)}(\alpha_i^I)$ in Eq. (2) has been taken into account. The stiffness matrix of the i-th *FE* is an interval matrix, formally analogous to the one pertaining to the deterministic *FE*, i.e.:

$$\mathbf{k}^{(i)}(\alpha_i^I) = \int_{V^{(i)}} \mathbf{B}^{(i)\mathrm{T}}(\mathbf{x})\mathbf{E}^{(i)}(\alpha_i^I)\mathbf{B}^{(i)}(\mathbf{x})\,\mathrm{d}V^{(i)} = \left(1+\Delta\alpha_i \hat{e}_i^I\right)\mathbf{k}_0^{(i)} \tag{5}$$

where $V^{(i)}$ is the volume of the i-th FE; $\mathbf{k}_0^{(i)} = \mathbf{k}^{(i)}(\alpha_i)\big|_{\alpha_i=0}$ is the nominal stiffness matrix and the interval elastic matrix is given by Eq. (2). In Eq. (5), the apex T denotes the transpose matrix operator.

Assuming for the sake of simplicity that all the quantities are referred to the global coordinate system, the standard assembly procedure yields the following interval global equations of motion:

$$\mathbf{M}\ddot{\mathbf{U}}(\boldsymbol{\alpha}^I,t) + \mathbf{C}(\boldsymbol{\alpha}^I)\dot{\mathbf{U}}(\boldsymbol{\alpha}^I,t) + \mathbf{K}(\boldsymbol{\alpha}^I)\mathbf{U}(\boldsymbol{\alpha}^I,t) = \mathbf{F}(t) \tag{6}$$

where a dot over a variable denotes differentiation with respect to time t; $\mathbf{M}$ is the mass matrix, and $\mathbf{K}(\boldsymbol{\alpha}^I)$ is the interval global stiffness defined as:

$$\mathbf{K}(\boldsymbol{\alpha}^I) = \mathbf{K}_0 + \sum_{i=1}^{r}\mathbf{K}_i \Delta\alpha_i \hat{e}_i^I; \quad \mathbf{K}_i = \frac{\partial \mathbf{K}(\boldsymbol{\alpha}^I)}{\partial \alpha_i}\bigg|_{\boldsymbol{\alpha}=\mathbf{0}} = \mathbf{L}^{(i)\mathrm{T}}\mathbf{k}_0^{(i)}\mathbf{L}^{(i)} \tag{7a,b}$$

where $\mathbf{K}_0 = \mathbf{K}(\boldsymbol{\alpha})\big|_{\boldsymbol{\alpha}=\mathbf{0}}$ and $\mathbf{K}_i$ are the nominal and deviation stiffness matrices, respectively. By adopting the Rayleigh model, the global damping matrix $\mathbf{C}(\boldsymbol{\alpha}^I)$ turns out to be an interval matrix as well, defined as:

$$\mathbf{C}(\boldsymbol{\alpha}^I) = c_0\mathbf{M} + c_1\mathbf{K}(\boldsymbol{\alpha}^I) = \mathbf{C}_0 + c_0\mathbf{M} + c_1\sum_{i=1}^{r}\mathbf{K}_i\Delta\alpha_i\hat{e}_i^I \tag{8}$$

where $\mathbf{C}_0 = c_0\mathbf{M} + c_1\mathbf{K}_0$ is the nominal damping matrix; c_0 and c_1 denote the Rayleigh damping constants, herein evaluated setting the uncertain parameters equal to their nominal values.

2.2. Interval Gaussian stochastic stationary response process

The external load vector $\mathbf{F}(t)$ in Eq. (6) can be expressed as sum of the mean-value $\boldsymbol{\mu}_{\mathbf{F}} = \mathrm{E}\langle \mathbf{F}(t)\rangle$, with $\mathrm{E}\langle\bullet\rangle$ denoting the stochastic average operator, plus a zero-mean random fluctuating component $\tilde{\mathbf{X}}_{\mathbf{F}}(t)$, i.e. $\mathbf{F}(t) = \boldsymbol{\mu}_{\mathbf{F}} + \tilde{\mathbf{X}}_{\mathbf{F}}(t)$. Thus, in the frequency domain, the full probabilistic characterization of the external load vector $\mathbf{F}(t)$ requires the knowledge of the mean-value vector, $\boldsymbol{\mu}_{\mathbf{F}} = \mathrm{E}\langle \mathbf{F}(t)\rangle$, and of the one-sided *Power Spectral Density* (*PSD*) function matrix $\mathbf{G}_{\tilde{\mathbf{X}}_\mathbf{F}\tilde{\mathbf{X}}_\mathbf{F}}(\omega)$ of the fluctuating component $\tilde{\mathbf{X}}_{\mathbf{F}}(t)$.

The interval stationary Gaussian stochastic response process $\mathbf{U}(\boldsymbol{\alpha}^I,t)$, ruled by the equations of motion in Eq.(6), is completely characterized in the frequency domain by the interval mean-value vector:

$$\boldsymbol{\mu}_{\mathbf{U}}(\boldsymbol{\alpha}^I) = \mathrm{E}\left\langle \mathbf{U}^I(t)\right\rangle = \mathbf{K}^{-1}(\boldsymbol{\alpha}^I)\boldsymbol{\mu}_{\mathbf{F}} \tag{9}$$

and by the interval one-sided *PSD* function matrix, $\mathbf{G}_{\mathbf{UU}}(\boldsymbol{\alpha}^I,\omega)$, defined as follows:

$$\mathbf{G}_{\mathbf{UU}}(\boldsymbol{\alpha}^I,\omega) = \mathbf{H}^*(\boldsymbol{\alpha}^I,\omega)\mathbf{G}_{\tilde{\mathbf{X}}_\mathbf{F}\tilde{\mathbf{X}}_\mathbf{F}}(\omega)\mathbf{H}^{\mathrm{T}}(\boldsymbol{\alpha}^I,\omega) \tag{10}$$

where the asterisk means complex conjugate, and $\mathbf{H}(\boldsymbol{\alpha}^I,\omega)$ is the interval *Frequency Response Function* (*FRF*) matrix given by:

$$\mathbf{H}(\boldsymbol{\alpha}^I,\omega) = \left[-\omega^2\mathbf{M} + \mathrm{j}\omega\mathbf{C}(\boldsymbol{\alpha}^I) + \mathbf{K}(\boldsymbol{\alpha}^I)\right]^{-1} \tag{11}$$

with $\mathrm{j} = \sqrt{-1}$ denoting the imaginary unit.

