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A Bimodal Distribution Function with Fuzzy Regression for Predicting Truck Load Population including Overloads

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Introduction

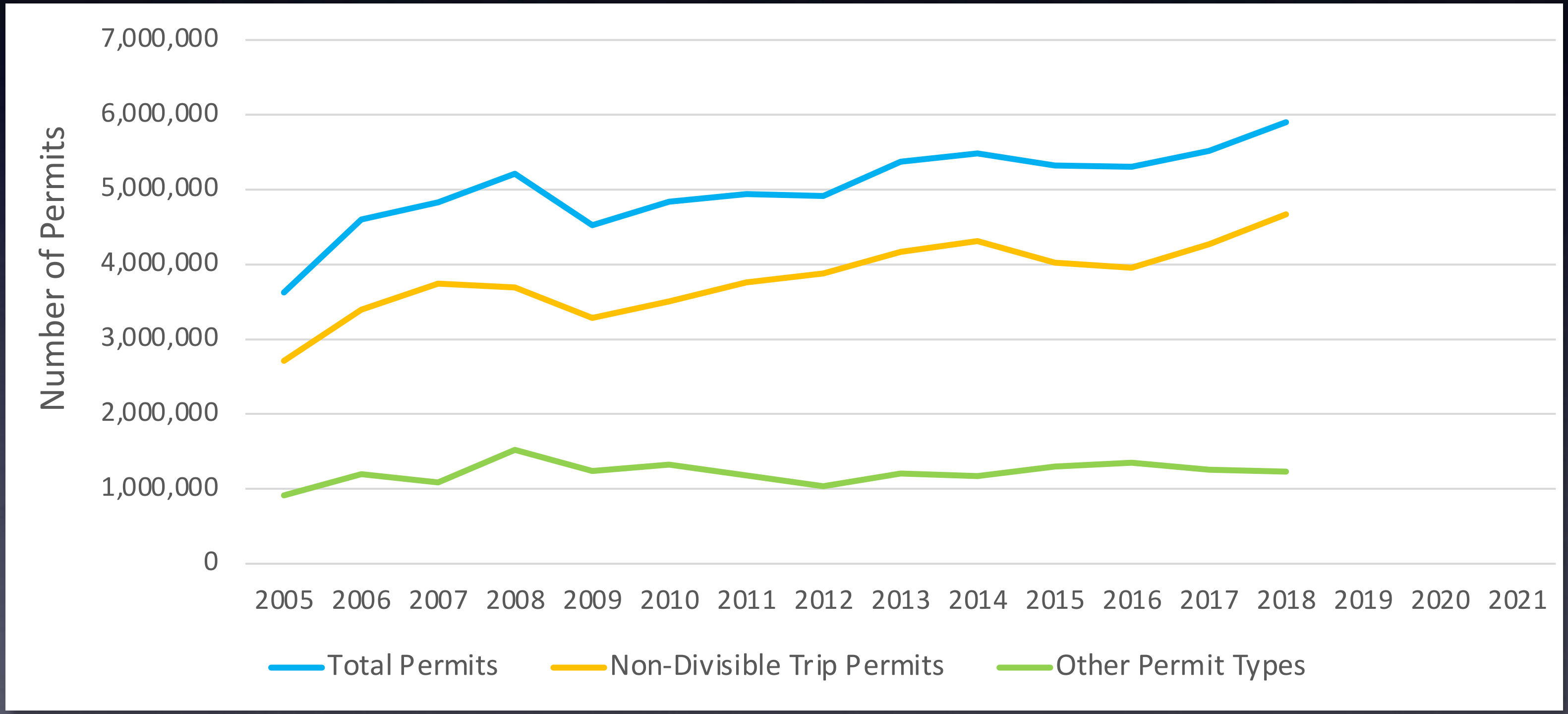
- ✓ Truck load data - an important role in condition assessment and life-cycle management of transportation systems
- ✓ Damage accumulation - a result of the repeated application of truck loads
- ✓ Damage assessment - using truck load data directly in a discrete format
- ✓ Discrete format - describing the frequencies of load occurrences with specific ranges

Introduction

- ✓ Some bridges have been built for low truck weights (some for as little as 240 kN (54 kips)) than current design load (356 kN (80 kips))
- ✓ Special attention to the potential for accelerated fatigue damage due to overloads on the bridge
- ✓ A more versatile and realistic approach in damage assessment - using a theoretical model that best represents the truck load data including overloads

Introduction

Number of permits counted annually:



Source: Freight Facts and Figures 2013, 2017, 2020

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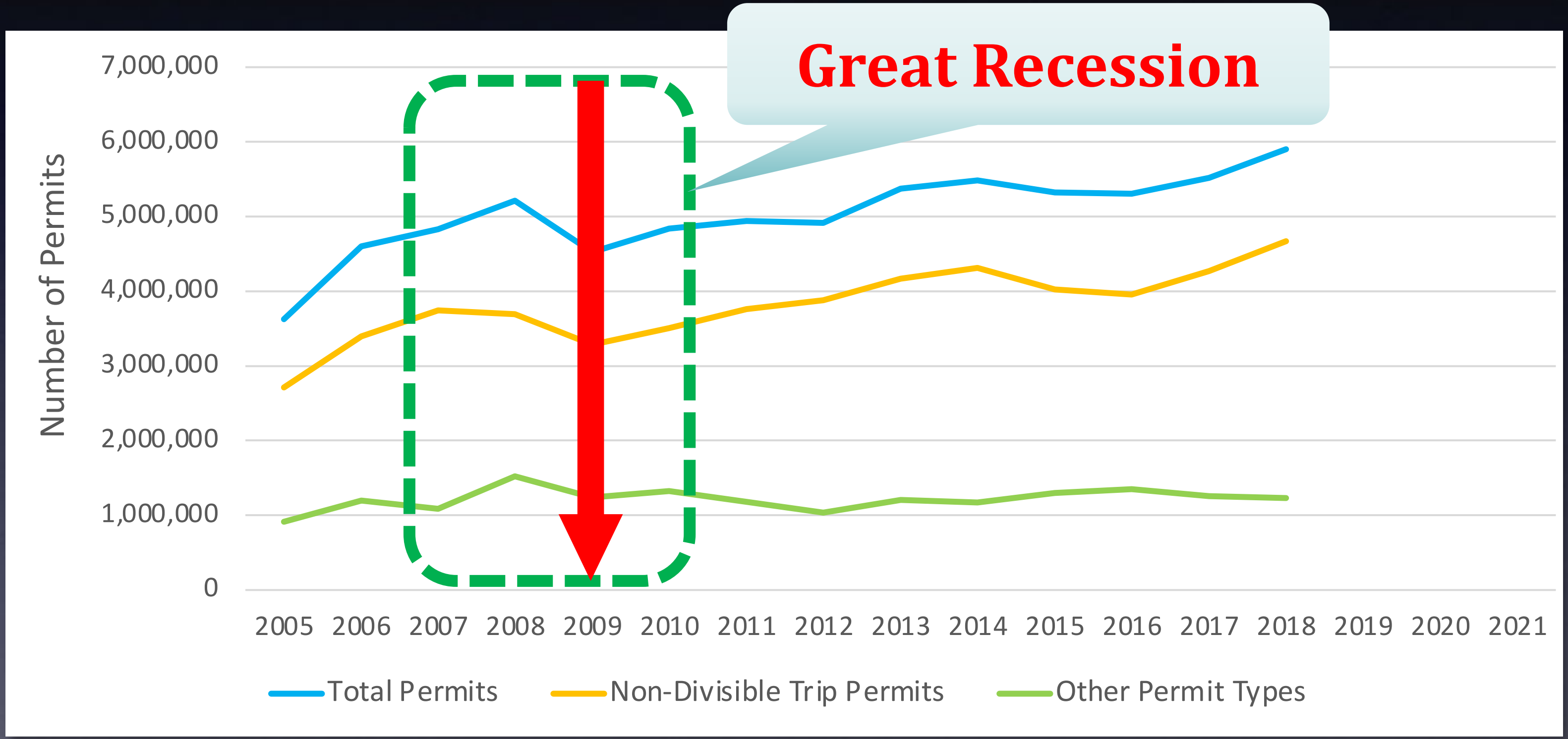

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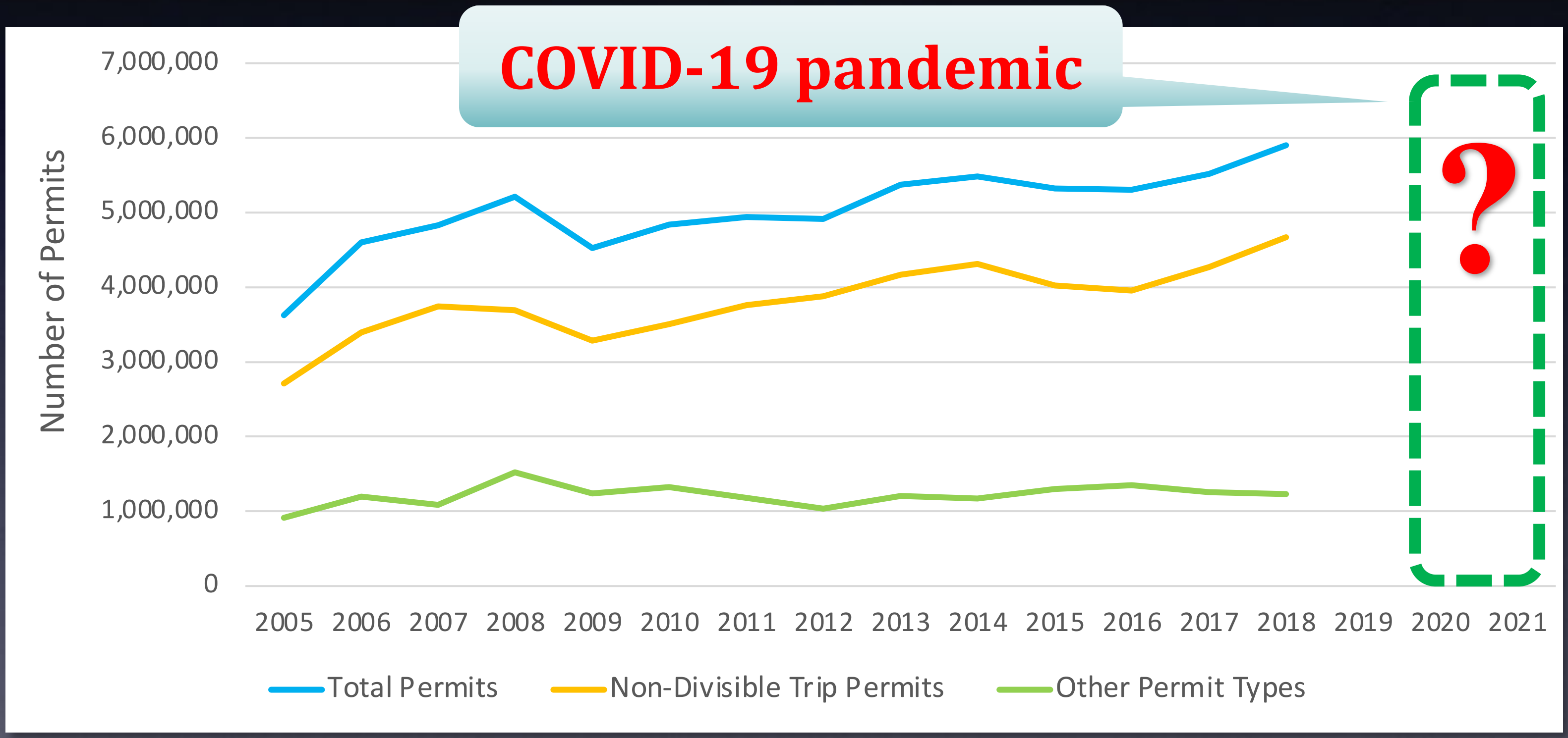

