ENABLING THE EVIDENCE THEORY THROUGH NON-INTRUSIVE PARAMETRIC MODEL ORDER REDUCTION FOR CRASH SIMULATIONS

9TH INTERNATIONAL WORKSHOP ON RELIABLE ENGINEERING COMPUTING

May 17-20, 2021

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BMW GROUP

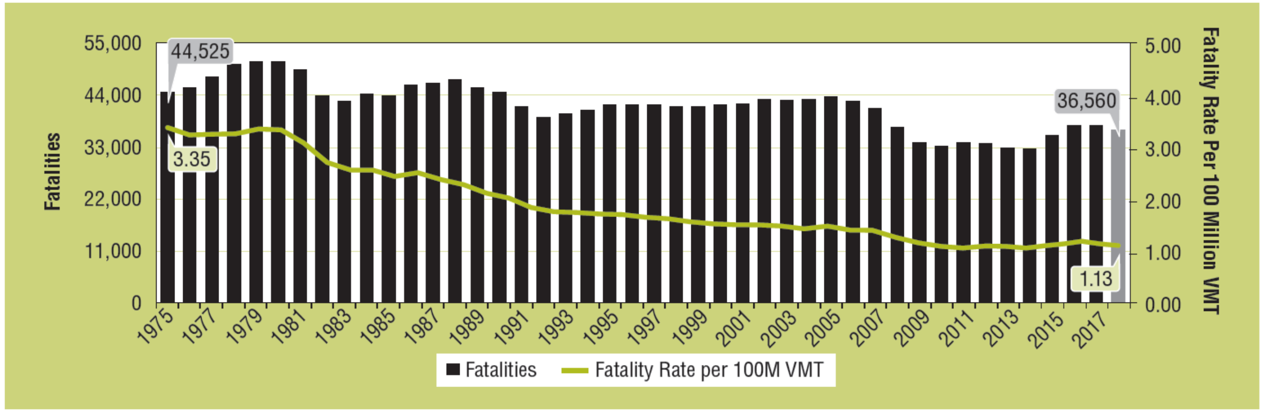
Universität der Bundeswehr München

MOTIVATION, INTRODUCTION, FRAMEWORK

MOTIVATION AND INTRODUCTION.

Global aim.

Improve the crashworthiness of cars through safety features to mitigate and avoid occupant injuries and fatalities.



[National Center for Statistics and Analysis (2019). 2018 fatal motor vehicle crashes: Overview. (Traffic Safety Facts Research Note. Report No. DOT HS 812 826). Washington, DC: National Highway Traffic Safety Administration.]

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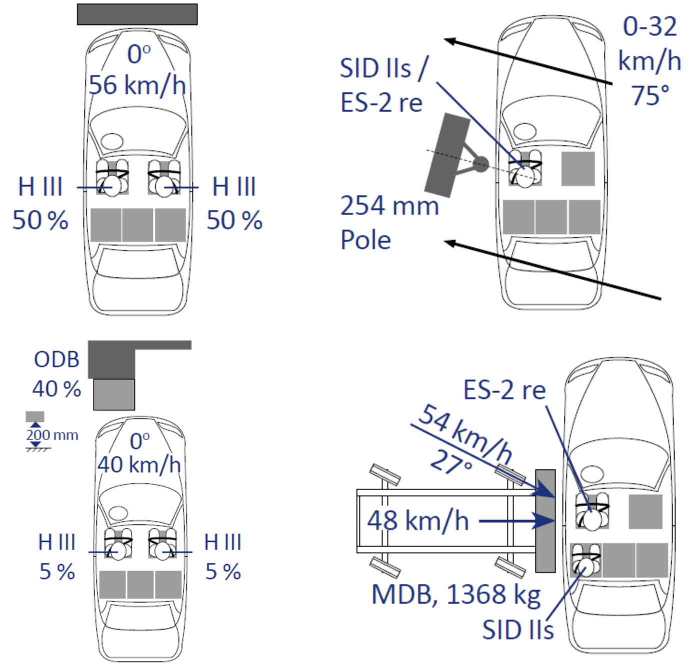
How to.

- Perform and analyze hardware crash tests.
  - High monetary costs, many disciplines, low repetitions.
- Use crash simulations to understand hardware phenomena.
  - Sensitivity Analysis,
  - Uncertainty Analysis,
  - Optimization, etc.



Pictures from [carhs (2021). Safety Companion.]

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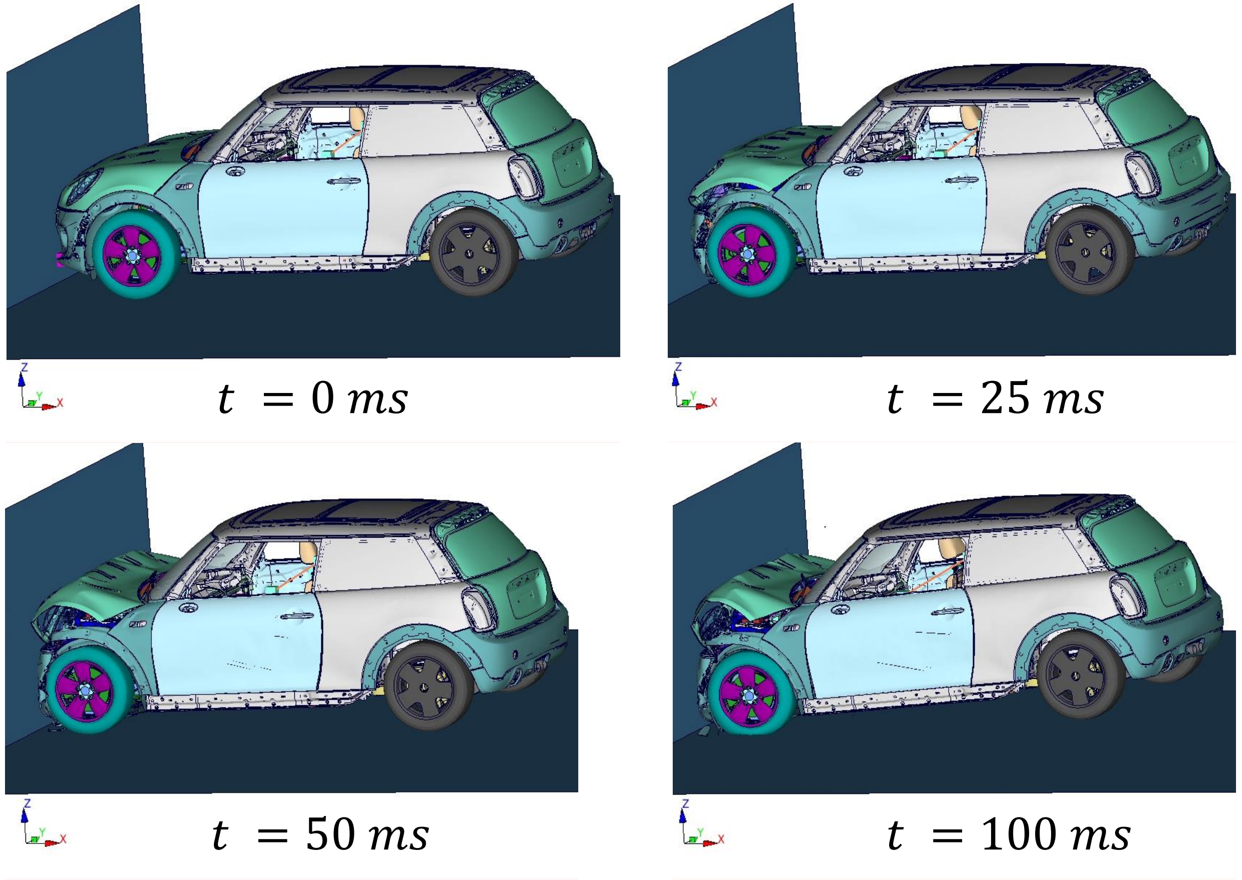


Internal finite element model of a front crash.

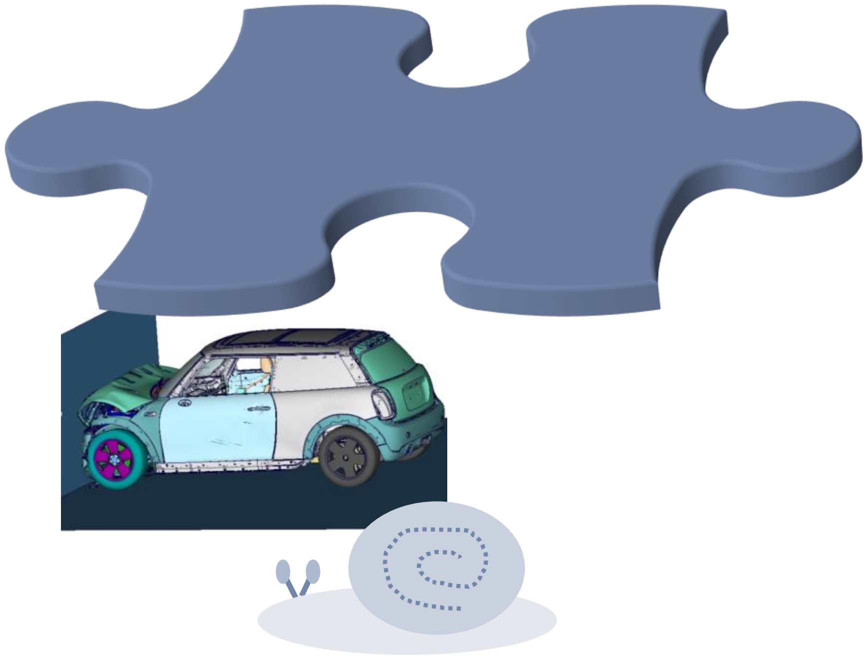
Challenges of crash simulations.

- Black box solver.
- Long computing times due to explicit computation (contact problem, complex geometry, etc.).
- Many diverse but independent inputs:
  - Test uncertainties: initial velocity, position of the dummy, etc.
  - Production uncertainties: material properties, wall thinknesses, etc.
  - Design parameters of safety systems: airbag and belt settings, etc.

PROPOSED UNCERTAINTY MANAGEMENT FRAMEWORK.

Finite Element Crash Simulation.

Long computing times, many inputs, black box character (non-intrusive), possibly numerical robustness issues



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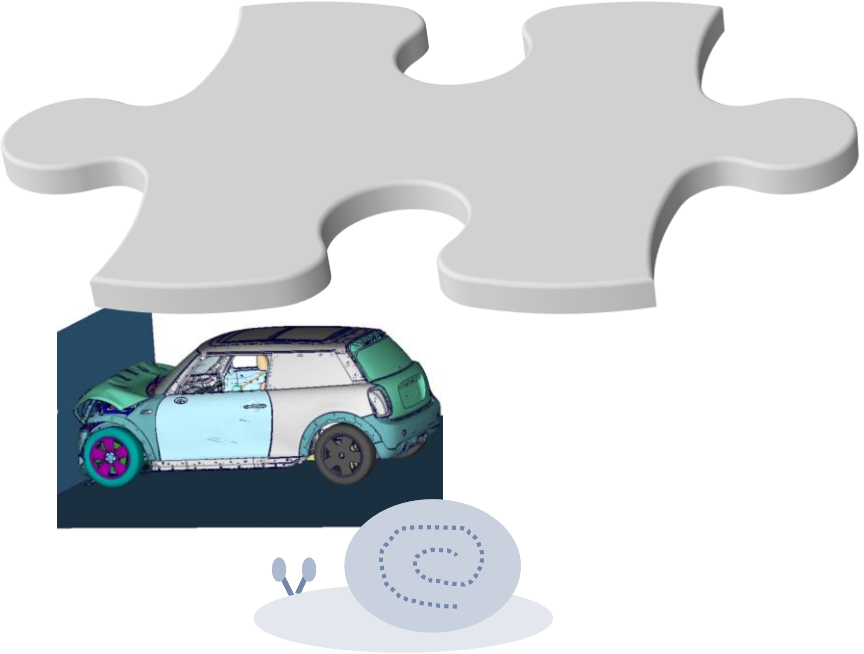
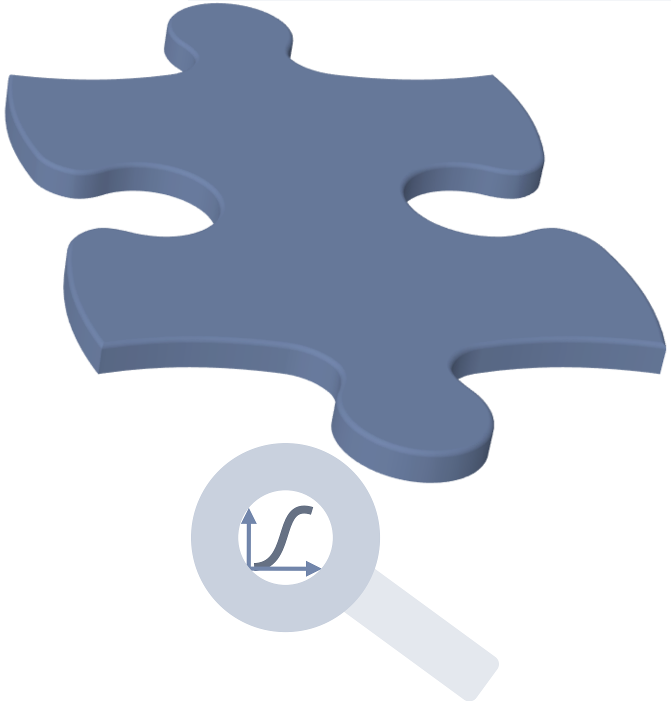
Propagating uncertainties through the system, visualization of the key results by a modified version of the evidence theory, making decisions.