Attention is herein focused on the j-th component of the interval stationary Gaussian random stress vector process $\boldsymbol{\sigma}^{(h)}(\boldsymbol{\alpha}^I,\mathbf{x},t)$ (see Eq.4b) at a given position $\mathbf{x}$ within the h-th *FE*:

$$Y_j^{(h)}(\boldsymbol{\alpha}^I,t) \equiv \sigma_j^{(h)}(\boldsymbol{\alpha}^I,\mathbf{x},t) = \left(1 + \Delta\alpha_h\hat{e}_h^I\right)\mathbf{r}_j^{(h)\mathrm{T}}(\mathbf{x})\mathbf{U}(\boldsymbol{\alpha}^I,t) \tag{12}$$

where $\mathbf{r}_j^{(h)\mathrm{T}}(\mathbf{x})$ is the j-th row of the $n\times n$ matrix $\mathbf{E}_0^{(h)}\mathbf{B}^{(h)}(\mathbf{x})\mathbf{L}^{(h)}$. To simplify the notation, the dependence of $\mathbf{r}_j^{(h)}$ on $\mathbf{x}$ is hereinafter omitted since the stress is evaluated at a given position. By inspection of Eq.(12), it is readily inferred that the interval variable $\alpha_h^I = \Delta\alpha_h \hat{e}_h^I$ occurs more than once. It follows that quantities related to interval stresses are more vulnerable to the *dependency phenomenon* than displacements.

The interval stationary Gaussian stress random process in Eq.(12) can be expressed as sum of the interval mean-value, $\mu_{Y_j^{(h)}}(\boldsymbol{\alpha}^I)$, plus a zero-mean random fluctuation, $\tilde{Y}_j^{(h)}(\boldsymbol{\alpha}^I,t)$, i.e. $Y_j^{(h)}(\boldsymbol{\alpha}^I,t) = \mu_{Y_j^{(h)}}(\boldsymbol{\alpha}^I) + \tilde{Y}_j^{(h)}(\boldsymbol{\alpha}^I,t)$. Its complete probabilistic characterization in the frequency domain thus requires the knowledge of the interval mean-value, $\mu_{Y_j^{(h)}}(\boldsymbol{\alpha}^I)$, and of the interval one-sided *PSD* function, $G_{\tilde{Y}_j^{(h)}\tilde{Y}_j^{(h)}}(\boldsymbol{\alpha}^I,\omega) \equiv G_{Y_j^{(h)}Y_j^{(h)}}(\boldsymbol{\alpha}^I,\omega)$, of the zero-mean random fluctuation process $\tilde{Y}_j^{(h)}(\boldsymbol{\alpha}^I,t)$. The interval mean-value $\mu_{Y_j^{(h)}}(\boldsymbol{\alpha}^I)$ can be evaluated by applying the stochastic average operator to both sides of Eq.(12), i.e.:

$$\mu_{Y_j^{(h)}}(\boldsymbol{\alpha}^I) = \mathrm{E}\left\langle Y_j^{(h)}(\boldsymbol{\alpha}^I,t)\right\rangle = \left(1+\Delta\alpha_h \hat{e}_h^I\right)\mathbf{r}_j^{(h)\mathrm{T}}\boldsymbol{\mu}_{\mathbf{U}}(\boldsymbol{\alpha}^I) \tag{13}$$

where $\boldsymbol{\mu}_{\mathbf{U}}(\boldsymbol{\alpha}^I)$ is the interval mean-value of the displacement vector given in Eq. (9).

Based on Eqs. (10) and (12), the interval one-sided *PSD* function $G_{Y_j^{(h)}Y_j^{(h)}}(\boldsymbol{\alpha}^I,\omega)$ of the interval stress random process $Y_j^{(h)}(\boldsymbol{\alpha}^I,t)$ takes the following form:

$$G_{Y_j^{(h)}Y_j^{(h)}}(\boldsymbol{\alpha}^I,\omega) = \left(1+\Delta\alpha_h \hat{e}_h^I\right)^2 \mathbf{r}_j^{(h)\mathrm{T}}\,\mathbf{H}^*(\boldsymbol{\alpha}^I,\omega)\mathbf{G}_{\tilde{\mathbf{X}}_\mathbf{F}\tilde{\mathbf{X}}_\mathbf{F}}(\omega)\mathbf{H}^{\mathrm{T}}(\boldsymbol{\alpha}^I,\omega)\mathbf{r}_j^{(h)}. \tag{14}$$

Notice that the same interval variables occur many times in the previous equation. This implies that the statistics of the interval stress random process may be affected by very serious overestimation due to the *dependency phenomenon*.

3. Bounds of the Interval Reliability Function

3.1. Interval reliability function

Failure or unsatisfactory performance of a structural system is herein identified with the *first-passage failure*, which occurs when the *extreme value* random process for some response measure (e.g., displacement, strain or stress) firstly exceeds a prescribed safe domain within a specified time interval $[0,T]$. Specifically, it is assumed that the structure fails in a *first-passage* sense if the j-th component of the interval stationary Gaussian random stress vector process $\boldsymbol{\sigma}^{(h)}(\boldsymbol{\alpha}^I,\mathbf{x},t)$ (see Eq.4b) at a given position $\mathbf{x}$ within the h-th *FE*, i.e. $Y_j^{(h)}(\boldsymbol{\alpha}^I,t)$ (see Eq. (12)), reaches a prescribed threshold.