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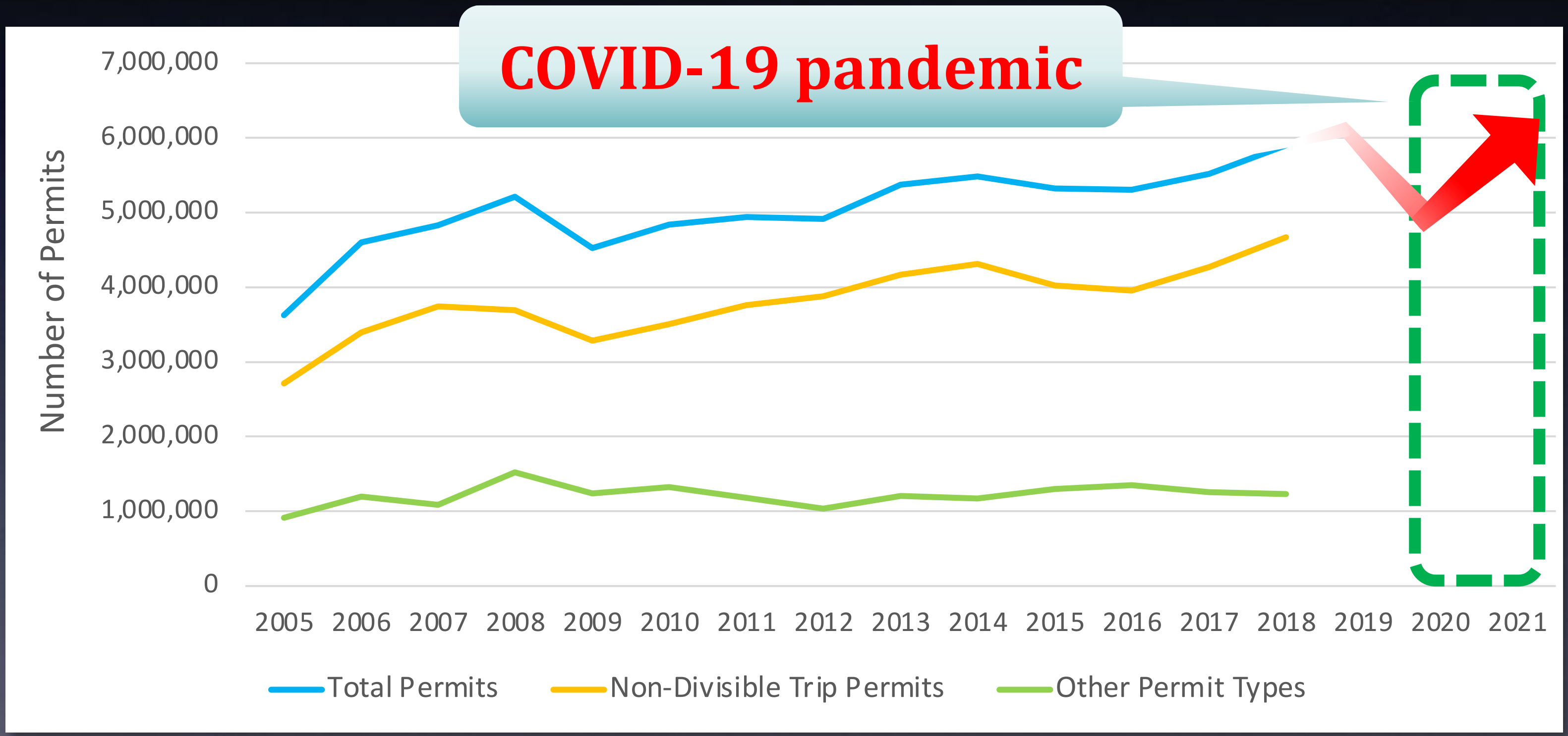


Source: Freight Facts and Figures 2013, 2017, 2020

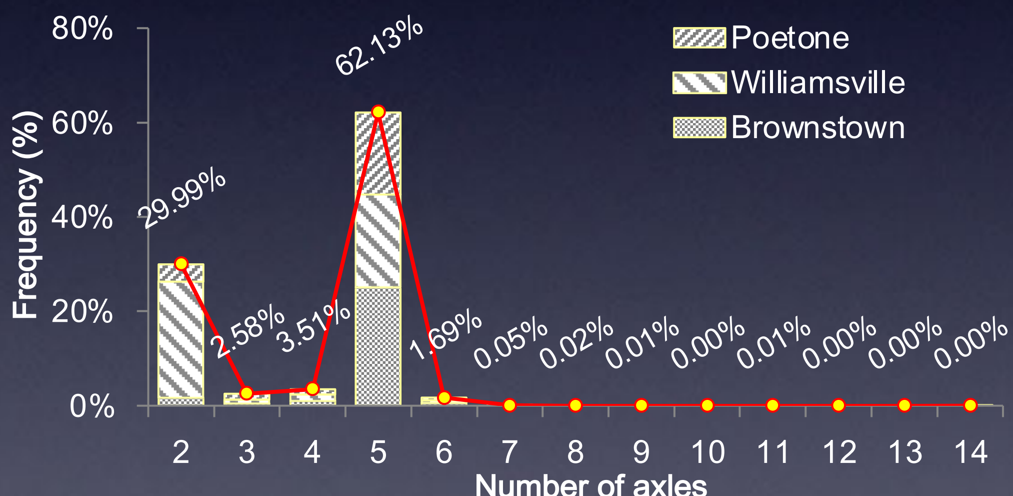
Introduction

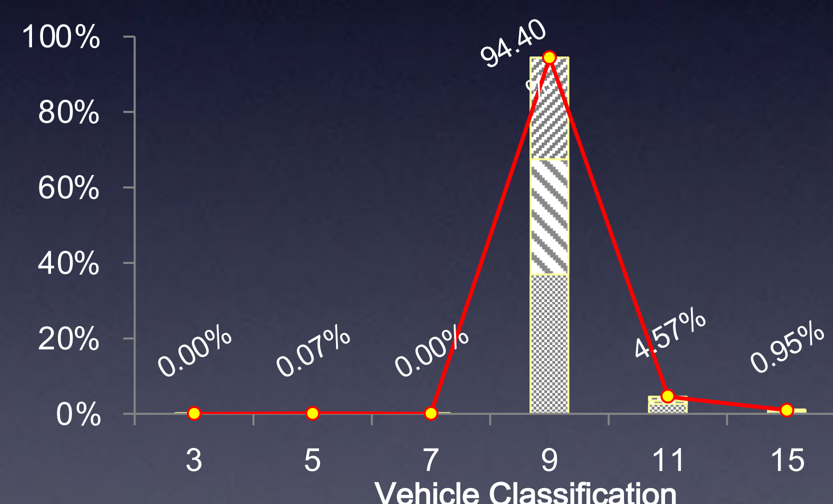
OBJECTIVE:

- ✓ To use samples of truck load data and develop a mathematical model to represent a reasonable distribution for overload trucks
- ✓ To predict the truck load population with limited data
- ✓ To suggest the use of models in applications where methods for transportation system damage assessment require information on the frequency and magnitude of loads, especially overloads

Truck Load Population

Truck Load Population and Overloads:



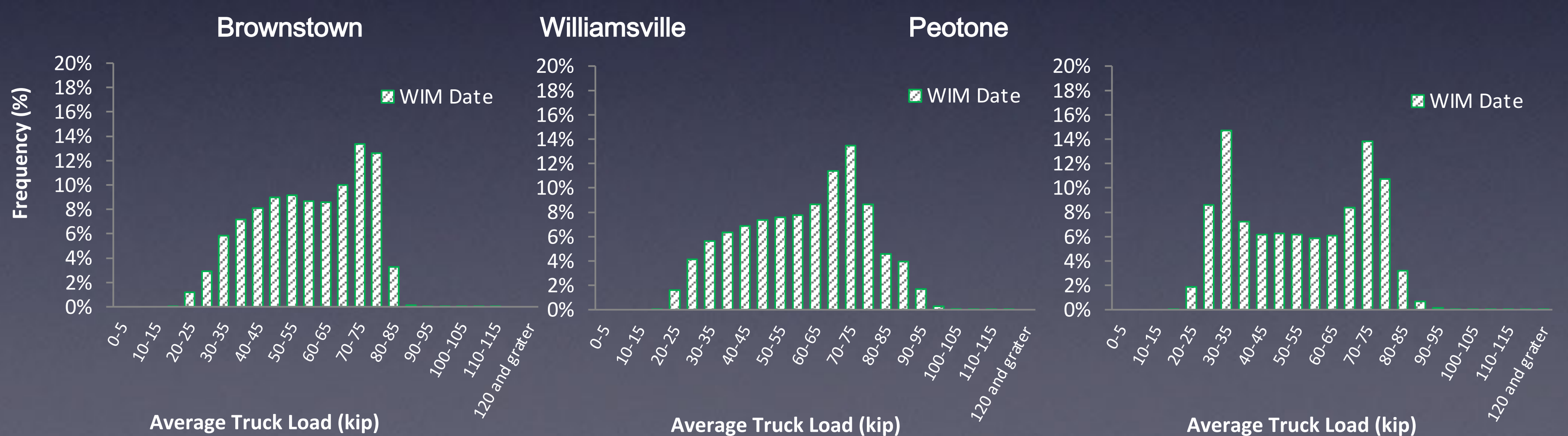


Overloads constitute total gross weight > 356 kN (80 kips)

Truck Load Population

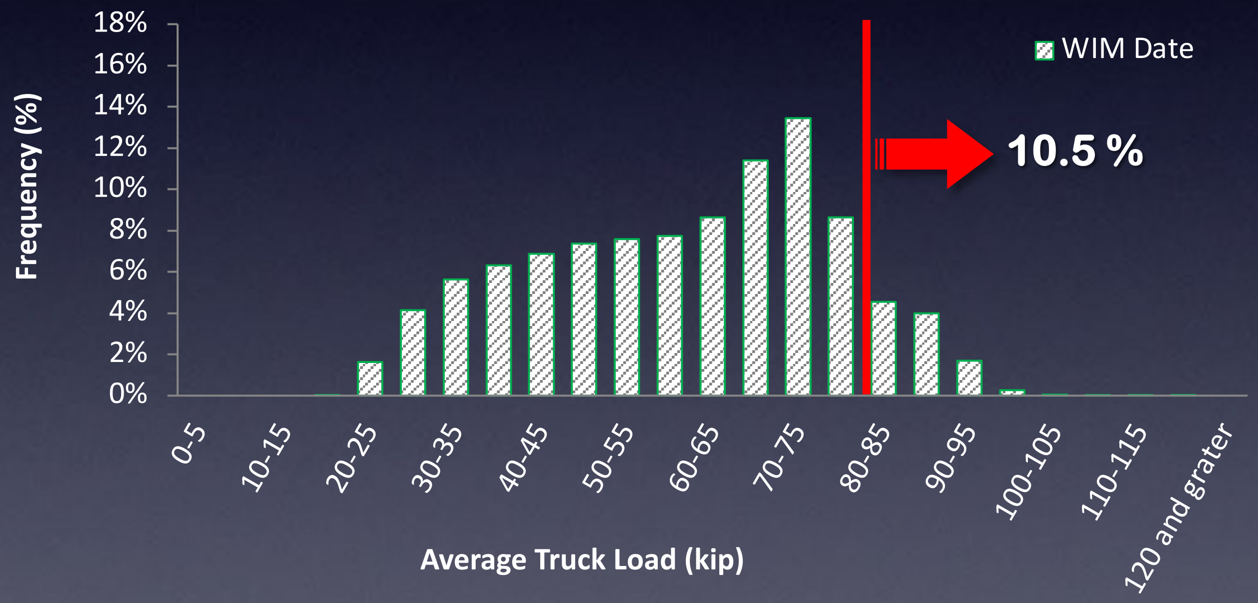
Truck Load Population and Overloads:

Truck Load Population for 5-axle truck from WIM data (adopted from IDOT & MDOT from June, 2014 to May, 2015)



Truck Load Population

Truck Load Population and Overloads:



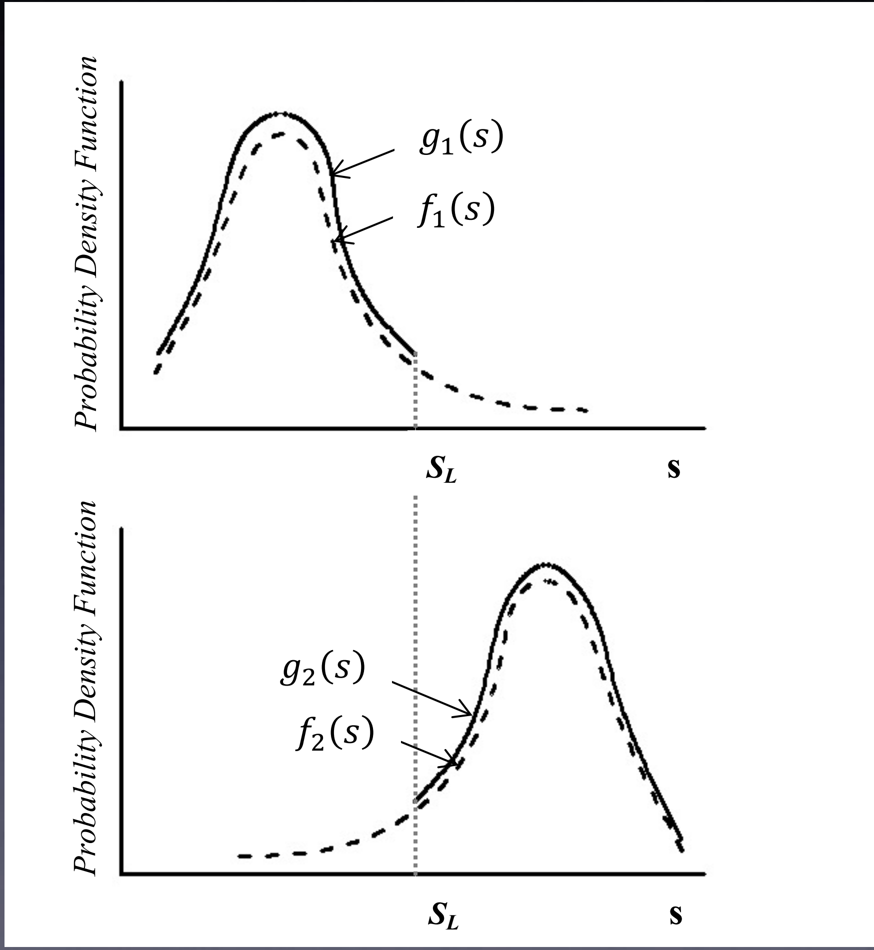
Unit Conversion

kip	kN
45	200
50	222
55	245
60	267
65	289
70	311

Overloads constitute total gross weight > 356 kN (80 kips)

Mixed Distribution Model

Schematic of mixed probability density function with consideration for the common load (S_L)



Mixed Distribution Model

Two-function mixed Model (bimodal distribution):

- ✓ A combination of two distinct probability density functions (f_1 and f_2)
- ✓ A suggestion of beta and log-normal distributions - (1) the fact that they only accept positive values and (2) others have also recommended them
- ✓ Several combinations of beta and lognormal distributions - beta-beta, lognormal-lognormal, and beta-lognormal distributions

Mixed Distribution Model

: Goodness-of-fit Test

Kolmogorov-Smirnov (K-S) test

$$D_n = \max_x |F_X(x) - S_n(x)|$$

where, $S_n(x)$ = the observed cumulative frequency function

$F_X(x)$ = the theoretical cumulative distribution function

Anderson-Darling (A-D) test

$$AD^2 = -\sum_{i=1}^{n} \left[\frac{(2i-1)\{lnF_X(x_i) + ln[1 - F_X(x_{n+1-i})]\}}{n} \right] - n$$

where, $F_X(x)$ = the theoretical cumulative distribution function

n = the sample size

Mixed Distribution Model
: Goodness-of-fit Test

Unit Conversion

kip	kN
45	200
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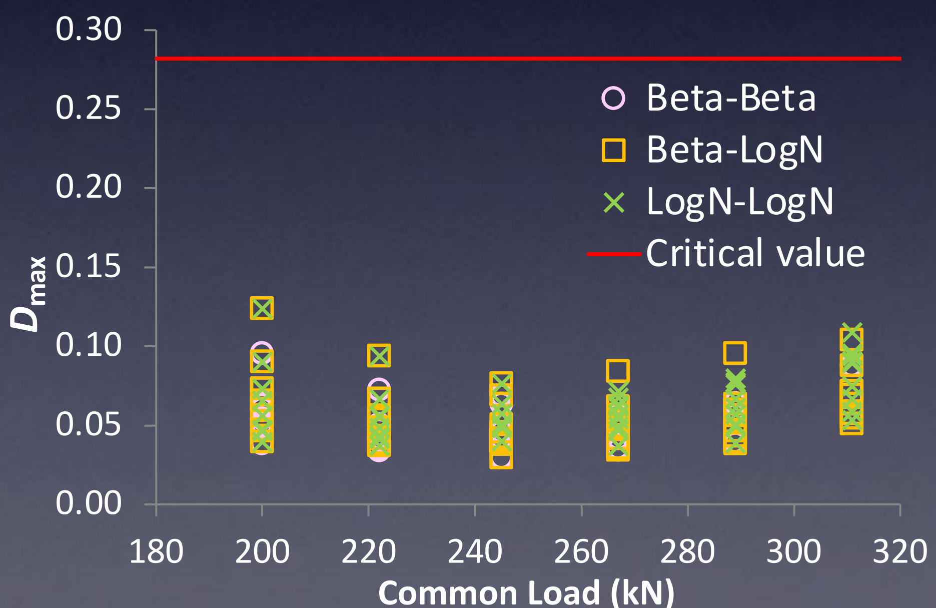
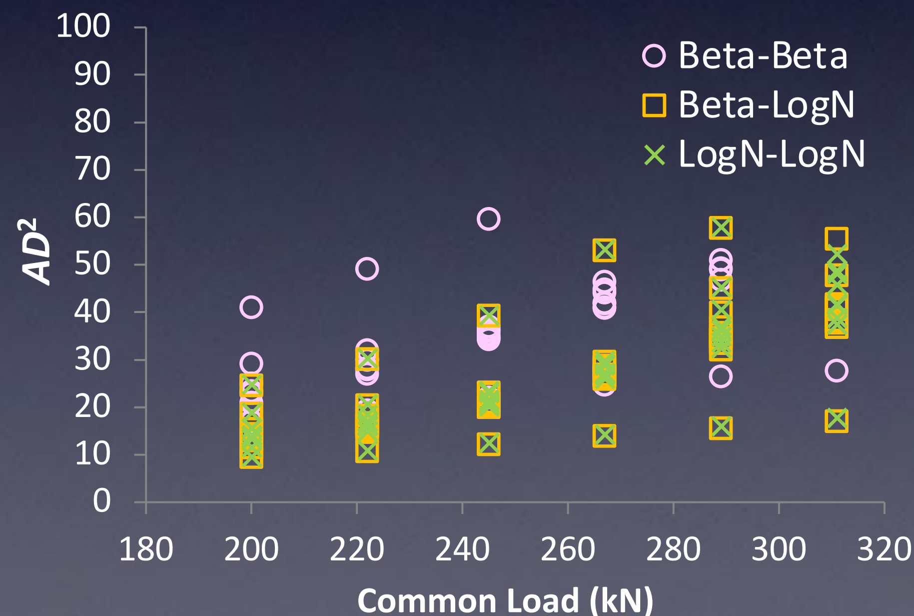
Results of K-S and A-D tests with various common loads and combinations of mixed distribution for the nine WIM stations