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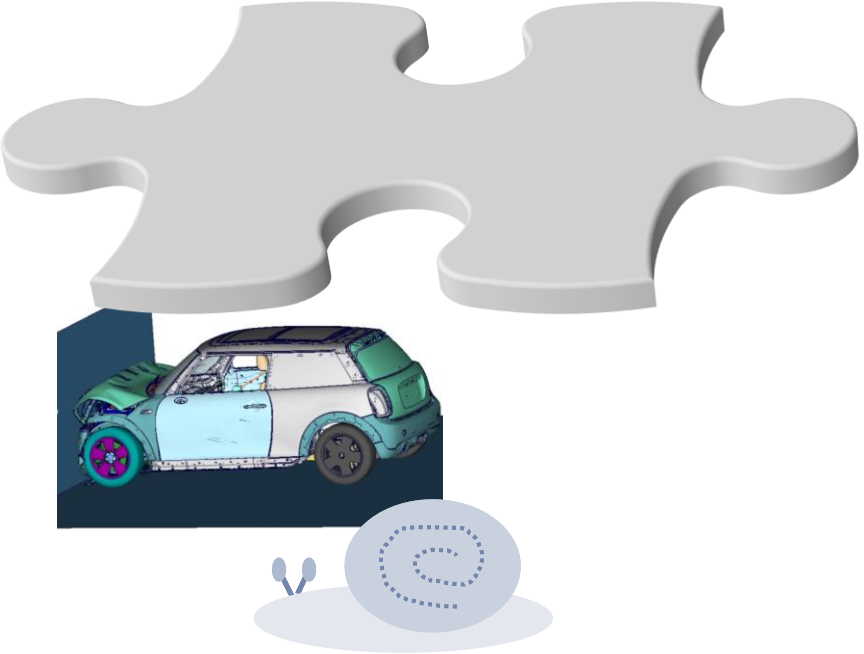

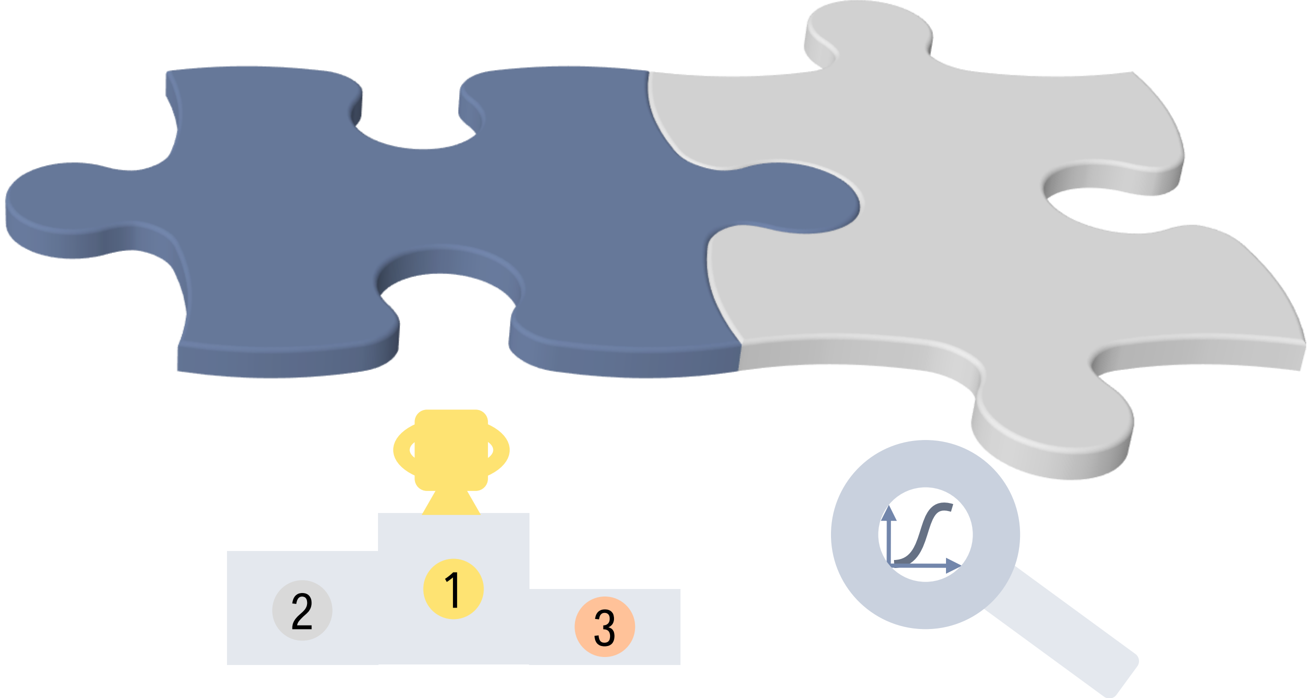

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Dimensionality Reduction and Metamodeling.

Screening, dimensionality reduction (kPCA), scalar (GP, PCE) and multi-target metamodels (NN), model order reduction

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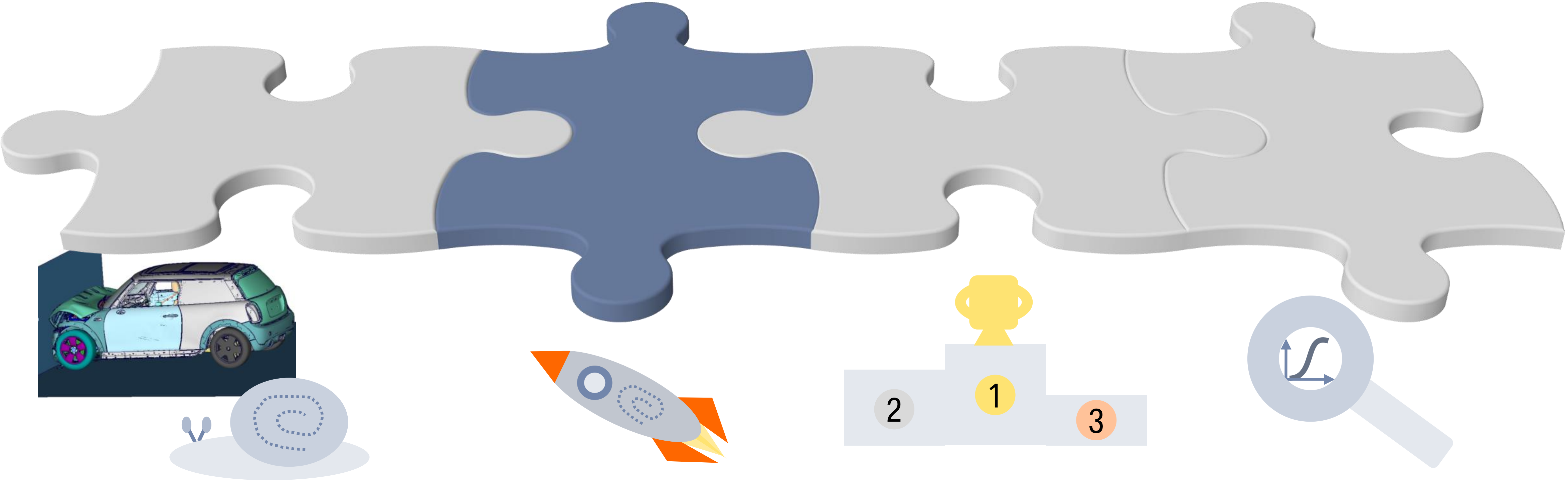

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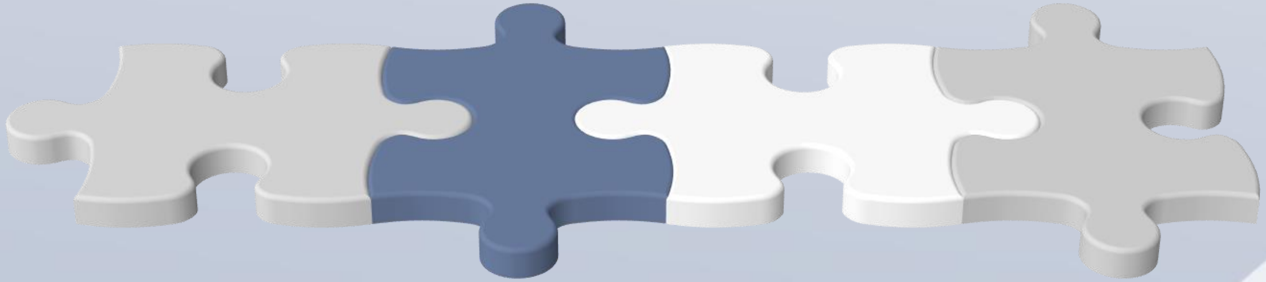
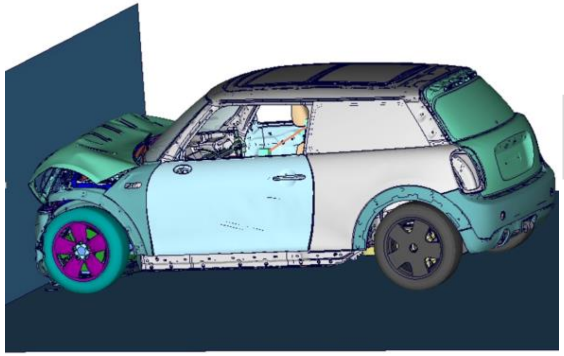
NON-INTRUSIVE PARAMETRIC MODEL ORDER REDUCTION (MOR).

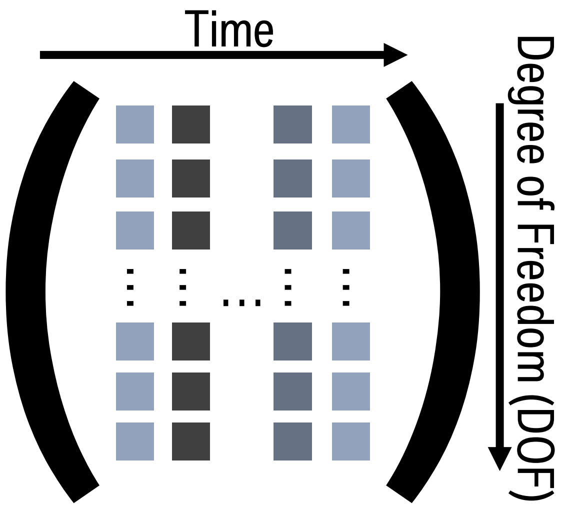


PROCEDURE OF THE MODEL ORDER REDUCTION METAMODEL.

1) Initialization and Training (Offline)



1 Snapshot = 1 Simulation
= 1 Parameter Configuration



Mathematical Form of the Snapshot

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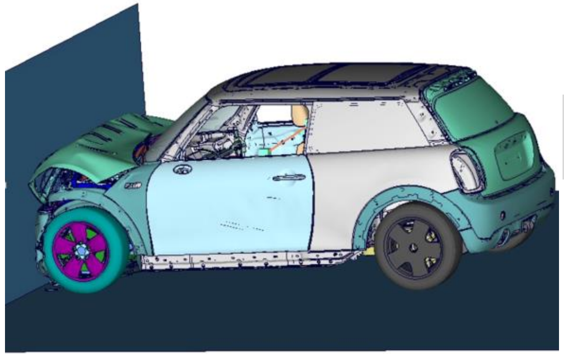

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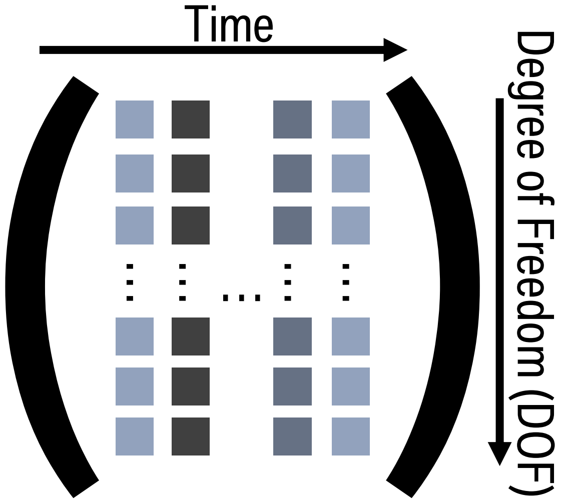
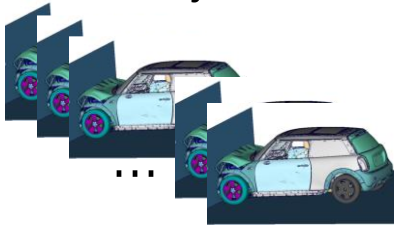


Mathematical Form of the Snapshot

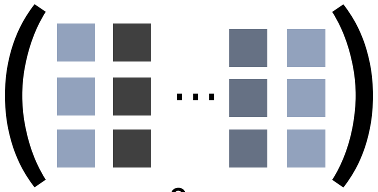
Set up **Snapshot Matrix** by stacking different Snapshots
↓

Dimension Reduction

via Projection Matrix obtained by SVD



Reduced Space



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Training **Metamodels** in the Reduced Space

Training must be done once!