The *extreme value* random process of $Y_j^{(h)}(\boldsymbol{\alpha}^I,t)$, over the time interval $[0,T]$, has an interval nature and is mathematically defined as:

$$Y_{j,\max}^{(h)}(\boldsymbol{\alpha}^I,T) = \max_{0\le t\le T}\left|Y_j^{(h)}(\boldsymbol{\alpha}^I,t)\right| \tag{15}$$

where the symbol $|\bullet|$ denotes absolute value. The probability that $Y^{(h)}_{j,\max}(\boldsymbol{\alpha}^I,T)$ is equal to or less than the critical level $b>0$ within the time interval $[0,T]$ is defined by the *cumulative distribution function* (*CDF*), also called *reliability function*, which has an interval nature as well:

$$L_{Y^{(h)}_{j,\max}}(\boldsymbol{\alpha}^I, b,T) = \Pr\left[Y^{(h)}_{j,\max}(\boldsymbol{\alpha}^I,T)\le b\right] = \left[\underline{L}_{Y^{(h)}_{j,\max}}(b,T),\quad \overline{L}_{Y^{(h)}_{j,\max}}(b,T)\right]. \tag{16}$$

The *LB* (or right bound) and *UB* (or left bound) of the interval *CDF* define a probability box (p-box) (Ferson et al., 2003) representative of the range of structural performance under prescribed variations of the uncertain parameters within their respective intervals.

If the Poisson assumption of independent up-crossings of a prescribed threshold is applied, then the *interval CDF* for unit initial probability can be expressed as (see e.g., Lutes and Sarkani, 1997):

$$L_{Y^{(h)}_{j,\max}}(\boldsymbol{\alpha}^I,b,T) = \Pr\left[Y^{(h)}_{j,\max}(\boldsymbol{\alpha}^I,T)\le b\right] \approx \exp\left\{-T\,\nu^+_{Y^{(h)}_j}(\boldsymbol{\alpha}^I)\exp\left[-\frac{\left(b-\left|\mu_{Y^{(h)}_j}(\boldsymbol{\alpha}^I)\right|\right)^2}{2\lambda_{0,Y^{(h)}_j}(\boldsymbol{\alpha}^I)}\right]\right\} \tag{17}$$

where

$$\nu^+_{Y^{(h)}_j}(\boldsymbol{\alpha}^I) = \frac{1}{2\,\pi}\sqrt{\frac{\lambda_{2,Y^{(h)}_j}(\boldsymbol{\alpha}^I)}{\lambda_{0,Y^{(h)}_j}(\boldsymbol{\alpha}^I)}} \tag{18}$$

is the mean up-crossing rate at level $\left|\mu_{Y^{(h)}_j}(\boldsymbol{\alpha}^I)\right|$; $\lambda_{0,Y^{(h)}_j}(\boldsymbol{\alpha}^I)$ and $\lambda_{2,Y^{(h)}_j}(\boldsymbol{\alpha}^I)$ are the interval spectral moments of zero- and second-order, respectively, of the interval stress random process, $Y^{(h)}_j(\boldsymbol{\alpha}^I,t)$, given by

$$\lambda_{\ell,Y^{(h)}_j}(\boldsymbol{\alpha}^I) = \int_0^\infty \omega^\ell\, G_{Y^{(h)}_j Y^{(h)}_j}(\boldsymbol{\alpha}^I,\omega)\,\mathrm{d}\omega, \quad (\ell = 0, 2) \tag{19}$$

where $G_{Y^{(h)}_j Y^{(h)}_j}(\boldsymbol{\alpha}^I,\omega)$ is the one-sided interval *PSD* function of $Y^{(h)}_j(\boldsymbol{\alpha}^I,t)$ defined in Eq.(14).

3.2. PROPOSED SENSITIVITY-BASED PROCEDURE

In the context of the *first-passage* theory, the aim of interval reliability analysis is the evaluation of the *LB* and *UB* of the *interval reliability function* defined by Eq. (17). Since interval stress-related quantities are more affected by overestimation than displacements, the bounds of the *interval reliability function* of the *extreme value* stress random process $Y^{(h)}_{j,\max}(\boldsymbol{\alpha}^I,T)$ are herein evaluated by applying a sensitivity-based procedure.

The key idea of this procedure is to perform a preliminary sensitivity analysis to identify suitable combinations of the values of the interval parameters which provide accurate estimates of the *LB* and *UB* of the interval *CDF*. This approach requires the evaluation of the sensitivity of the *CDF* of the random process $Y^{(h)}_{j,\max}(\boldsymbol{\alpha}^I,T)$ with respect to the uncertain parameters $\alpha_i \in \alpha_i^I$ $(i=1,2,\ldots,r)$ by direct differentiation of Eq. (17) which leads to (Sofi et al., 2020):

$$
\begin{aligned}
S_{L_{Y_{j,\max}^{(h)}},i}(b,T) &= \left.\frac{\partial L_{Y_{j,\max}^{(h)}}(\boldsymbol{\alpha},b,T)}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}} \\
&= C_{Y_j^{(h)}}(b,T)\left\{2\left(b-\left|\mu_{Y_j^{(h)}}^{(0)}\right|\right)\frac{\left|\mu_{Y_j^{(h)}}^{(0)}\right|}{\mu_{Y_j^{(h)}}^{(0)}}S_{\mu_{Y_j^{(h)}},i}+\left[\frac{\left(b-\left|\mu_{Y_j^{(h)}}^{(0)}\right|\right)^2}{\lambda_{0,Y_j^{(h)}}^{(0)}}-1\right]S_{\lambda_{0,Y_j^{(h)}},i}+\frac{\lambda_{0,Y_j^{(h)}}^{(0)}}{\lambda_{2,Y_j^{(h)}}^{(0)}}S_{\lambda_{2,Y_j^{(h)}},i}\right\}
\end{aligned}
\tag{20}
$$

where $\mu_{Y_j^{(h)}}^{(0)}$ and $\lambda_{\ell,Y_j^{(h)}}^{(0)}$ $(\ell=0,2)$ denote the nominal mean-value and spectral moments of the selected stress process, given by Eqs. (13) and (19) with $\boldsymbol{\alpha}=\mathbf{0}$; the function $C_{Y_j^{(h)}}(b,T)$ is defined as follows:

$$
C_{Y_j^{(h)}}(b,T)=-\frac{T}{4\,\pi}\frac{\lambda_{2,Y_j^{(h)}}^{(0)}}{\lambda_{0,Y_j^{(h)}}^{(0)}}\frac{L_{Y_{j,\max}^{(h)}}^{(0)}(b,T)}{\sqrt{\lambda_{0,Y_j^{(h)}}^{(0)}\;\lambda_{2,Y_j^{(h)}}^{(0)}}}\exp\left[-\frac{\left(b-\left|\mu_{Y_j^{(h)}}^{(0)}\right|\right)^2}{2\lambda_{0,Y_j^{(h)}}^{(0)}}\right]
\tag{21}
$$