Proposed Bimodal Distribution

Select Common Load, S_L

Determine statistical parameters for two PDFs, f_1 is load $s < S_L$ and f_2 is $s > S_L$

1st PDF, $f_1(s)$

Compute μ_1, σ_1, and a_1 from WIM data

Compute parameters: p, q Determine $f_1(s)$ where,

$$f(s) = \frac{1}{B(q,r)} \frac{s^{q-1}(s_0 - s)^{r-1}}{{s_0}^{q+r-1}}$$

$$B(q,r) = \int_0^1 x^{q-1}(1-x)^{r-1}dx$$

Determine adjusted, $f_1(s)$, $g_1(s)$ $\quad g_1(s) = \frac{f_1(s)}{F_1(S_L)}$

2nd PDF, $f_2\,(s)$

Compute μ_2, σ_2, and a_2 from WIM data

Compute parameters: ξ and λ

Determine $f_2(s)$

$$f(s) = \frac{1}{\sqrt{2\pi}(\zeta s)} exp\left[-\frac{1}{2}\left(\frac{\ln s - \lambda}{\zeta}\right)^2\right]$$

Determine adjusted, $f_2(s)$, $g_2(s)$ $\quad g_2(s) = \frac{f_2(s)}{1 - F_2(S_L)}$

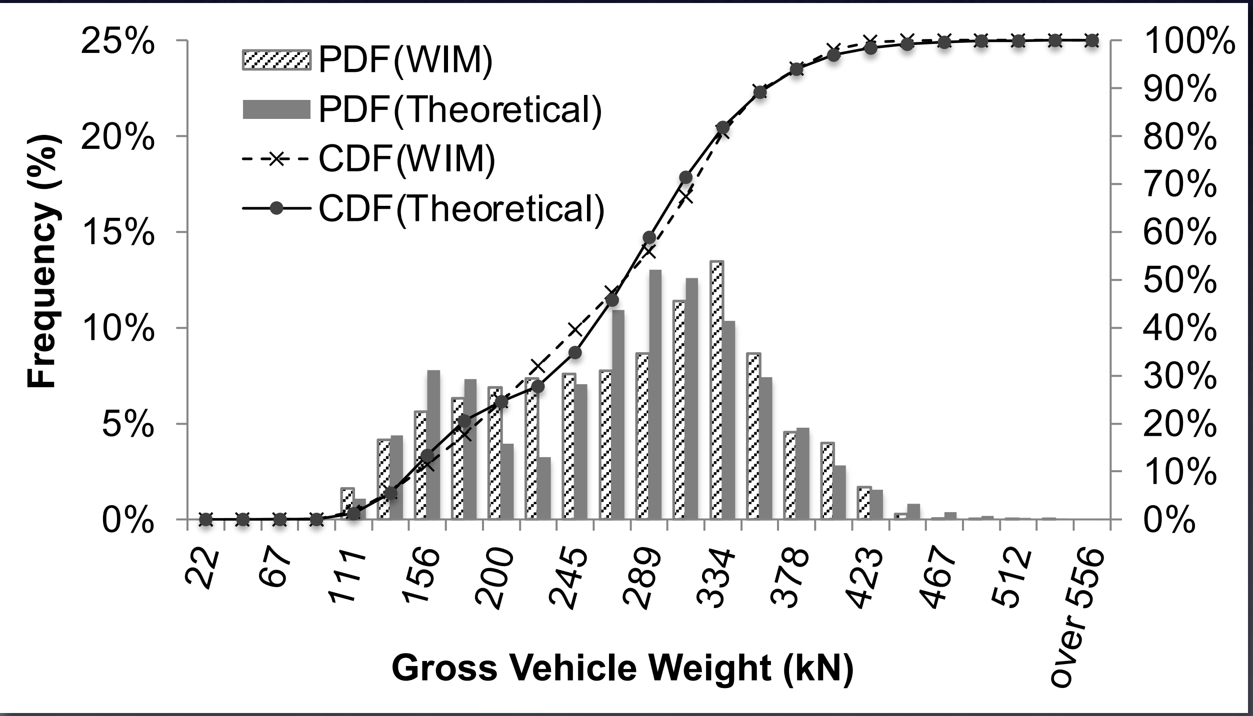
Combine and obtain a theoretical distribution model

Note that $\alpha_1 + \alpha_2$ =1.

$$g(s) = \begin{cases} \alpha_1 g_1(s), & for\ \ s \le S_L \\ \alpha_2 g_2(s), & for\ \ s > S_L \end{cases}$$

Proposed Bimodal Distribution

Frequency and cumulative probability distribution for the observed WIM data and the theoretical model



Predicting Truck Load Population

WIM Data is not Available:

- ✓ Average daily truck traffic (ADTT)
- ✓ The number of overload permits

$$OL^{*} = \frac{the\ number\ of\ permits\ per\ year}{ADTT \times 365}$$

where, OL^{*} = the minimum percentage of overloads in the entire truck load population