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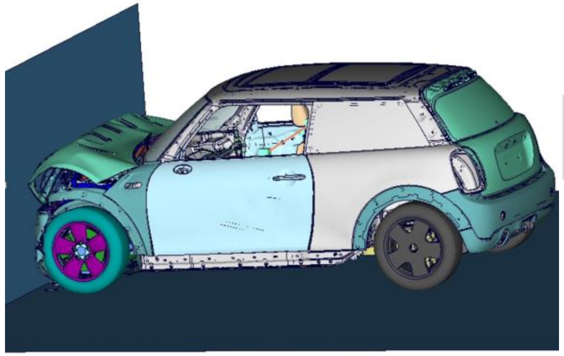

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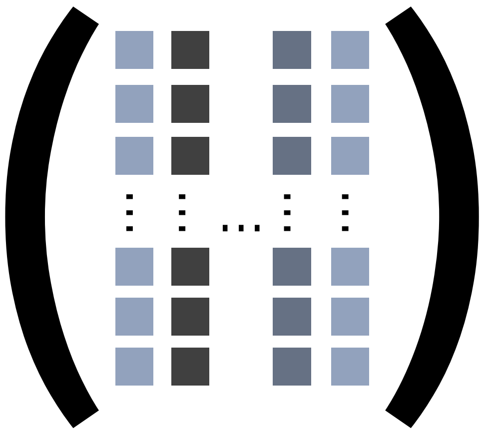
Time



Degree of Freedom (DOF)

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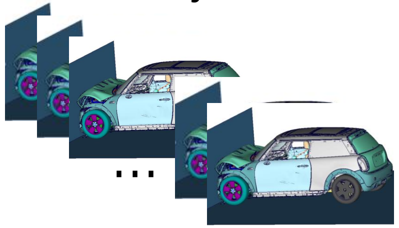

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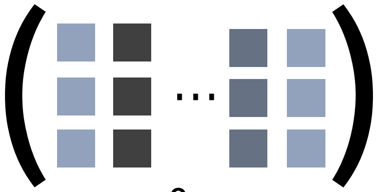

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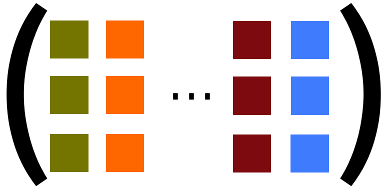
Making Predictions



in the Reduced Space by evaluating the Metamodels

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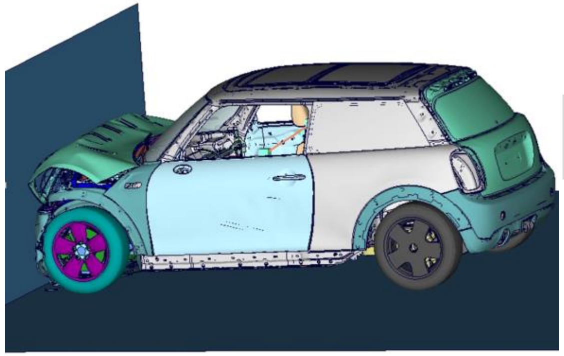

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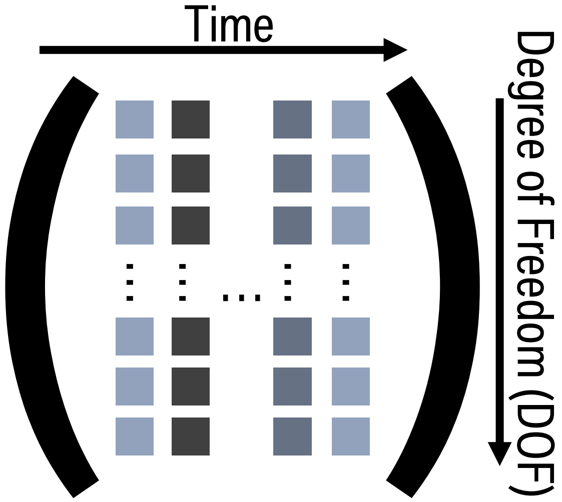

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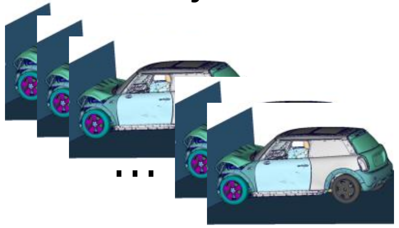

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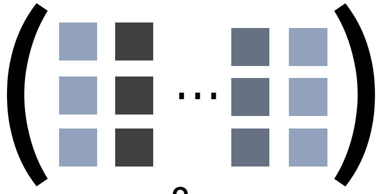

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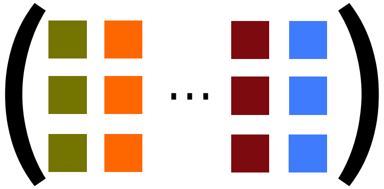
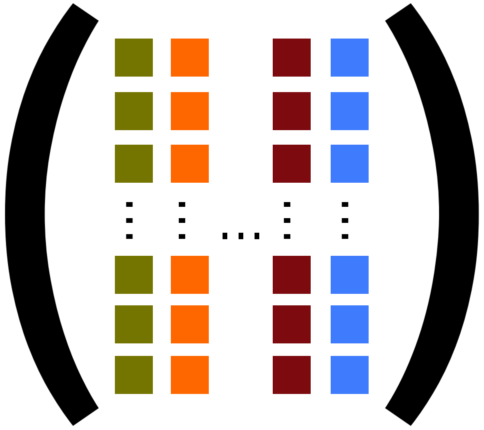


in the Reduced Space by evaluating the Metamodels

Projection into full space
via Projection Matrix



Approximated Snapshot

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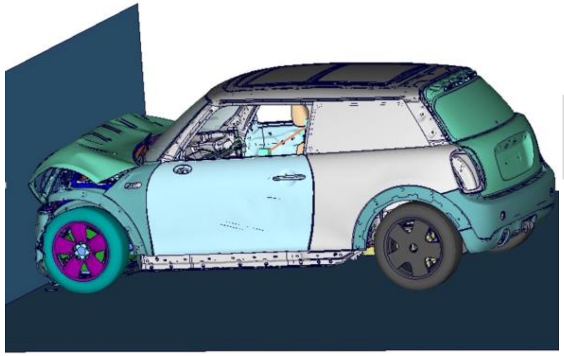

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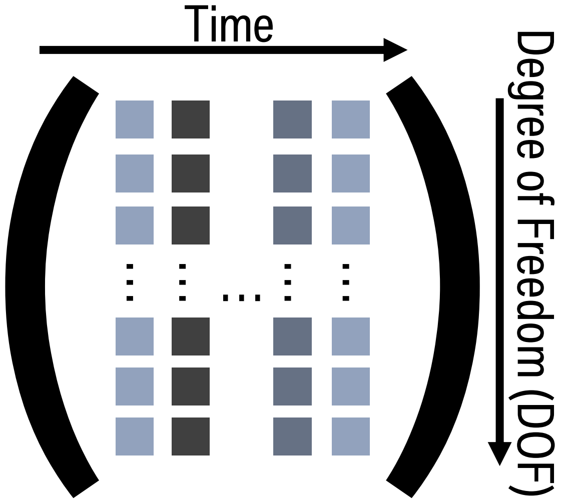

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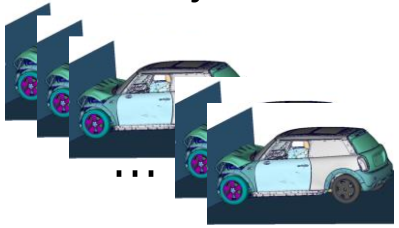

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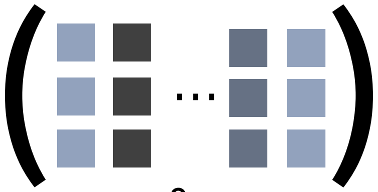

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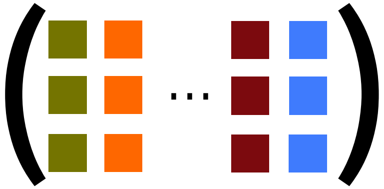

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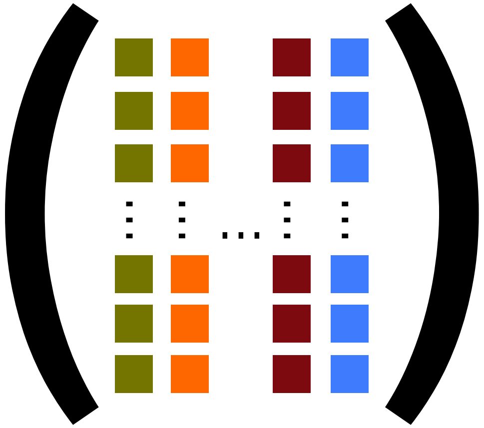
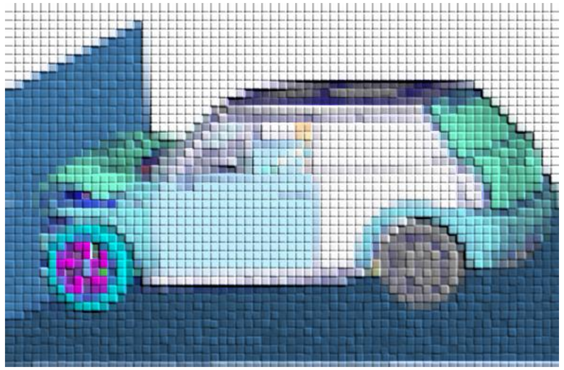
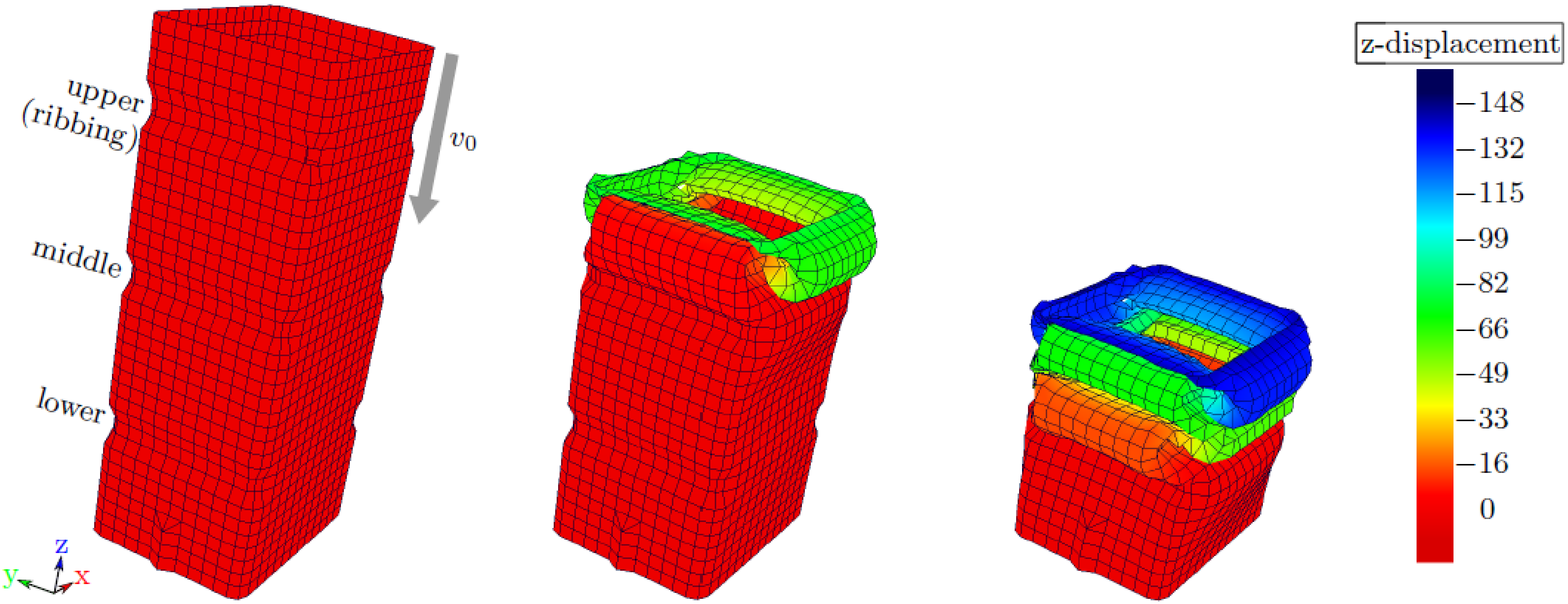


Approximated Snapshot



Approximated Simulation

APPLICATION EXAMPLE: LS-DYNA CRASHBOX WITH FIVE INPUT PARAMETERS.