with $L_{Y_{j,\max}^{(h)}}^{(0)}(b,T)=L_{Y_{j,\max}^{(h)}}(\boldsymbol{\alpha},b,T)\Big|_{\boldsymbol{\alpha}=\mathbf{0}}$ denoting the nominal *CDF*. Furthermore, in Eq.(20) $S_{\mu_{Y_j^{(h)}},i}$ is the sensitivity of the mean-value $\mu_{Y_j^{(h)}}(\boldsymbol{\alpha})$ of the interval stress random process (see Eq. (13)) with respect to the uncertain parameter α_i, which can be evaluated as:

$$
S_{\mu_{Y_j^{(h)}},i}=\begin{cases}
\left.\dfrac{\partial \mu_{Y_j^{(h)}}(\boldsymbol{\alpha})}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}} = \mu_{Y_j^{(h)}}^{(0)}-\mathbf{r}_j^{(h)\mathrm{T}}\,\mathbf{K}_0^{-1}\mathbf{K}_h\,\mathbf{K}_0^{-1}\,\boldsymbol{\mu}_{\mathbf{F}}, & \text{if } i=h \\
\left.\dfrac{\partial \mu_{Y_j^{(h)}}(\boldsymbol{\alpha})}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}} = -\mathbf{r}_j^{(h)\mathrm{T}}\,\mathbf{K}_0^{-1}\mathbf{K}_i\,\mathbf{K}_0^{-1}\,\boldsymbol{\mu}_{\mathbf{F}}, & \text{if } i\neq h
\end{cases}
\tag{22a,b}
$$

where $\mathbf{K}_0$ is the nominal stiffness matrix; $\mathbf{K}_i$ is given by Eq. (7b); and $\mathbf{K}_0^{-1}\mathbf{K}_i\,\mathbf{K}_0^{-1}\,\boldsymbol{\mu}_{\mathbf{F}}$ is the $i-$th sensitivity of $\boldsymbol{\mu}_{\mathbf{U}}(\boldsymbol{\alpha})$ (derived by Eq. (9)). Finally, in Eq.(20), $S_{\lambda_{\ell,Y_j^{(h)}},i}$ denotes the sensitivity of the spectral moment of order ℓ of the interval stress random process $Y_j^{(h)}(\boldsymbol{\alpha}^I,t)$ with respect to the $i-$th parameter α_i:

$$
S_{\lambda_{\ell,Y_j^{(h)}},i}=\left.\frac{\partial \lambda_{\ell,Y_j^{(h)}}(\boldsymbol{\alpha})}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}}=\int_0^{\infty}\omega^{\ell}S_{G_{Y_j^{(h)}Y_j^{(h)}},i}(\omega)\,\mathrm{d}\omega,\quad (\ell=0,2).
\tag{23}
$$

In the previous expression, $S_{G_{Y_j^{(h)}Y_j^{(h)}},i}(\omega)$ is the $i-$th sensitivity of the one-sided *PSD* function $G_{Y_j^{(h)}Y_j^{(h)}}(\boldsymbol{\alpha}^I,\omega)$ (see Eq. (14)) which can be evaluated as:

$$S_{G_{Y_j^{(h)}Y_j^{(h)}},i}(\omega)=\begin{cases}\left.\dfrac{\partial G_{Y_j^{(h)}Y_j^{(h)}}(\boldsymbol{\alpha},\omega)}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}}=2G^{(0)}_{Y_j^{(h)}Y_j^{(h)}}(\omega)+\mathbf{r}_j^{(h)\mathrm{T}}\mathbf{P}_h(\omega)\mathbf{r}_j^{(h)}, & \text{if } i=h;\\ \left.\dfrac{\partial G_{Y_j^{(h)}Y_j^{(h)}}(\boldsymbol{\alpha},\omega)}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}}=\mathbf{r}_j^{(h)\mathrm{T}}\mathbf{P}_i(\omega)\mathbf{r}_j^{(h)}, & \text{if } i\neq h\end{cases}\qquad(24\text{a,b})$$

where $G^{(0)}_{Y_j^{(h)}Y_j^{(h)}}(\omega)$ is the nominal one-sided *PSD* function of the selected stress process, given by Eq. (14) with $\boldsymbol{\alpha}=\mathbf{0}$. Furthermore, in the previous equations the matrix $\mathbf{P}_i(\omega)$ is defined as:

$$\mathbf{P}_i(\omega)=\mathbf{S}_i^*(\omega)\mathbf{G}_{\tilde{\mathbf{X}}_\mathbf{F}\tilde{\mathbf{X}}_\mathbf{F}}(\omega)\mathbf{H}_0^{\mathrm{T}}(\omega)+\mathbf{H}_0^*(\omega)\mathbf{G}_{\tilde{\mathbf{X}}_\mathbf{F}\tilde{\mathbf{X}}_\mathbf{F}}(\omega)\mathbf{S}_i^{\mathrm{T}}(\omega)\qquad(25)$$

with

$$\mathbf{S}_i(\omega)=\left.\frac{\partial\mathbf{H}(\boldsymbol{\alpha},\omega)}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}}=-(1+\mathrm{j}c_1\,\omega)\,\mathbf{H}_0(\omega)\,\mathbf{K}_i\,\mathbf{H}_0(\omega)\qquad(26)$$

where $\mathbf{K}_i$ is given by Eq.(7b), and $\mathbf{H}_0(\omega)$ is the nominal *FRF* matrix given as:

$$\mathbf{H}_0(\omega)=\left[-\omega^2\mathbf{M}+\mathrm{j}\omega\mathbf{C}_0+\mathbf{K}_0\right]^{-1}.\qquad(27)$$