$ADTT$ = the annual daily truck traffic

Predicting Truck Load Population

Fuzzy Regression Analysis:

- ✓ A fuzzy regression model - the fuzzy logic to treat the imprecise and uncertain phenomenon in a data set
- ✓ Minimizing the data fuzziness (and thus uncertainty) using a linear programming method
- ✓ A classical fuzzy regression analysis with crisp-input (i.e., the percentage of overload, *OL*) and fuzzy-output (i.e., statistical parameters required in the Bimodal distribution for predicting the truck load population)

Predicting Truck Load Population

The general form of fuzzy linear output:

$$\widetilde{Y}_i = \widetilde{A_0} + \sum_{j=1}^{m} \widetilde{A_j} x_{ij}$$

where, $\widetilde{Y}_i$ = the fuzzy linear output with i = 1, 2, …, n

$\widetilde{A_0} = (a_0, c_0), \widetilde{A_j} = (a_j, c_j)$ = fuzzy coefficient with j = 1, 2, …, m

$x_{ij} = [x_{1j}, x_{2j}, \ldots, x_{nj}]$ =a vector of input variables with respect to j th fuzzy coefficient

(a_j, c_j) = the fuzzy coefficient in the membership function (*MF*)

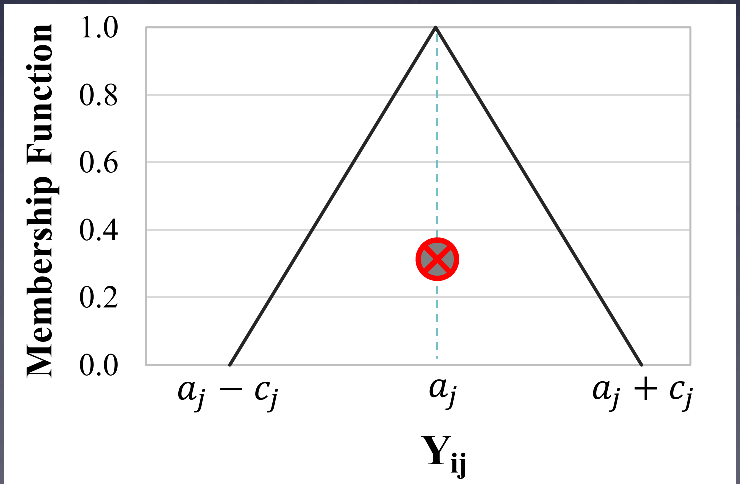
Predicting Truck Load Population

Membership function (*MF*) - a symmetric triangular:

$$MF(a_j) = \begin{cases} max\left(1 - \frac{|a_j - c_j|}{c_j}\right), & a_j - c_j \le a_j \le a_j + c_j \\ 0, & otherwise \end{cases}$$

where, a_j = the mode in the membership function (*MF*)

c_j = the spread in the membership function (*MF*)



Predicting Truck Load Population

Fuzzy linear regression analysis with crisp-input and fuzzy-output:

Minimize

$$\sum_{i}^{n}\left(c_0 + \sum_{j=1}^{m} c_j \left|x_{ij}\right|\right)$$

Subject to

$$a_0 + \sum_{j=1}^{m} a_j x_{ij} + (1-h)\left[c_0 + \sum_{j=1}^{m} c_j \left|x_{ij}\right|\right] \geq y_i + (1-h)e_i$$

$$a_0 + \sum_{j=1}^{m} a_j x_{ij} - (1-h)\left[c_0 + \sum_{j=1}^{m} c_j \left|x_{ij}\right|\right] \leq y_i - (1-h)e_i$$

$$c_j \geq 0, \quad 0 \leq h \leq 1, \quad i = 1,2,\dots,n, \quad j = 1,2,\dots,m$$

where, y_j = the mode in a symmetric triangular membership function for the fuzzy output

e_j = the spread in a symmetric triangular membership function for the fuzzy output

h = a factor to control the level of support (spread) in the membership function

Predicting Truck Load Population

Select Common Load, S_L

Is the truck load population available? (e.g. WIM data)

Yes

- CASE I: WIM data
- Compute μ_1, σ_1, μ_2, and σ_2 from WIM data
- Compute α_1 and α_2 from WIM data

No

- CASE II: ADTT & # of overload permits
- Calculate OL^*

  $$OL^* = \frac{\text{the number of permits per year}}{ADTT \times 365}$$

- Determine the fuzzy regression model of μ_1^*, σ_1^*, μ_2^*, σ_2^*, and $\alpha_{2\text{-}OL^*}$ for the estimated OL^* and the selected S_L
- Determine the membership function of μ_1^*, σ_1^*, μ_2^*, σ_2^*, and $\alpha_{2\text{-}OL^*}$ with the selected level of h
- Estimate the value of μ_1^*, σ_1^*, μ_2^*, σ_2^*, and $\alpha_{2\text{-}OL^*}$ from defuzzification of their membership function
- Calculate α_1 and α_2 using $\alpha_{2\text{-}OL^*}$