(a) Time state 1 (b) Time state 11 (c) Time state 22

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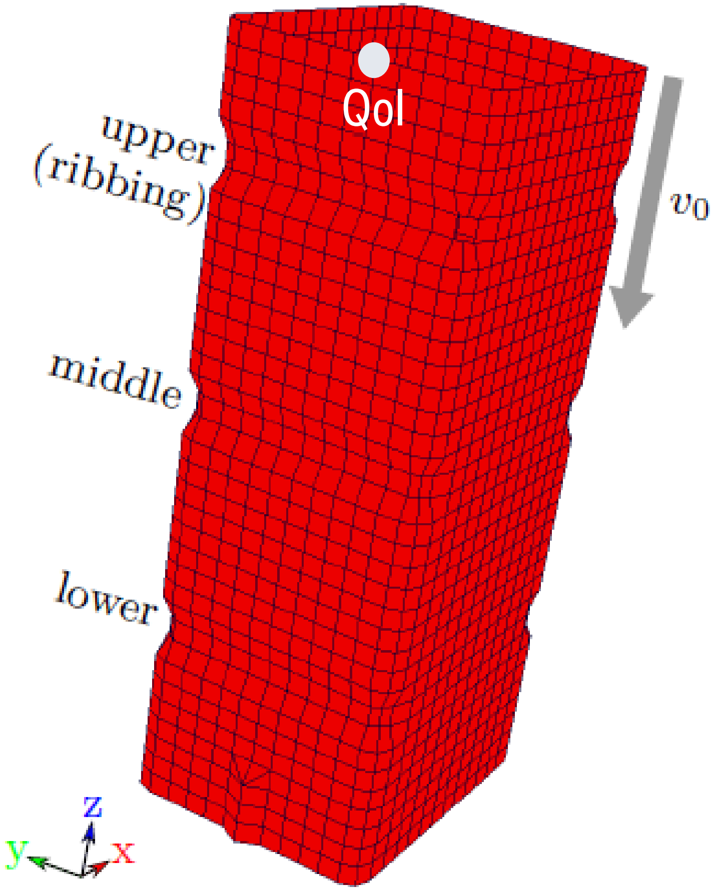


- Inputs of the LS-DYNA Crashbox Model:

Input	Interval	Unit	Uncertainty type	Description
μ_1	$[2, 3]$	mm	epistemic	wall thickness
μ_2	$[8.5, 9]$	$\frac{m}{s}$	aleatory	initial velocity
μ_3	$[-3, 1]$	mm	epistemic	relative upper ribbings depth
μ_4	$[-3, 1]$	mm	epistemic	relative middle ribbings depth
μ_5	$[-3, 1]$	mm	epistemic	relative lower ribbings depth

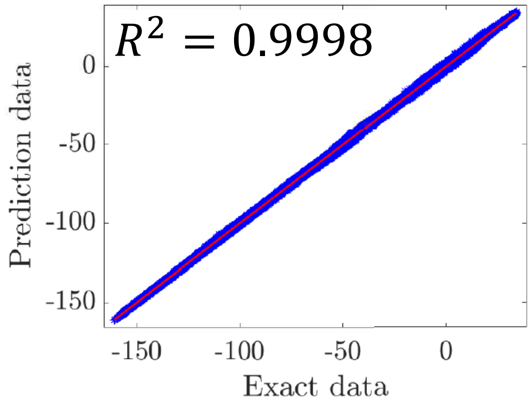
- Output: All DOF Displacement Histories of the Crashbox
- Considered QoI for further Analysis: Maximum Displacement of one Top Node

- Details of the FE Model:
  - 22 seconds Computing Time
  - 1925 nodes, i.e. $3 \cdot 1925 = 5775$ DOFs (x-,y-,z-Direction)

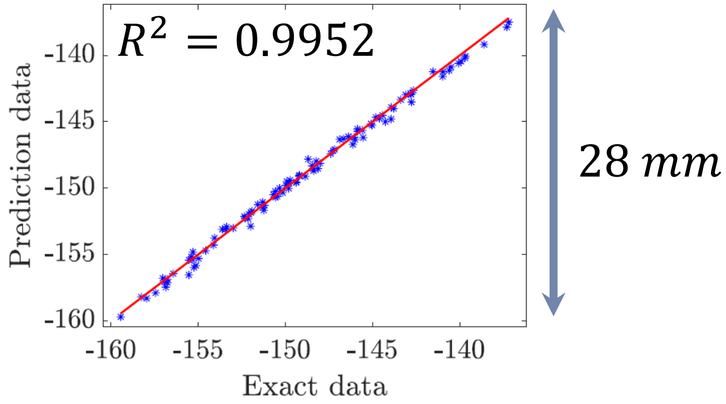
- MOR Metamodel:
  - Trained with 100 Simulations (20 for the Dimension Reduction, 80 for the Kriging Metamodel) created by Latin Hypercube Sampling
  - Validated on another 100 Simulations

APPLICATION EXAMPLE: LS-DYNA CRASHBOX WITH FIVE INPUT PARAMETERS.

Actual vs. Predicted Plots



Plot for all DOFs of the system.



Plot for the QoI values.

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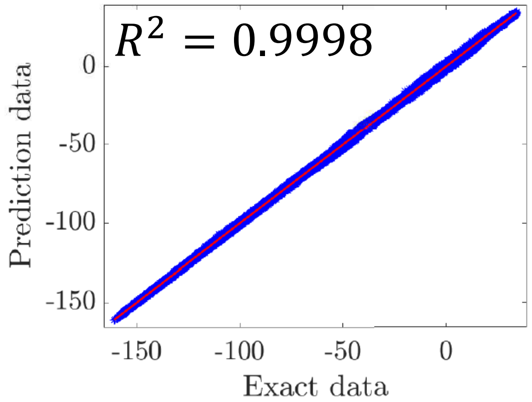

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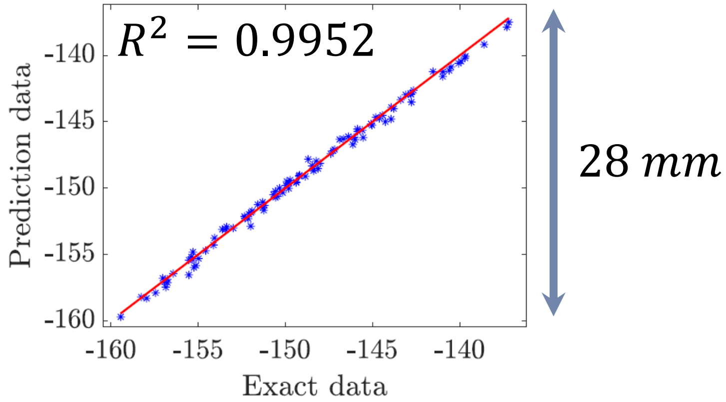
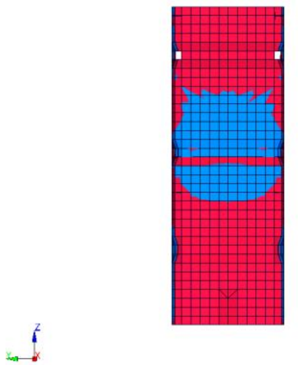
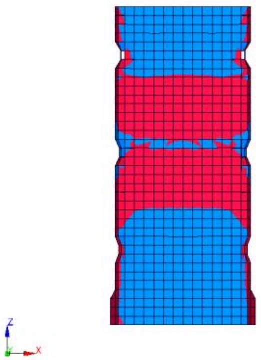
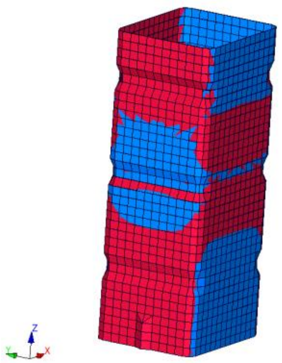


Plot for the QoI values.

Comparison of the exact and approximated simulation video.

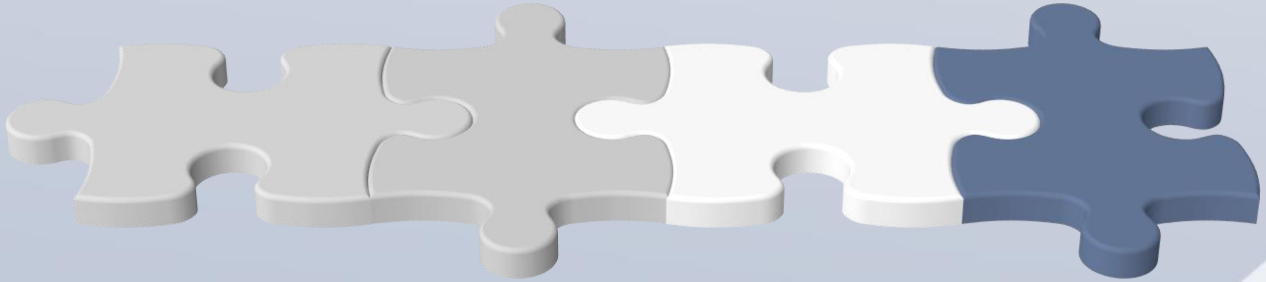






Red: Exact (22s, Finite Element Simulation),
Blue: Predicted (≪1s, MOR Metamodel)

UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY.



UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY USING THE MOR MODEL.

Why we chose the evidence theory.

- Easy to set and test different rough or fine uncertainties for input variables (Experts‘ task).

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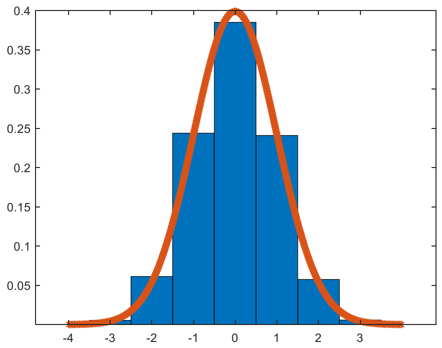

PDF of standard normal distribution

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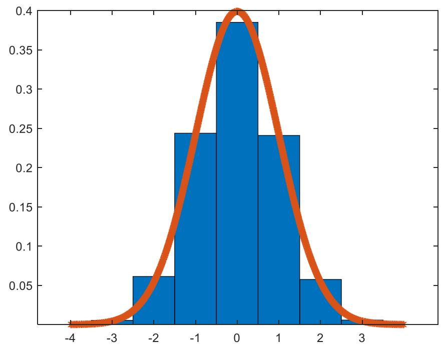

PDF of standard normal distribution

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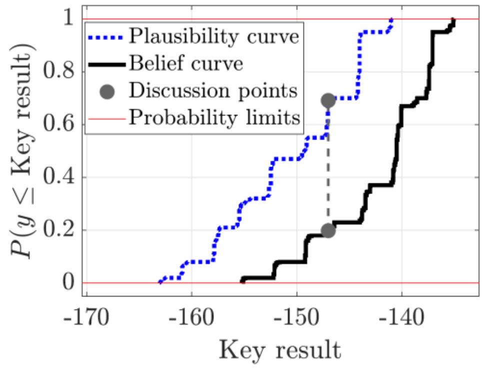
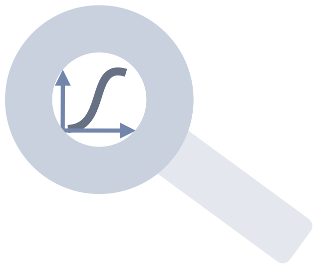
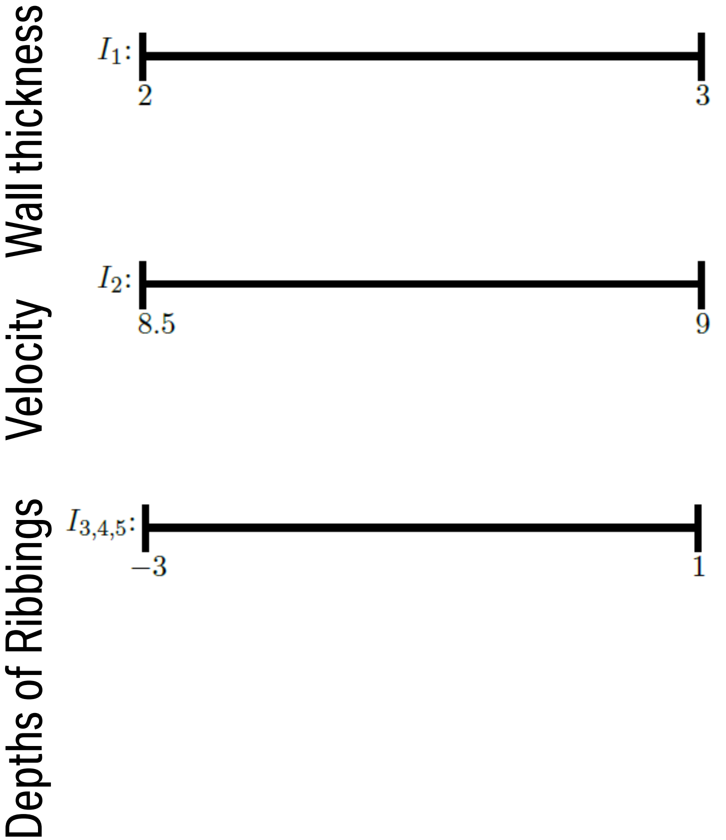
- Understandable visualization of a corridor for the cumulative distribution function (CDF).
- Visualization can be used for decisions.
  - ➢ Do we have to improve or
  - ➢ is the risk low enough?





UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY USING THE MOR MODEL.