The knowledge of the sensitivity $S_{L_{Y_{j,\max}^{(h)}},i}$ defined in Eq.(20) allows us to predict the influence of a small variation of the $i-$th uncertain parameter α_i on the *interval reliability function* $L_{Y_{j,\max}^{(h)}}(\boldsymbol{\alpha}^I,b,T)$. Specifically, within a small range around $\boldsymbol{\alpha}=\mathbf{0}$, $L_{Y_{j,\max}^{(h)}}(\boldsymbol{\alpha}^I,b,T)$ is a monotonic increasing or decreasing function of α_i depending on whether $S_{L_{Y_{j,\max}^{(h)}},i}>0$ or $S_{L_{Y_{j,\max}^{(h)}},i}<0$, and its bounds, therefore, correspond to suitable combinations of the endpoints of the uncertain parameters. Relying on the monotonic increasing or decreasing behaviour predicted by studying the sign of sensitivities, the combinations of the extreme values of the uncertain parameters, which yield accurate estimates of the *LB* and *UB* of the *interval reliability function* $L_{Y_{j,\max}^{(h)}}(\boldsymbol{\alpha}^I,b,T)$, denoted as $\alpha^{(\mathrm{LB})}_{Y_{j,\max}^{(h)},i}$ and $\alpha^{(\mathrm{UB})}_{Y_{j,\max}^{(h)},i}$, $(i=1,2,\ldots,r)$, can be identified as follows:

$$\begin{aligned}&\text{if } S_{L_{Y_{j,\max}^{(h)}},i}>0,\ \text{ then }\ \alpha^{(\mathrm{UB})}_{Y_{j,\max}^{(h)},i}=\overline{\alpha}_i,\quad \alpha^{(\mathrm{LB})}_{Y_{j,\max}^{(h)},i}=\underline{\alpha}_i;\\&\text{if } S_{L_{Y_{j,\max}^{(h)}},i}<0,\ \text{ then }\ \alpha^{(\mathrm{UB})}_{Y_{j,\max}^{(h)},i}=\underline{\alpha}_i,\quad \alpha^{(\mathrm{LB})}_{Y_{j,\max}^{(h)},i}=\overline{\alpha}_i.\end{aligned}\qquad(28\text{a,b})$$

Such combinations can be collected into the following two vectors:

$$\begin{aligned}\boldsymbol{\alpha}^{(\mathrm{LB})}_{Y_{j,\max}^{(h)}}&=\left[\alpha^{(\mathrm{LB})}_{Y_{j,\max}^{(h)},1}\quad\alpha^{(\mathrm{LB})}_{Y_{j,\max}^{(h)},2}\quad\cdots\quad\alpha^{(\mathrm{LB})}_{Y_{j,\max}^{(h)},r}\right]^{\mathrm{T}};\\\boldsymbol{\alpha}^{(\mathrm{UB})}_{Y_{j,\max}^{(h)}}&=\left[\alpha^{(\mathrm{UB})}_{Y_{j,\max}^{(h)},1}\quad\alpha^{(\mathrm{UB})}_{Y_{j,\max}^{(h)},2}\quad\cdots\quad\alpha^{(\mathrm{UB})}_{Y_{j,\max}^{(h)},r}\right]^{\mathrm{T}}.\end{aligned}\qquad(29\text{a,b})$$

Finally, the *LB* and *UB* of the *interval reliability function* for the *extreme value* stress random process $Y_{j,\max}^{(h)}(\boldsymbol{\alpha}^I,T)$ can be obtained by evaluating Eq.(17) for $\boldsymbol{\alpha}=\boldsymbol{\alpha}^{(\mathrm{LB})}_{Y_{j,\max}^{(h)}}$ and $\boldsymbol{\alpha}=\boldsymbol{\alpha}^{(\mathrm{UB})}_{Y_{j,\max}^{(h)}}$, respectively:

$$\underline{L}_{Y_{j,\max}^{(h)}}\left(b,T\right)=L_{Y_{j,\max}^{(h)}}(\boldsymbol{\alpha},b,T)\Big|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{LB})}};$$
$$\overline{L}_{Y_{j,\max}^{(h)}}\left(b,T\right)=L_{Y_{j,\max}^{(h)}}(\boldsymbol{\alpha},b,T)\Big|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{UB})}}. \tag{30a,b}$$

Summarizing, the sensitivity-based procedure requires only two stochastic analyses of the structure for assigned values of the uncertain parameters given by Eqs (29a,b) in order to evaluate the mean-value and spectral moments of zero- and second-order of the interval random stress process $Y_j^{(h)}(\boldsymbol{\alpha}^I,t)$ entering the definition of the *CDF* (see Eq. (17)).

Under the assumption that the spectral moments are monotonic functions of the interval parameters, the proposed procedure requires preliminarily the evaluation of the interval global stiffness matrix (see Eqs.(7)), and of the interval damping matrix (see Eq.(8)). Then, the following sensitivities for the *extreme value* stress random process $Y_{j,\max}^{(h)}(\boldsymbol{\alpha}^I,T)$, with respect to the uncertain parameters $\alpha_i \in \alpha_i^I \quad (i=1,2,\ldots,r)$ are required: i) sensitivities of mean-value (see Eqs.(22)), ii) sensitivities of spectral moments (see Eq.(23)); iii) sensitivities of the *CDF* (see Eq. (20)). Finally, once the sensitivities of the *CDF* are determined, the vector $\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{LB})}$ (see Eq. (29a)) has to be defined in order to evaluate the *LB* of the *interval CDF* (see Eq. (30a)) which defines the worst case scenario for a conservative design.

The knowledge of the sensitivities of the *interval CDF* of the *extreme value* stress random process $Y_{j,\max}^{(h)}(\boldsymbol{\alpha}^I,T)$ in Eq. (20) can also be exploited to enhance the computational efficiency of the proposed procedure. As known, sensitivity analysis allows us to identify the most influential parameters on the response quantity of interest. To this aim, the so-called *function of sensitivity* of the *interval CDF*, $L_{Y_{j,\max}^{(h)}}(\boldsymbol{\alpha}^I,b,T)$, is evaluated:

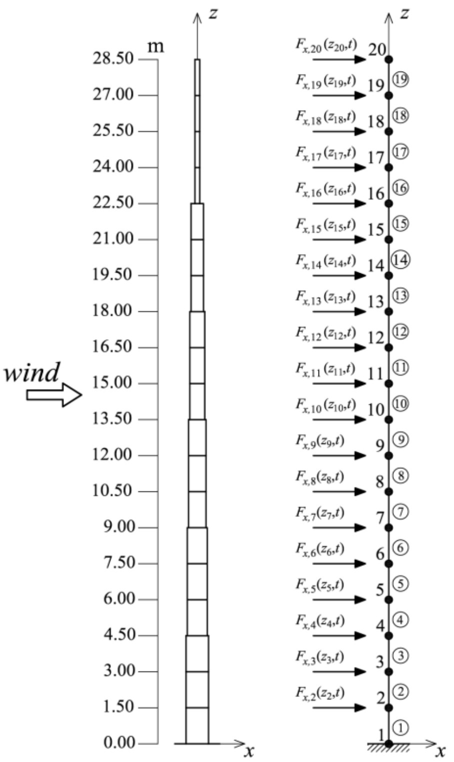
$$\varphi_{i,L_{Y_{j,\max}^{(h)}}}(b,T)(\%)=\frac{S_{L_{Y_{j,\max}^{(h)}},i}(b,T)}{L_{Y_{j,\max}^{(h)}}^{(0)}(b,T)}\Delta\alpha_i\times 100, \tag{31}$$

where $S_{L_{Y_{j,\max}^{(h)}},i}(b,T)$ is the $i-$th sensitivity of the *interval CDF*, $L_{Y_{j,\max}^{(h)}}(\boldsymbol{\alpha}^I,b,T)$, defined in Eq. (20); $L_{Y_{j,\max}^{(h)}}^{(0)}(b,T)$ is the *CDF* pertaining to the nominal system. The *function of sensitivity* represents a percentage measure of the influence of the generic interval variable α_i^I on the *CDF* of the selected *extreme value* stress process. This implies that the crucial uncertain parameters are those characterized by higher values of the *function of sensitivity*. The least influential parameters can be reasonably assumed deterministic and set equal to their nominal values.

4. Numerical Application

The effectiveness of the proposed procedure is assessed by performing *first-passage* reliability analysis of a steel telecommunication antenna mast with interval stiffness uncertainties subjected to wind excitation modelled as a stationary Gaussian multi-correlated random process.

The steel telecommunication antenna mast, 28.50 m high, depicted in Figure1a, is discretized into $N^{(e)} = 19$ two-node Euler-Bernoulli beam FEs resulting in a $n = 38$ DOFs system (Fig. 1b). A lumped mass model is assumed. The properties of the FE model of the antenna are listed in Table I, where the masses lumped at nodes and the tributary areas A_i entering the definition of wind loads $F_{x,i}(z_i, t)$ (see Eq. (32)) are also reported. The nominal Young's modulus is set equal to $E_0 = 210\,\text{GPa}$ for the whole structure. The values $c_0 = 0.149575\,\text{s}^{-1}$ and $c_1 = 0.00316358\,\text{s}$ are assumed for the Rayleigh damping constants in Eq. (8) in such a way that the modal damping ratio of the first and third modes of the nominal structure is $\zeta_0 = 0.01$. The fundamental period of the nominal structure is $T_0 = 0.723\,\text{s}$.



a) b)

Figure 1. Steel telecommunication antenna mast under wind excitation: a) 2D model; b) *FE* model.

Table I. Properties of the telecommunication antenna mast					
Node	**Height [m]**	**Outer diameter [m]**	**Thickness [m]**	**Lumped mass [t]**	**Tributary area [m^2]**
2	1.50	0.9148	0.008	0.3050	1.3722
3	3.00	0.9148	0.008	0.3050	1.3722
4	4.50	0.9148	0.008	0.2754	1.2957
5	6.00	0.8128	0.0071	0.2458	1.2192
6	7.50	0.8128	0.0071	0.2458	1.2192
7	9.00	0.8128	0.0071	0.2211	1.143
8	10.50	0.7112	0.0063	0.1964	1.0668
9	12.00	0.7112	0.0063	0.1964	1.066
10	13.50	0.7112	0.0063	0.1760	0.9906
11	15.00	0.6096	0.0056	0.1530	0.9144
12	16.50	0.6096	0.0056	0.1505	0.9144
13	18.00	0.6096	0.0056	0.1288	0.8382
14	19.50	0.508	0.005	0.2671	0.762
15	21.00	0.508	0.005	0.1071	0.762
16	22.50	0.508	0.005	0.1650	0.526275
17	24.00	0.1937	0.0045	0.0429	0.29055
18	25.50	0.1937	0.0045	0.1329	0.29055
19	27.00	0.1937	0.0045	0.0429	0.29055
20	28.50	0.1937	0.0045	0.0214	0.145275

Young's modulus of the material is assumed to be affected by uncertainty. In a first stage, in order to allow comparisons with the *vertex method* (Dong and Shah, 1987), only the first $r = 12$ FEs (see Fig. 1b) are assumed to be characterized by interval Young's moduli i.e. $E^{(i)I} = E_0\left(1 + \Delta\alpha \hat{e}_i^I\right)$, $(i = 1, 2, \ldots, r = 12)$, where the same deviation amplitude is considered. Under this assumption, the application of the *vertex method* involves $2^r = 2^{12}$ stochastic analyses of the structure while the proposed method requires to evaluate the *reliability function* only twice, regardless of the number of uncertain parameters. Indeed, the *vertex method* requires to evaluate the *reliability function* for all the 2^r possible combinations of the endpoints of the r uncertain parameters. Then, for each barrier level b, the *LB* and *UB* of the *interval reliability function* are obtained as the minimum and maximum among the 2^r values pertaining to the *vertex analysis*.

The maximum interval axial stress, $Y_1^{(1)}(\boldsymbol{\alpha}^I, t)$, at the antenna base section, i.e. at node 1 of *FE* 1, is assumed as the response quantity responsible of structural failure. To predict the range of structural performance, the bounds of the *interval reliability function*, $L_{Y_{1,\max}^{(1)}}(\boldsymbol{\alpha}^I, b, T)$, (see Eq. (17)) are evaluated. The observation time is assumed equal to $T = 1000T_0$, T_0 being the fundamental period of the structure with nominal Young's moduli.