  $$\alpha_2 = \alpha_{2-OL^*} + OL^*$$

  $$\alpha_1 = 1 - \alpha_2$$

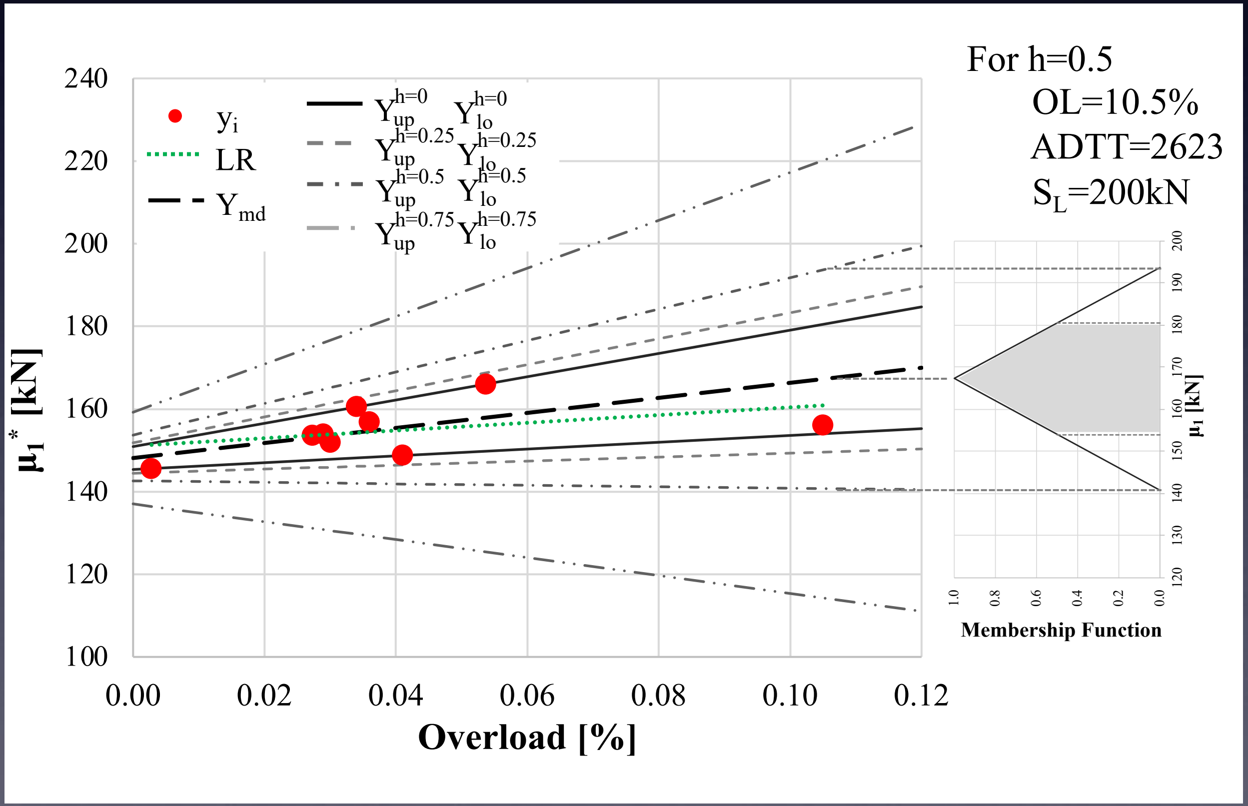
Estimate parameters: p, q, ξ, and λ

Obtain a theoretical distribution model

Predicting Truck Load Population

Numerical Example of Fuzzy linear Regression Model

- ✓ μ_1*
- ✓ S_L = 200 kN (45 kips)
- ✓ A triangular *MF*
- ✓ h = 0, 0.25, 0.5, 0.75



Predicting Truck Load Population

Limitations of Proposed Model:

- ✓ The model developed is exclusive to the sources from which WIM data was acquired.
- ✓ The study utilized the WIM data as received from the sources.
- ✓ A key parameter in the proposed Bimodal distribution model is the “common load.”
- ✓ In the fuzzy regression analysis, various type of membership functions may be employed.

Conclusions

- ✓ A mixed distribution function - the distribution often appears with two distinct peaks
- ✓ A bimodal distributions - beta-lognormal combination to present a favorable mixed distribution model
- ✓ The statistical test - using A-D test in addition to the more traditional K-S test
- ✓ No or limited WIM data - Using the proposed Bimodal distribution (with fuzzy regression model)
- ✓ A load distribution is needed for damage assessment - using the Bimodal distribution model for better estimation

Acknowledgement

The assistance provided by David C. Thompson of the Illinois and Thomas Foltz of the Michigan Department of Transportation, respectively, in making the WIM data available to the authors is appreciated.

Thank you for your attention!

References

Anderson, T.W. and Darling, D.A. 1954. A Test of Goodness of Fit. *Journal of the American Statistical Association*, 49(268), 765-769.

Ang, A.H. and Tang, W.H. 2007. *Probability Concepts in Engineering – Emphasis on Applications to Civil and Environmental Engineering*. John Wiley & Sons, Inc.

Jang, B. and Mohammadi, J. 2017. Bridge rating modification to incorporate fatigue damage from truck overloads. *Bridge Structures*, 13(2-3), 101-107.

Jang, B. and Mohammadi, J. 2018. Significance of fatigue damage from overload trucks on bridge load rating. *MATEC Web of Conferences*, 165, 10005.

Mohammadi, J. and Shah, N. 1992. Statistical Evaluation of Truck overloads. *Journal of Transportation Engineering*, 118(5), 651-665.

Stephens, M.A. 1974. EDF Statistics for Goodness of Fit and Some Comparisons. *Journal of the American Statistical Association*, 69(347), 730-737.

Tanaka, H., Uejima, S., and Asai, K. Linear Regression Analysis with Fuzzy Model. *IEEE Transactions on Systems, Man, An Cybernetics*, Smc-12(6):903-907, 1982.

Timm, D.H., Tisdale, S.M., and Turochy, R.E. 2005. Axle load spectra characterization by mixed distribution modeling. *Journal of Transportation Engineering*, 132(2), 83-88.

Appendix

Beta Distribution

$$f(s) = \frac{1}{B(q,r)} \frac{s^{q-1}(s_0 - s)^{r-1}}{{s_0}^{q+r-1}}$$

$$B(q,r) = \int_0^1 x^{q-1}(1-x)^{r-1}dx$$

Lognormal Distribution

$$f(s) = \frac{1}{\sqrt{2\pi}(\zeta s)} exp\left[-\frac{1}{2}\left(\frac{\ln s - \lambda}{\zeta}\right)^2\right]$$

Mixed Density Function

$$g_1(s) = \frac{f_1(s)}{F_1(S_L)}$$

$$g_2(s) = \frac{f_2(s)}{1 - F_2(S_L)}$$

$$g(s) = \begin{cases} \alpha_1 g_1(s), & for \ \ s \le S_L \\ \alpha_2 g_2(s), & for \ \ s > S_L \end{cases}$$

$$\alpha_1 + \alpha_2 = 1$$

Appendix

The fuzzy linear regression model for σ_1*, μ_2*, σ_2*, and $a_{2\text{-OL}}$* for S_L = 200 kN (45 kips)