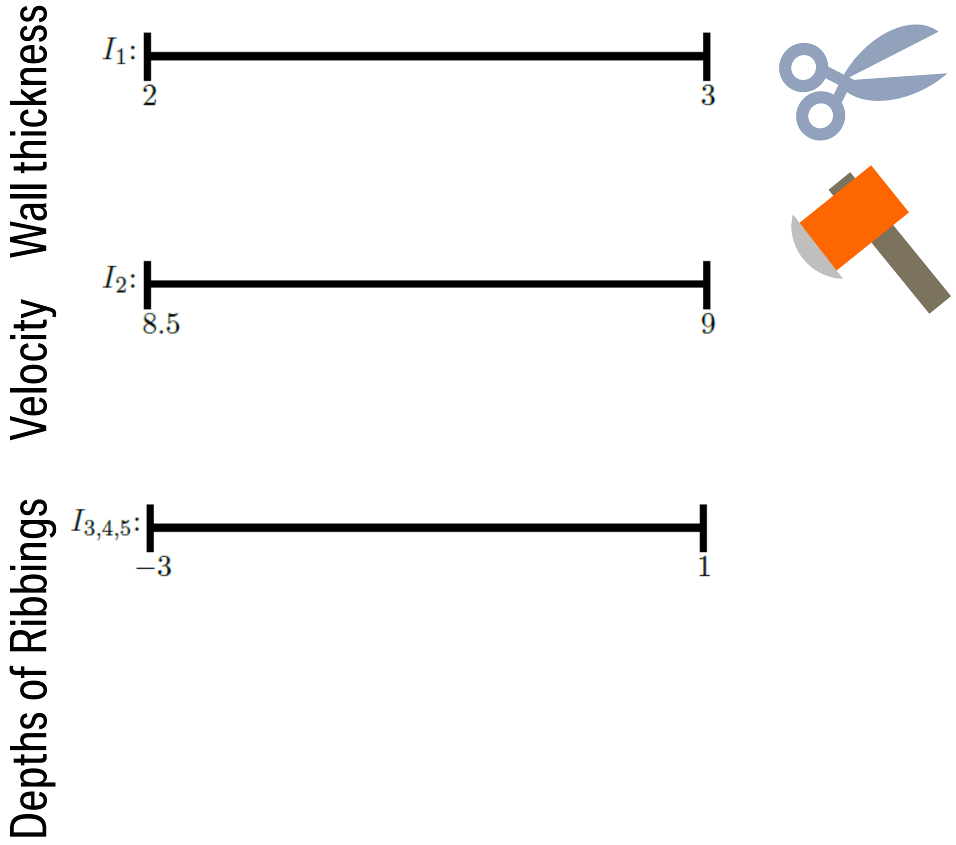
Illustrative description of the evidence theory – Initial position.



Fictive allocation!

UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY USING THE MOR MODEL.

Illustrative description of the evidence theory – Split into subintervals (Experts).



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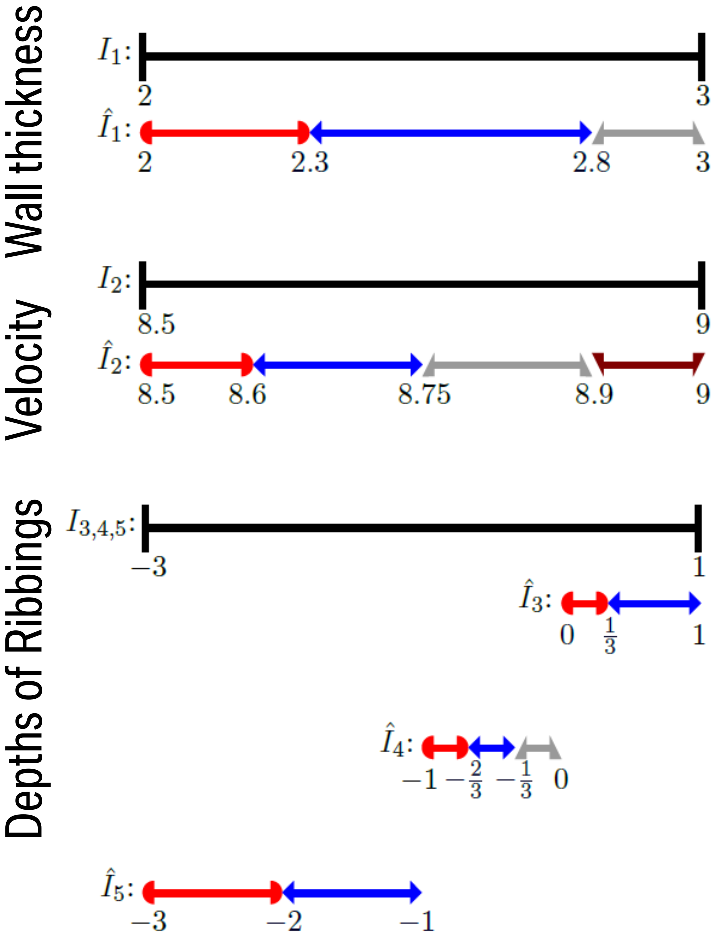
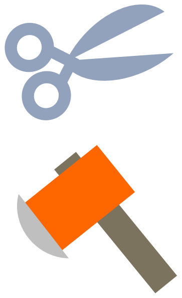
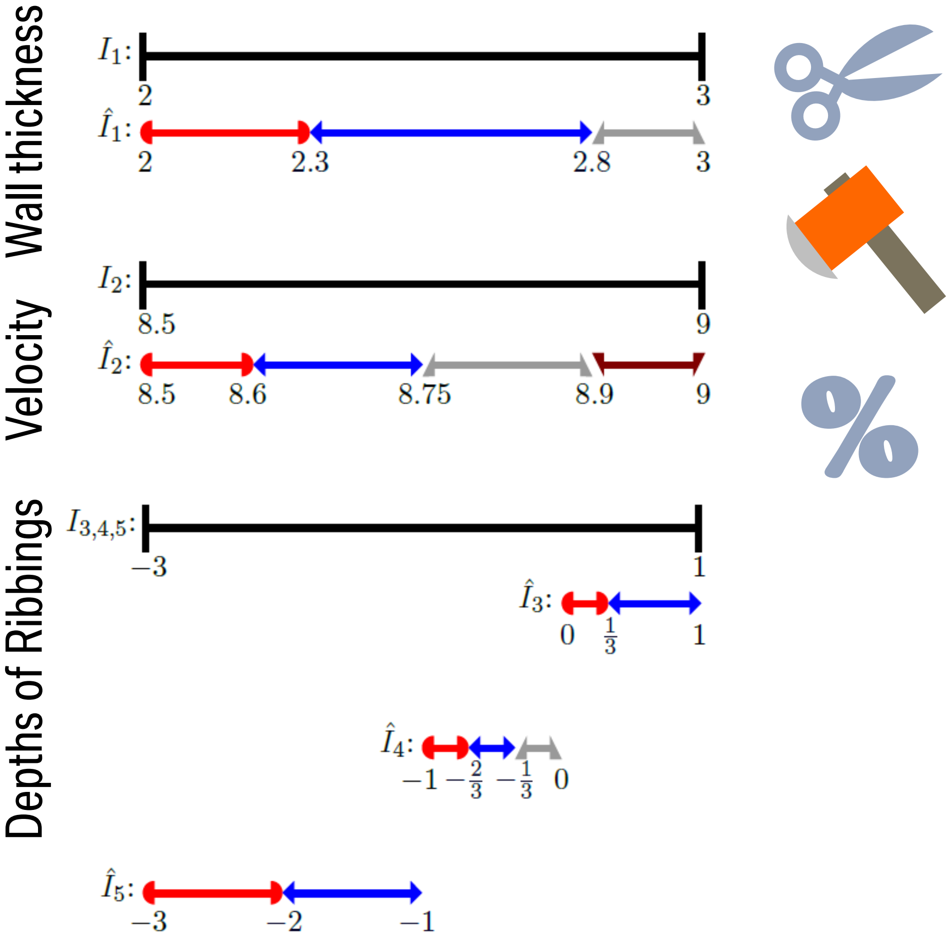




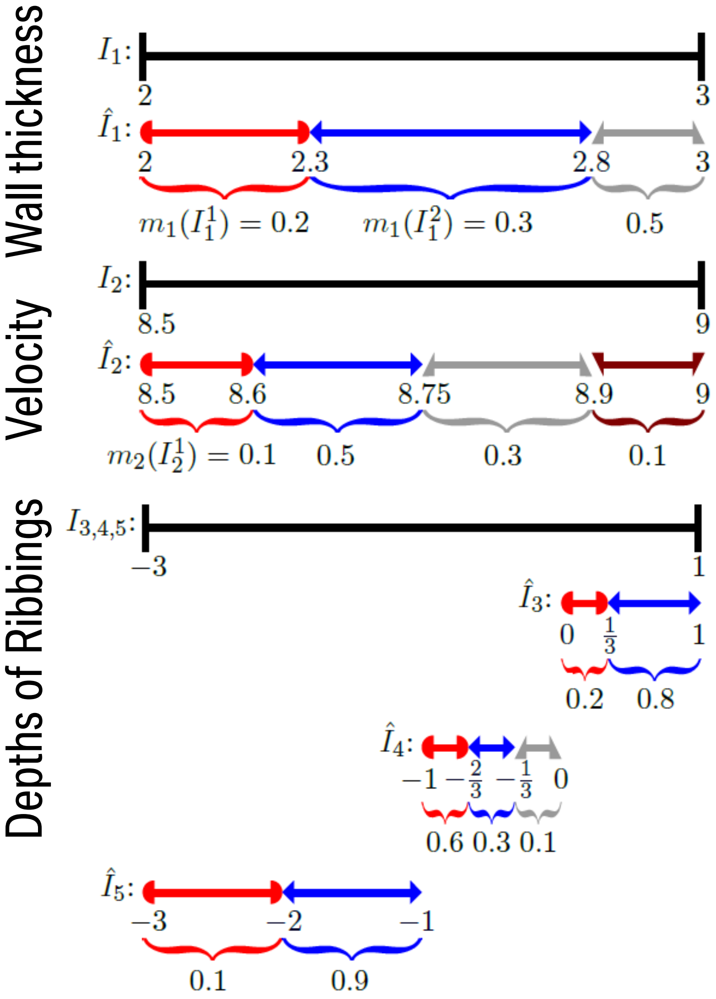
UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY USING THE MOR MODEL.

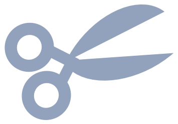
Illustrative description of the evidence theory – Basic Probability Assignment (Experts).

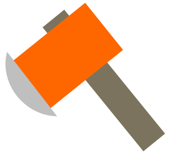


UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY USING THE MOR MODEL.

Illustrative description of the evidence theory – Basic Probability Assignment (Experts).

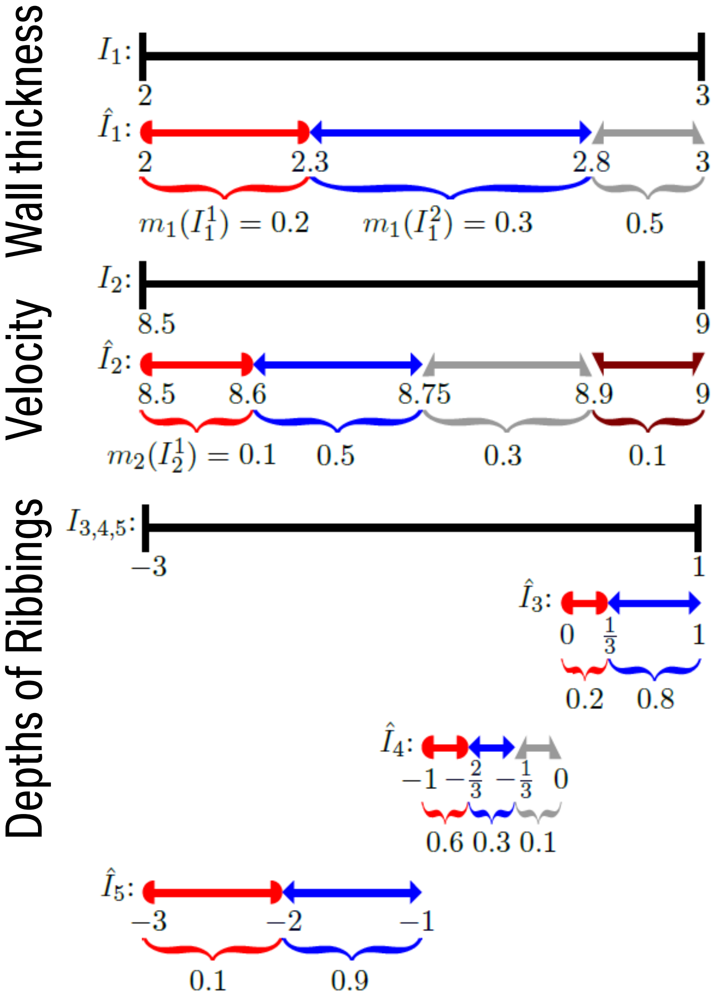


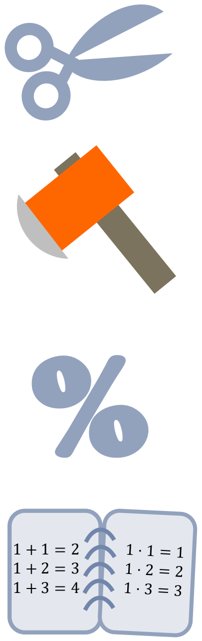




UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY USING THE MOR MODEL.