The along wind force (x-direction) exerted on the i-th node at height z_i of the discretized structure is defined by the well-known formula (Simiu and Scanlan, 1996):

$$F_{x,i}(z_i, t) = \frac{1}{2}\rho C_D A_i w_s^2(z_i) + \rho C_D A_i \tilde{W}(z_i, t) w_s(z_i) \tag{32}$$

where the contribution of the nodal velocities of the structure and the square of the fluctuating component of wind speed have been neglected. In Eq. (32), $\rho = 1.25$ kg/m^3 is the air density; C_D is the drag coefficient; A_i is the tributary area of the i-th node; $w_s(z) = w_{s,10}\left(z/10\right)^{\gamma}$ is the mean wind velocity which is assumed to vary with the elevation z following a power law; $w_{s,10}$ is the mean wind speed measured at height $z = 10$ m above ground and γ is a coefficient depending on surface roughness, herein taken equal to $w_{s,10} = 25$ m/s and $\gamma = 0.3$, respectively. Furthermore, $\tilde{W}(z,t)$ denotes the fluctuating component of wind velocity field, which is modelled as a zero-mean stationary Gaussian random field, fully described from a probabilistic point of view by the one-sided *PSD* function proposed by Davenport (1961):

$$G_{\tilde{W}\tilde{W}}(\omega) = 4K_0 w_{s,10}^2 \frac{\chi^2}{\omega\left(1+\chi^2\right)^{4/3}} \tag{33}$$

where K_0 is the non-dimensional roughness coefficient, herein set equal to $K_0 = 0.03$, and $\chi = b_1\omega/(\pi w_{s,10})$ with $b_1 = 600$ m. The vector $\tilde{\mathbf{W}}(t)$ collecting wind velocity fluctuations at the wind-exposed nodes located at different heights z_i is characterized from a probabilistic point of view by the one-sided *PSD* function matrix $\mathbf{G}_{\tilde{\mathbf{W}}\tilde{\mathbf{W}}}(\omega)$. If the imaginary part (*q*-spectrum) is neglected, the one-sided cross-*PSD* components of $\mathbf{G}_{\tilde{\mathbf{W}}\tilde{\mathbf{W}}}(\omega)$ can be expressed as follows (Simiu and Scanlan, 1996):

$$G_{\tilde{W}_i\tilde{W}_j}(z_i, z_j; \omega) = G_{\tilde{W}\tilde{W}}(\omega) f_{ij}(\omega) = G_{\tilde{W}\tilde{W}}(\omega)\mathrm{Exp}\left\{-\frac{|\omega|\sqrt{C_z^2(z_i - z_j)^2}}{\pi\left[w_s(z_i) + w_s(z_j)\right]}\right\} \tag{34}$$

where $f_{ij}(\omega)$ is the so-called *coherence function* with $C_z = 10$ denoting an appropriate decay coefficient.

In Figure 2, the proposed *LB* and *UB* of $L_{Y_{1,\max}^{(1)}}(\boldsymbol{\alpha}^I, b, T)$ are compared with those provided by the *vertex method*. Two different deviation amplitudes of the uncertain parameters, $\Delta\alpha = 0.10$ and $\Delta\alpha = 0.20$, are considered. The nominal *CDF*, $L_{Y_{1,\max}^{(1)}}^{(0)}(b, T)$, is also reported. An excellent agreement between the proposed sensitivity-based procedure and the *vertex method* can be observed. As expected, the p-box describing structural performance becomes wider as the degree of uncertainty increases. The deviation of the left and right bounds of the p-box from the nominal solution proves that assuming the nominal value of Young's moduli may lead to inaccurate predictions of the structural safety level. As already mentioned, a conservative design should rely on the *LB* of the *interval reliability function*.

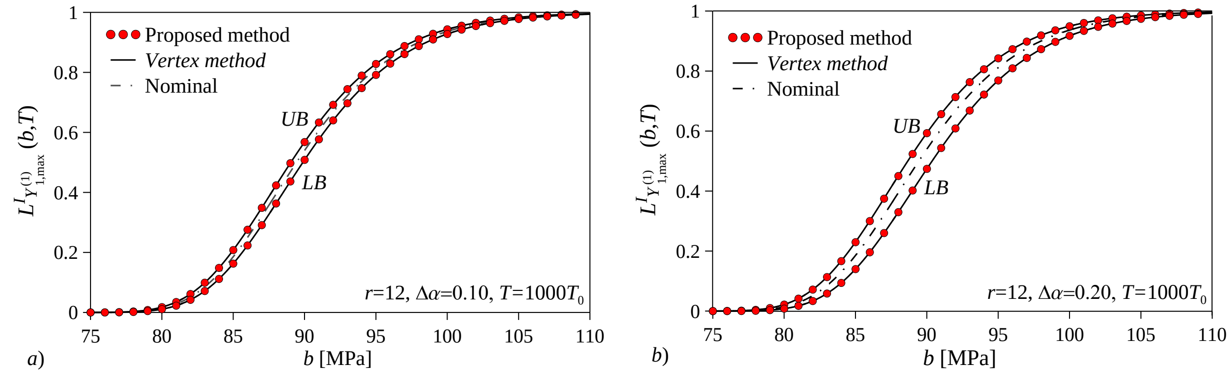


Figure 2. Bounds of the *interval CDF* of the *extreme value* axial stress process $Y^{(1)I}_{1,\max}(T)$ of the telecommunication antenna with uncertain Young's moduli $E^{(i)I} = E_0\left(1+\Delta\alpha\hat{e}^I_i\right)$, $(i=1,2,\ldots,r=12)$: comparison between the proposed procedure and the *vertex method* for *a*) $\Delta\alpha = 0.10$ and *b*) $\Delta\alpha = 0.20$.

In order to predict the influence of a small change of Young's moduli on the performance of the telecommunication antenna mast, the *function of sensitivity* of the *CDF* $L_{Y^{(1)}_{1,\max}}(\boldsymbol{\alpha}^I, b, T)$ is evaluated (see Eq. (31)) under the assumption that all Young's moduli are described by intervals. In Figure 3, the *functions of sensitivity* $\varphi_{i,L_{Y^{(1)}_{1,\max}}}(b,T)$ of $L_{Y^{(1)}_{1,\max}}(\boldsymbol{\alpha}^I, b, T)$ with respect to the fluctuations $\alpha^I_i = \Delta\alpha\hat{e}^I_i$ of a selected number of interval Young's moduli $E^I_i = E_0(1+\alpha^I_i) = E_0(1+\Delta\alpha\hat{e}^I_i)$, $(i=1,2,\ldots,8,12,13,14,16,17)$ versus the deterministic barrier level b are plotted ($\Delta\alpha = 0.10$).