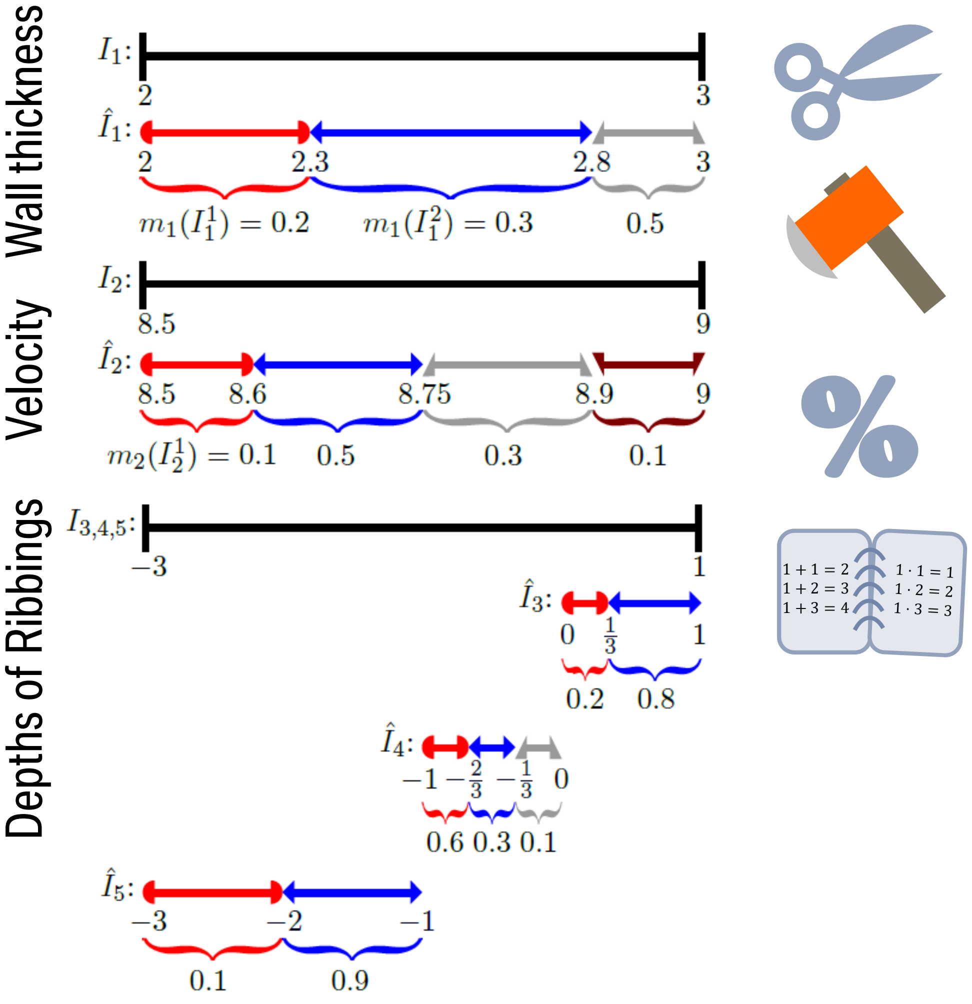
Illustrative description of the evidence theory – Propagate Uncertainties.





UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY USING THE MOR MODEL.

Illustrative description of the evidence theory – Propagate Uncertainties.



1. Build interval cells (combinations of subintervals, in total 144).

$$C^1 = I_1^1 \times I_2^1 \times I_3^1 \times I_4^1 \times I_5^1 = \bullet \times \bullet \times \bullet \times \bullet \times \bullet$$
$$C^2 = I_1^1 \times I_2^1 \times I_3^1 \times I_4^1 \times I_5^2 = \bullet \times \bullet \times \bullet \times \bullet \times \blacklozenge$$
$$\vdots$$
$$C^{144} = I_1^3 \times I_2^4 \times I_3^2 \times I_4^3 \times I_5^2 = \blacktriangle \times \blacktriangledown \times \blacklozenge \times \blacktriangle \times \blacklozenge$$

2. Find minimum, L_i, and maximum, U_i, of the system function (**MOR Metamodel**) for every interval cell C^i, $i = \{1, \dots, 144\}$.

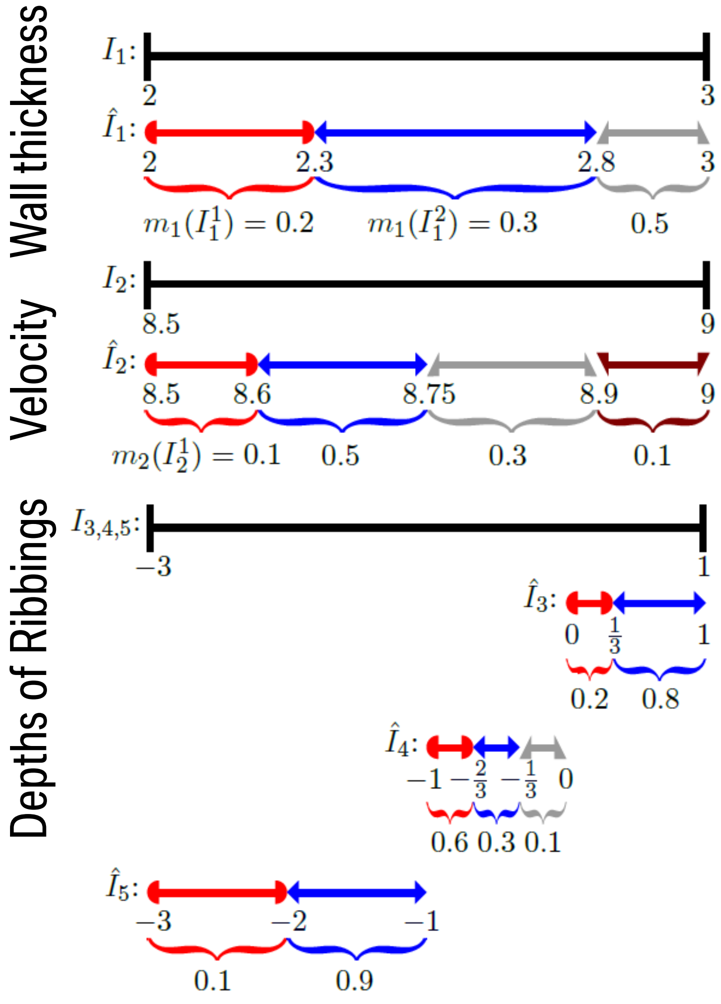
3. Calculate probabilities and allocate them to L_i and U_i.

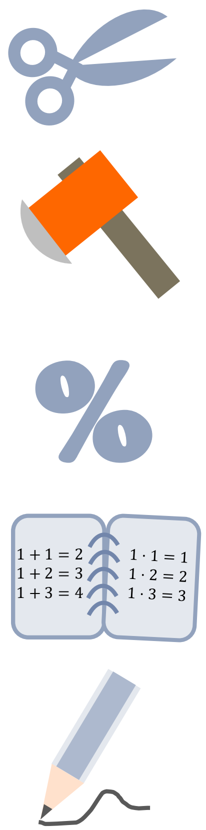
$$p^1 = 0.2 \cdot 0.1 \cdot 0.2 \cdot 0.6 \cdot 0.1 = 2.4 \cdot 10^{-4}, \quad (\bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet)$$
$$p^2 = 0.2 \cdot 0.1 \cdot 0.2 \cdot 0.6 \cdot 0.9 = 0.0022, \quad (\bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \blacklozenge)$$
$$\vdots$$
$$p^{144} = 0.5 \cdot 0.1 \cdot 0.8 \cdot 0.1 \cdot 0.9 = 0.0036. \quad (\blacktriangle \cdot \blacktriangledown \cdot \blacklozenge \cdot \blacktriangle \cdot \blacklozenge)$$

4. Sort vectors L and U and cumulate probabilities.

UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY USING THE MOR MODEL.

Illustrative description of the evidence theory – Plot Results.





1. Build interval cells (combinations of subintervals, in total 144).

$$C^1 = I_1^1 \times I_2^1 \times I_3^1 \times I_4^1 \times I_5^1 = ● \times ● \times ● \times ● \times ●$$
$$C^2 = I_1^1 \times I_2^1 \times I_3^1 \times I_4^1 \times I_5^2 = ● \times ● \times ● \times ● \times ◆$$
$$\vdots$$
$$C^{144} = I_1^3 \times I_2^4 \times I_3^2 \times I_4^3 \times I_5^2 = ▲ \times ▼ \times ◆ \times ▲ \times ◆$$

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3. Calculate probabilities and allocate them to L_i and U_i.

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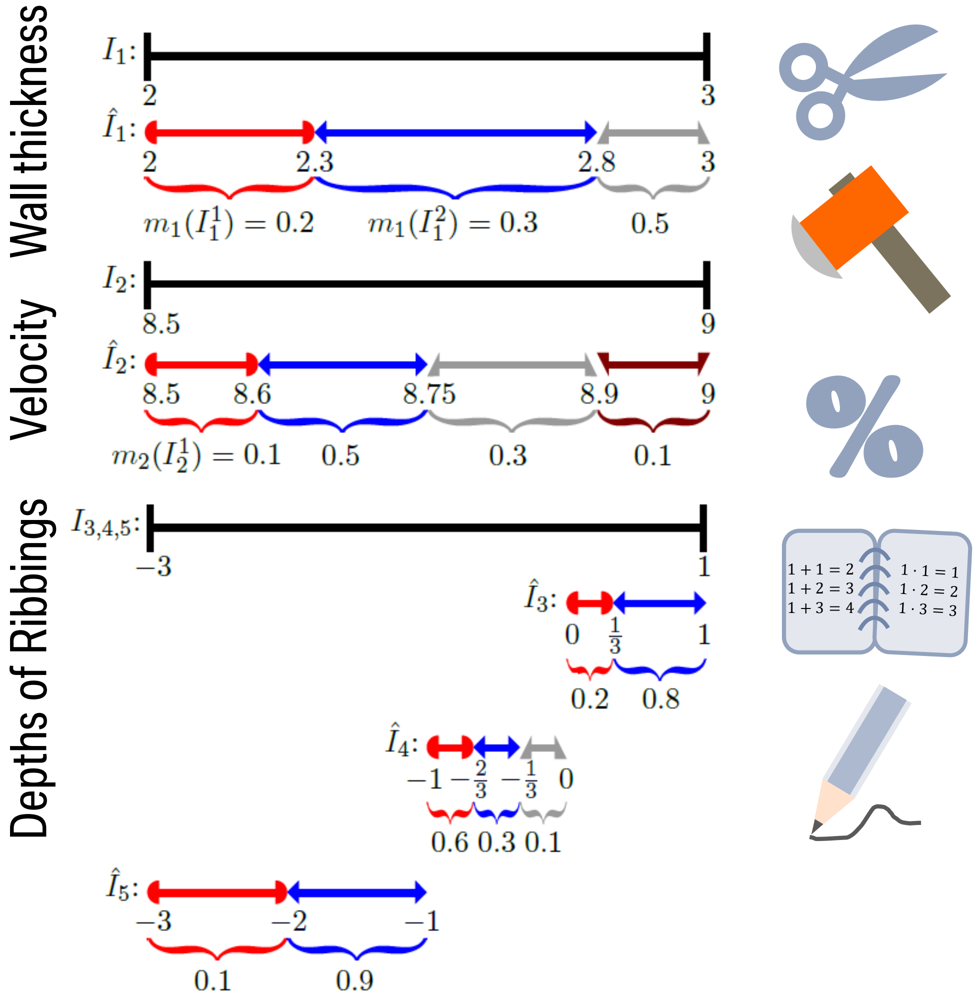
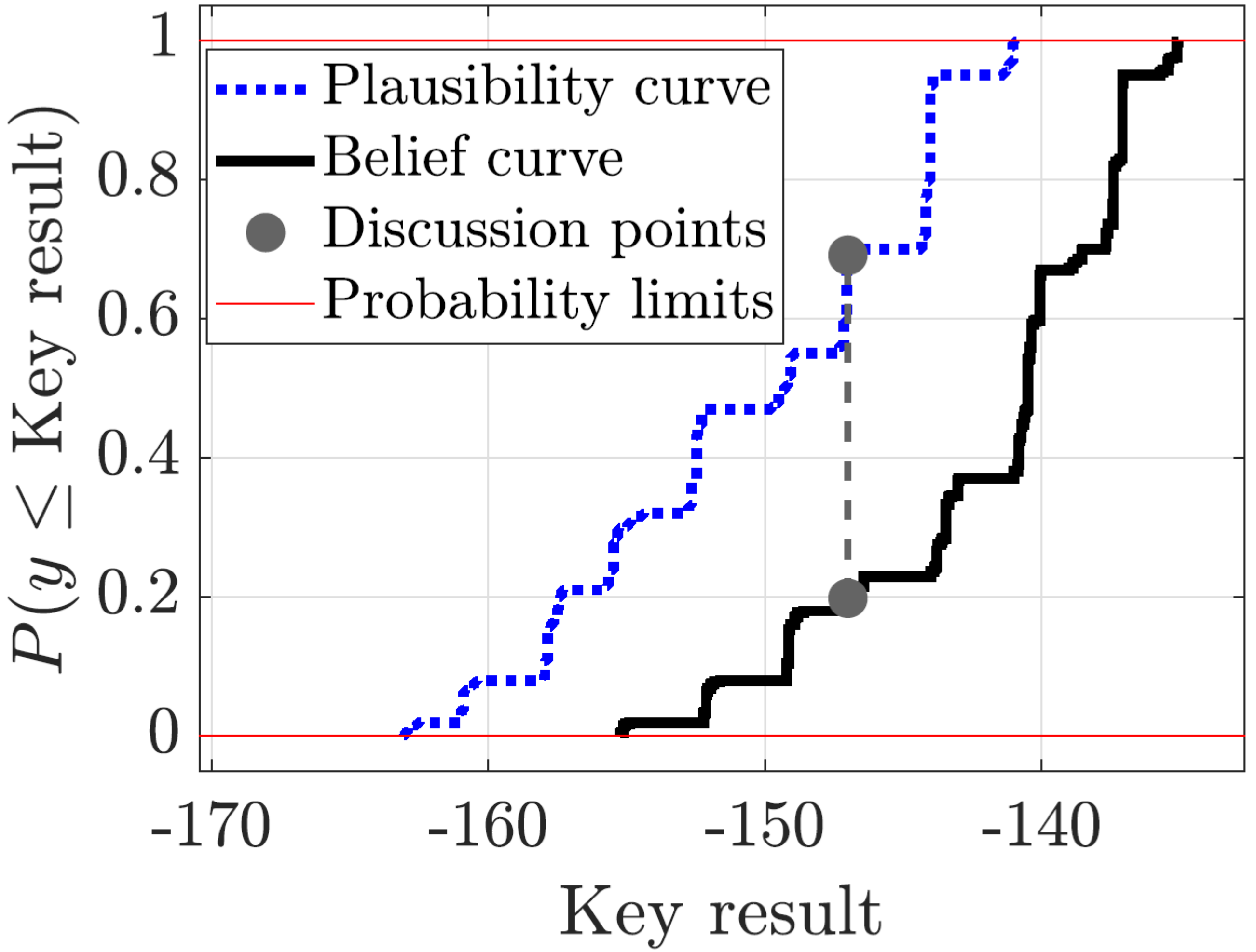
$$p^2 = 0.2 \cdot 0.1 \cdot 0.2 \cdot 0.6 \cdot 0.9 = 0.0022, \quad (● \cdot ● \cdot ● \cdot ● \cdot ◆)$$
$$\vdots$$
$$p^{144} = 0.5 \cdot 0.1 \cdot 0.8 \cdot 0.1 \cdot 0.9 = 0.0036. \quad (▲ \cdot ▼ \cdot ◆ \cdot ▲ \cdot ◆)$$

4. Sort vectors L and U and cumulate probabilities.

UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY USING THE MOR MODEL.

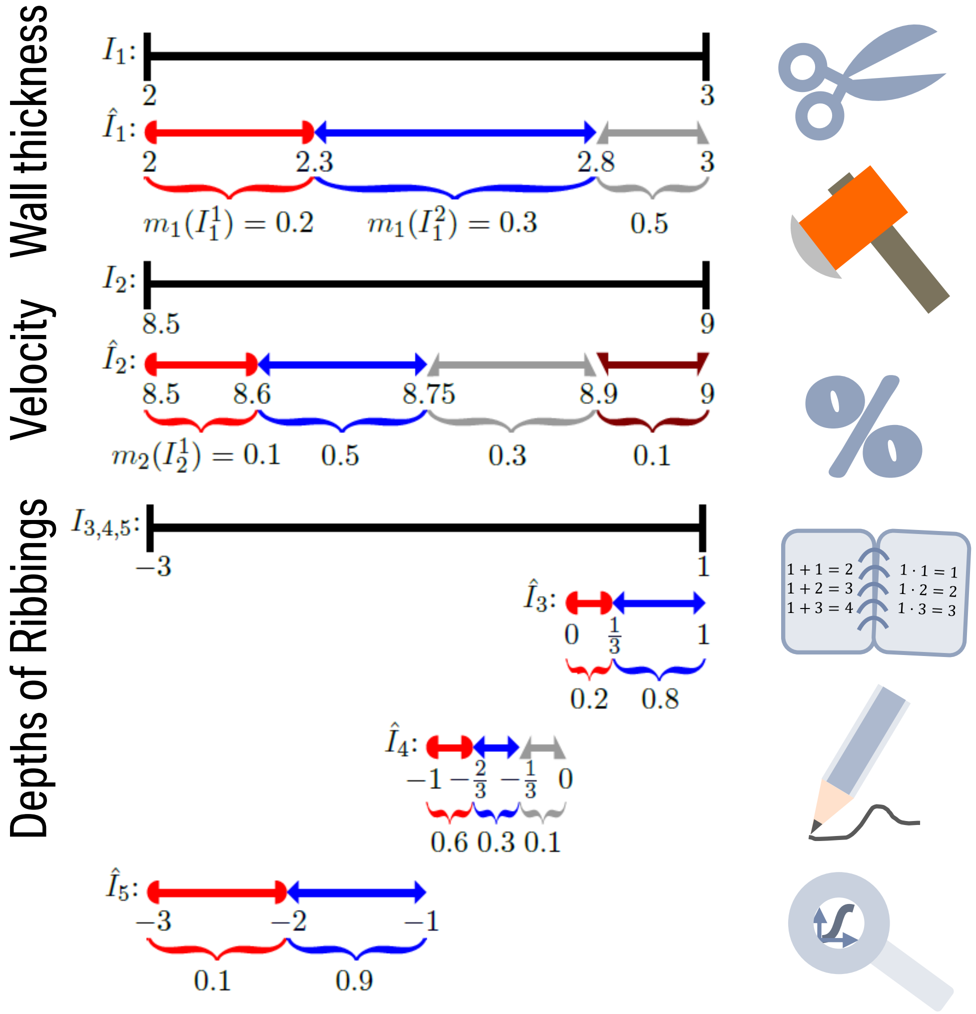
Illustrative description of the evidence theory – Plot Results.

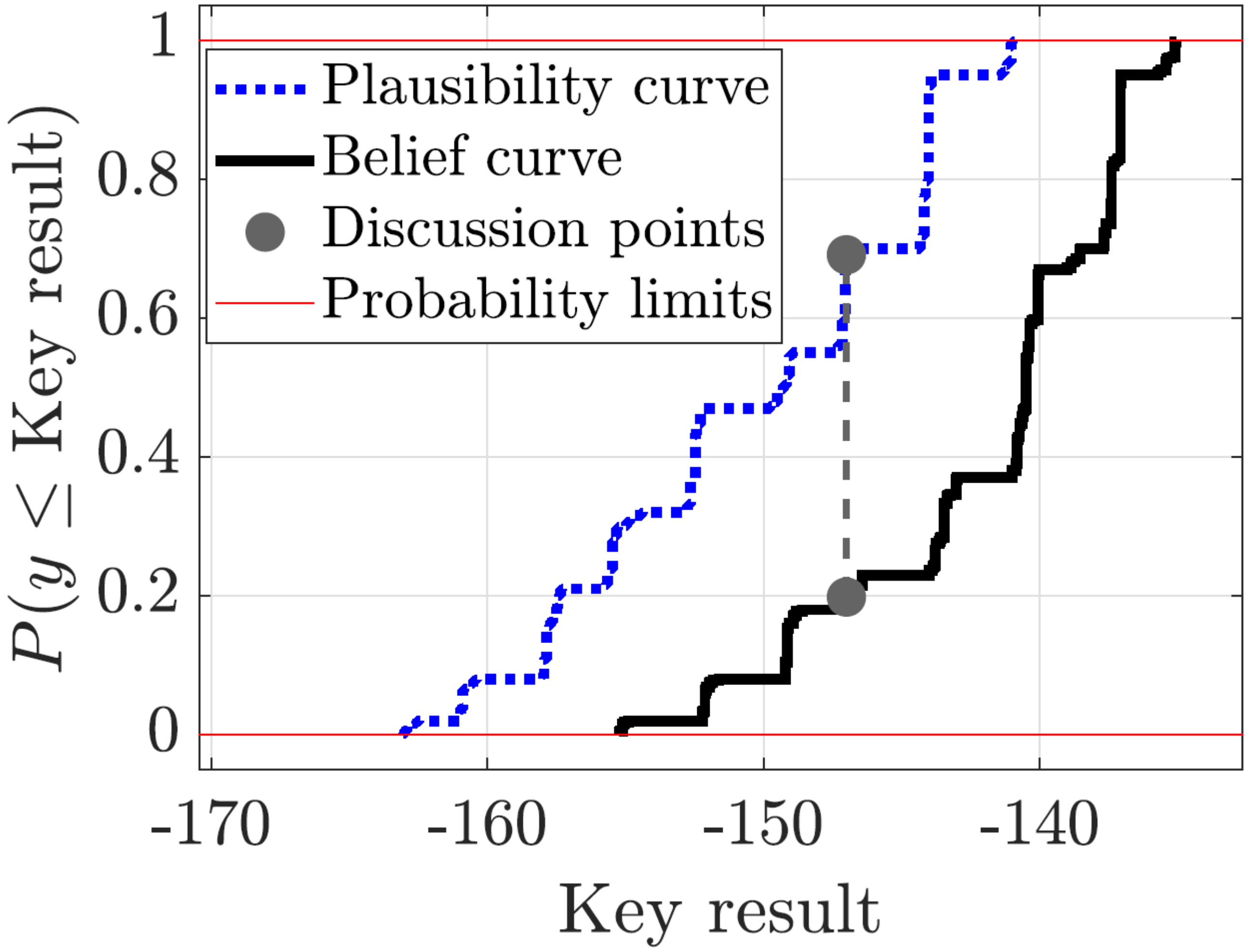




UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY USING THE MOR MODEL.

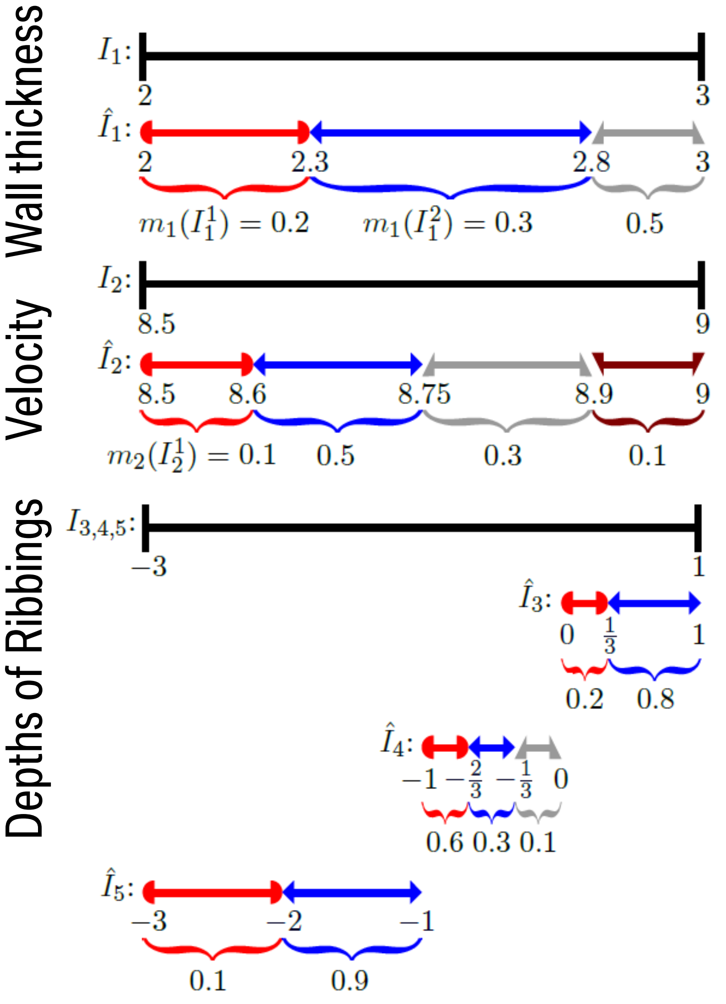
Illustrative description of the evidence theory – Discuss Results.

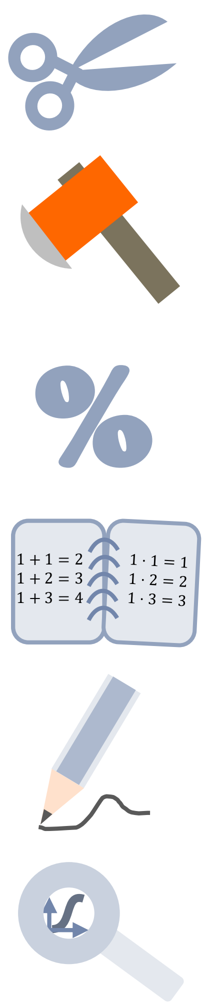


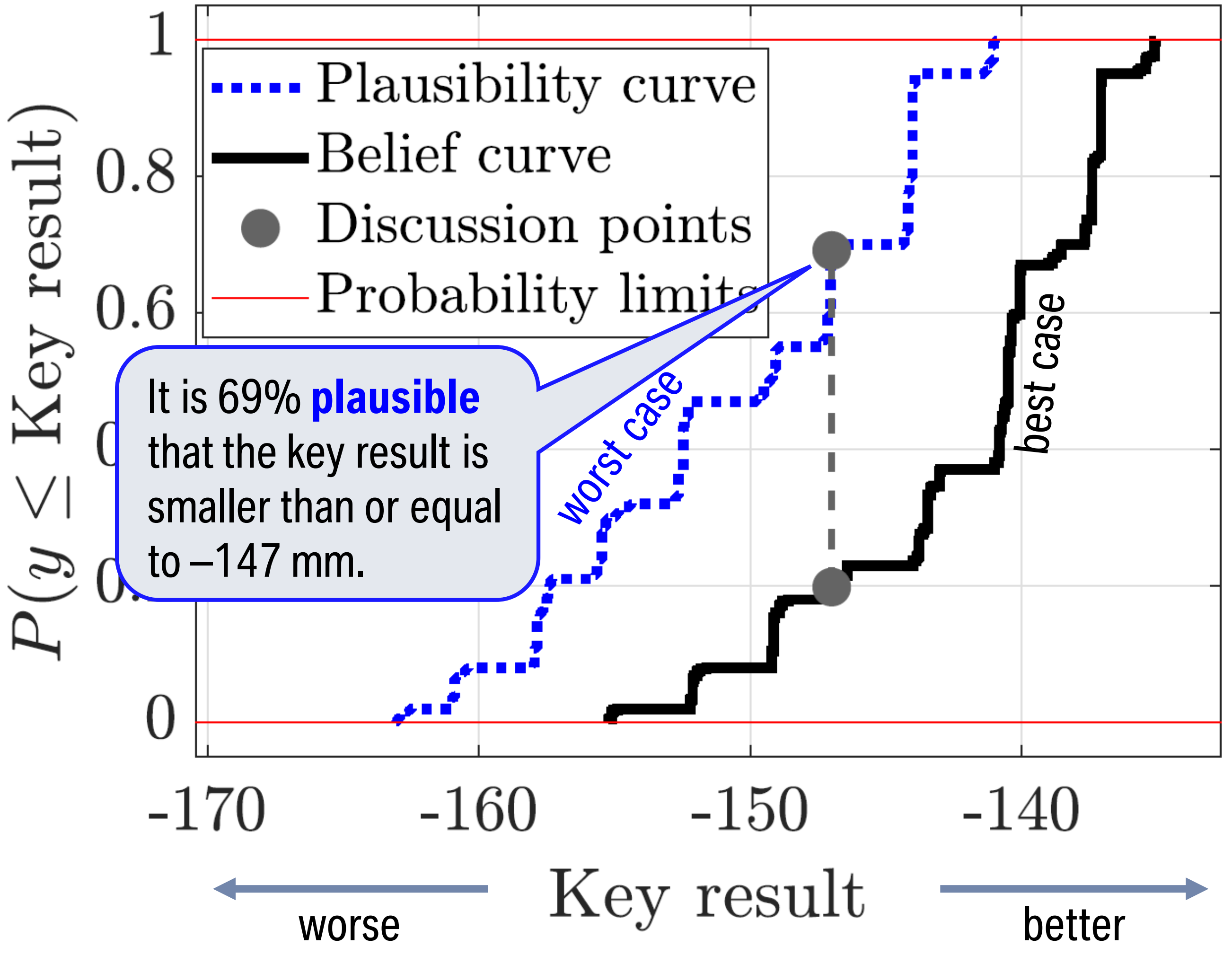


UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY USING THE MOR MODEL.