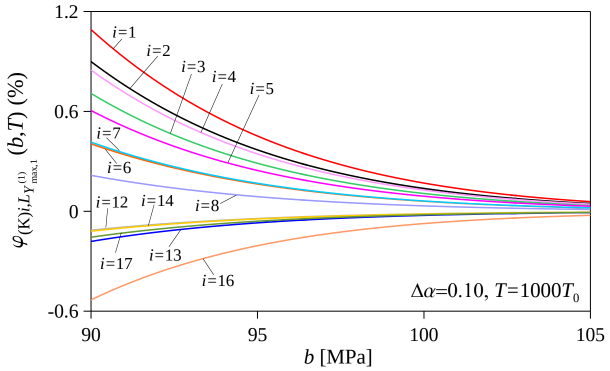


Figure 3. Functions of sensitivity of the *interval reliability function* of the *extreme value* axial stress process $Y^{(1)I}_{1,\max}(T)$ of the telecommunication antenna with respect to the fluctuations of Young's moduli $E^{(i)I} = E_0\left(1+\Delta\alpha\hat{e}^I_i\right)$, $(i=1,2,\ldots,8,12,13,14,16,17)$, versus the deterministic barrier level b ($\Delta\alpha = 0.10$, $T = 1000T_0$).

For the sake of clarity, the *functions of sensitivity* with respect to the fluctuations of the remaining Young's moduli are omitted. It is observed that the most influential Young's moduli are those of *FE*s 1 and 16 for any value of the barrier level. A close inspection of Figure 3 also shows that a small increase of Young's moduli of the first eight *FE*s would produce an increment of structural reliability since the pertinent *functions of sensitivity* are positive. Conversely, a small increase of Young's moduli of *FE*s $12,13,14,16,17$ would lead to a lower safety level.

Finally, in Figure 4, the bounds of the *interval reliability function* $L^I_{Y^{(1)}_{1,\max}}(b,T)$ evaluated by applying the proposed sensitivity-based approach considering Young's moduli of all the $r = N^{(e)} = 19$ *FE*s as uncertain (full) are contrasted with the ones computed retaining only the $r = 13$ most influential uncertain parameters (reduced) identified by sensitivity analysis. For comparison purpose, the bounds pertaining to the structure with the first $r = 6$ most influential uncertain Young's moduli $E^{(i)I} = E_0\left(1+\Delta\alpha\hat{e}^I_i\right)$, $(i = 1,2,3,4,5,16)$, are also plotted. Also in this case, two different deviation amplitudes of the uncertain parameters, $\Delta\alpha = 0.10$ and $\Delta\alpha = 0.20$, are considered. It can be observed that the left and right bounds of the p-box describing structural performance obtained considering only the first $r = 6$ uncertain parameters are enclosed by the bounds pertaining to full uncertainty. This entails that some of the neglected parameters play a crucial role on the prediction of the safety level. Conversely, the region of the *interval CDF* predicted retaining $r = 13$ uncertain Young's moduli is almost coincident with the one obtained performing full uncertainty analysis.

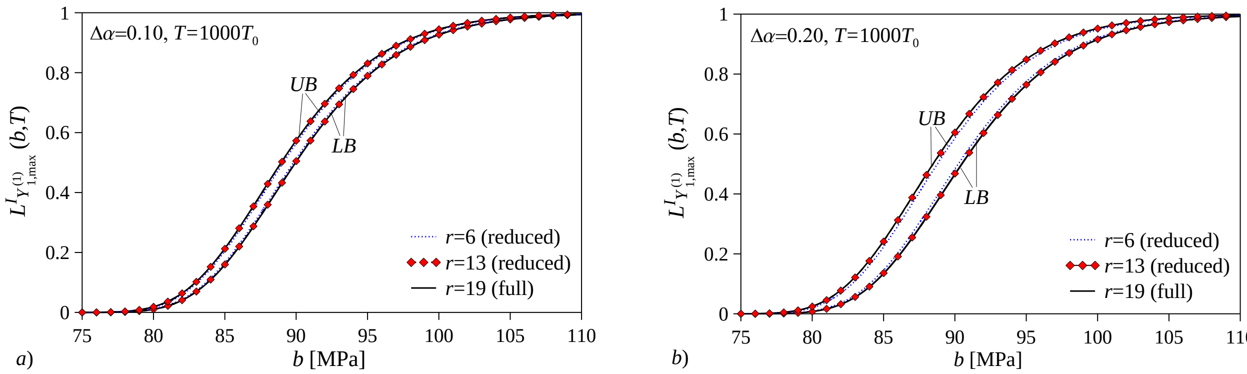


Figure 4. Bounds of the *interval CDF* of the *extreme value* axial stress process $Y^{(1)I}_{1,\max}(T)$ of the telecommunication antenna provided by the proposed procedure considering Young's moduli of all the 19 *FE*s as uncertain (full) and retaining only the first $r = 6$ and $r = 13$ most influential uncertain parameters (reduced): a) $\Delta\alpha = 0.10$ and b) $\Delta\alpha = 0.20$ ($T = 1000T_0$).

5. Summary and Conclusions

A sensitivity-based procedure for reliability analysis of finite element modeled structures with interval stiffness subjected to stationary Gaussian multi-correlated random excitation is presented. The formulation is developed in the context of the *first-passage* theory under the Poisson assumption of independent up-crossings of a prescribed threshold. The presented procedure basically consists in identifying suitable combinations of the endpoints of the uncertain structural parameters which yield accurate estimates of the bounds of the *interval reliability function* of the selected *extreme value* stress random process. This task is pursued by performing a preliminary sensitivity analysis of the reliability function.

The main features of the proposed sensitivity-based procedure may be summarized as follows: i) the bounds of the *interval reliability function* of the selected stress process are obtained by performing only two stochastic analyses of the structure wherein the uncertain parameters are set equal to the pertinent combinations identified by sensitivity analysis; ii) the presented procedure yields results in excellent agreement with the ones provided by the *vertex method* in spite of the much higher computational efficiency; iii) reliability analysis of arbitrary finite element modeled structures involving stiffness uncertainties can be performed; iv) sensitivity analysis provides a deep insight into the impact of stiffness fluctuations on structural performance allowing the identification of the least influential parameters which may be set equal to the nominal values to enhance the computational efficiency.

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