Illustrative description of the evidence theory – Discuss Results.

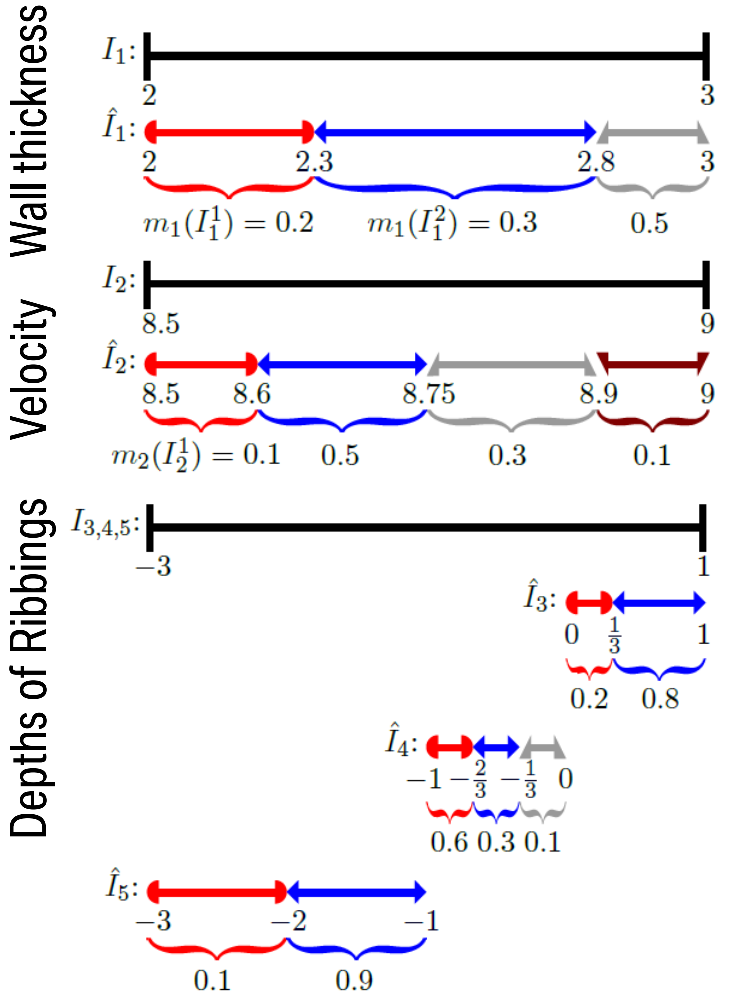


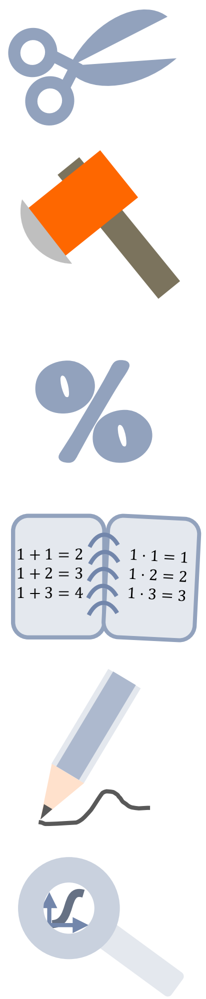


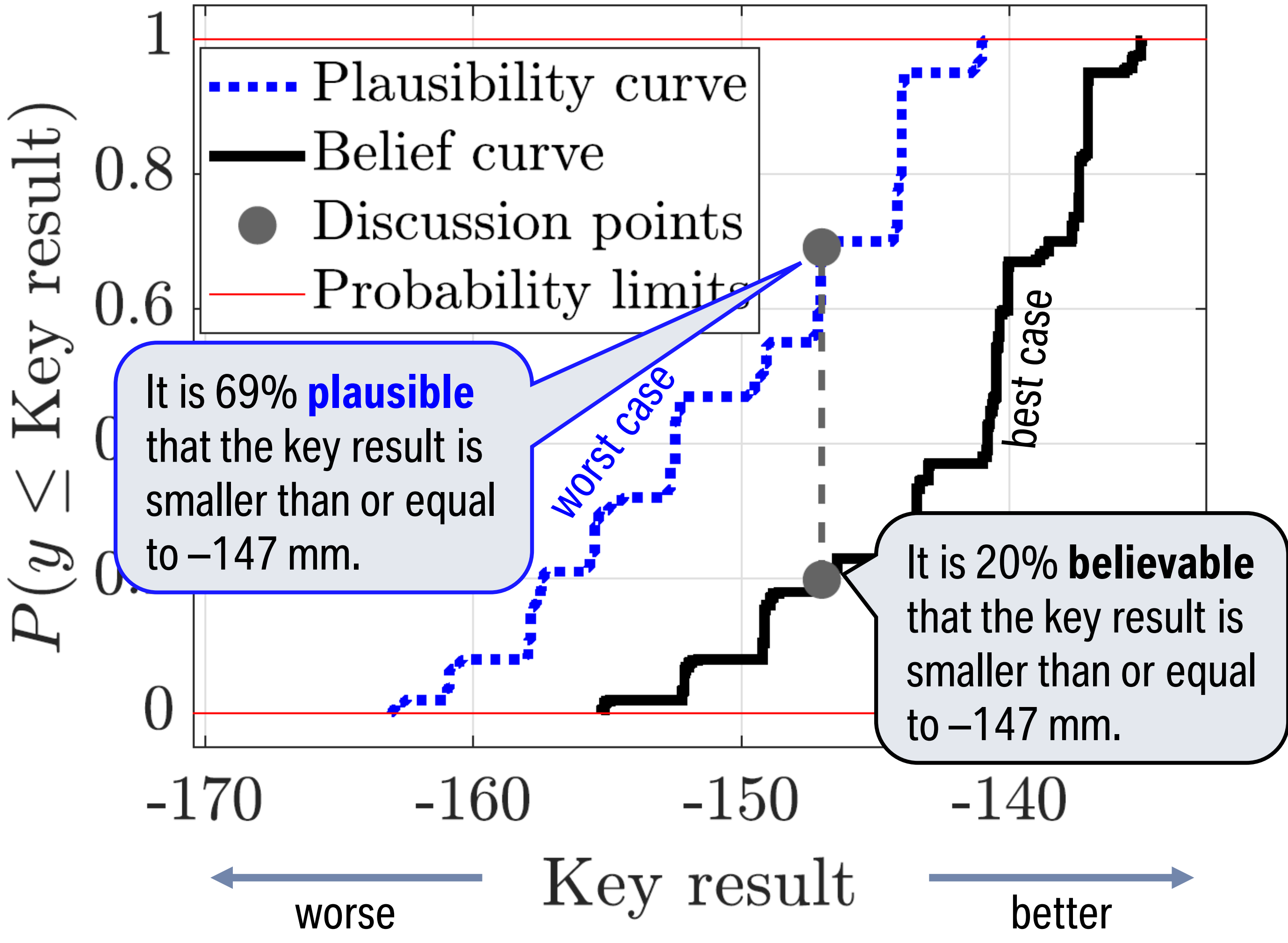


UNCERTAINTY PROPAGATION VIA EVIDENCE THEORY USING THE MOR MODEL.

Illustrative description of the evidence theory – Discuss Results.



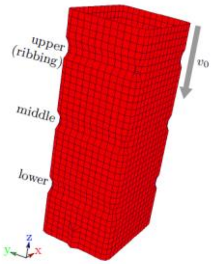




CONCLUSION.

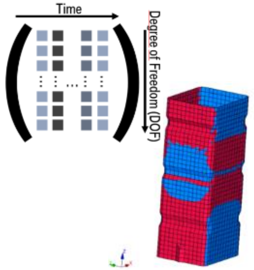
PROPOSED UNCERTAINTY MANAGEMENT FRAMEWORK.

Finite Element Crash Simulation.



Crashbox Example introduced ✓

Dimensionality Reduction and Metamodeling.

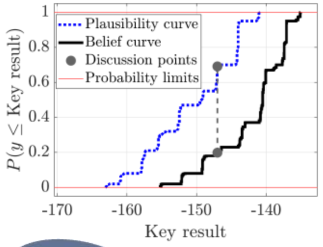


MOR Metamodel created ✓

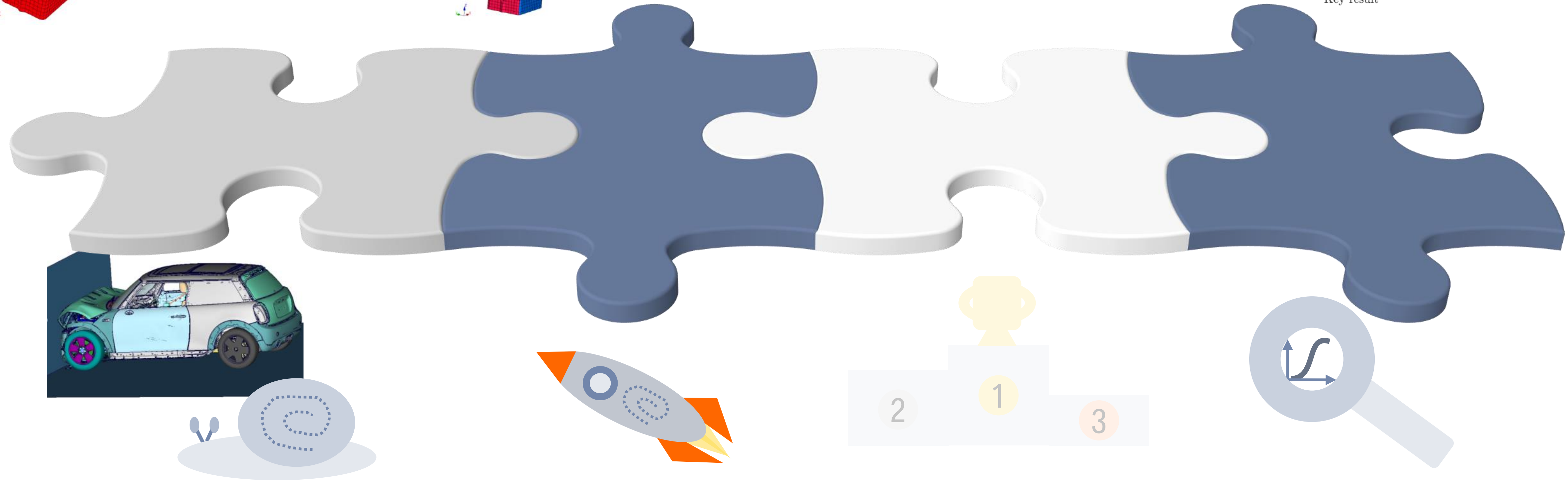
Input Investigation by Sensitivity Analysis.

Only a few parameters analyzed

Output Assessment by Uncertainty Propagation.



Evidence Theory applied ✓



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Basis for MOR

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Basis for Evidence Theory

THANK YOU FOR YOUR ATTENTION!

PROPOSED UNCERTAINTY MANAGEMENT FRAMEWORK.

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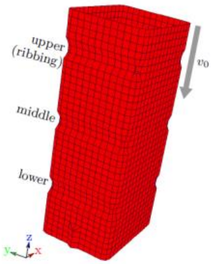

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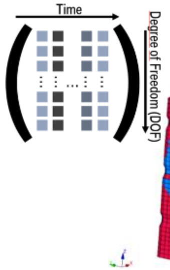

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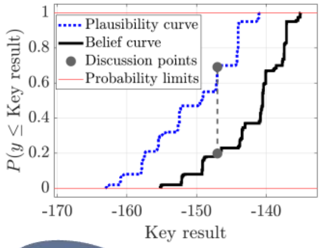

Evidence Theory applied ✓