An Integrated Interval Neural Network for Uncertainty Modeling in Inhomogeneous Materials

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Overview

- Interval neural network (INN)
  - Trained by interval-valued data
- Substitution for conventional constitutive models
  - Inhomogeneous materials
- Numerical experiments

Introduction

- Uncertainty in engineering structures
- Decision making
- Interval-valued data
  - Uncertainty in measurements
- Interval Finite Element Method (IFEM)
  - Applied loads, material properties, etc.
- Constitutive material model

Introduction

- Constitutive material model
  - Mathematical formulation
  - Fitting experimental measurements
- Limitations
  - Establishing an exact closed form model is not always feasible
  - Uncertainty in measurements

Introduction

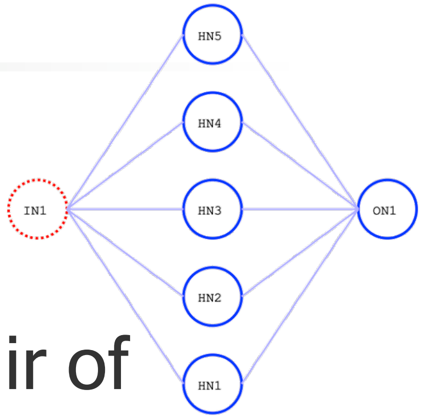
- This research
  - Substitution of conventional material models with artificial neural networks
  - Accounting for uncertainty
  - Employing an incorporation of interval-valued data with neural networks
  - Integrating the trained INN-based material model in IFEM framework

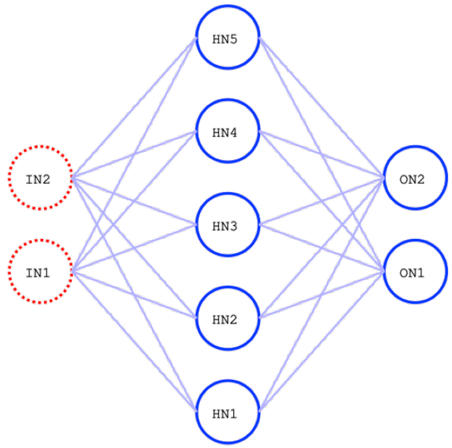
Interval Neural Network

- Interval neural network
  - Inputs, outputs, assigned weights
- The universal approximation theorem
  - Single hidden layer, finite number of neurons,
  - Can represent arbitrary complex but smooth approximation of any input-output mapping

Interval Neural Network

- Extremal approach
  - On top of MLP
  - Interval-valued numbers as a pair of numbers,
    - Lower bound and upper bound
    - Center value and radius
  - $\boldsymbol{x_i} = [\underline{x}_i, \overline{x}_i] = \langle x_i^c, x_i^r \rangle$





Computational Framework

- INN constitutive model
  - Mechanical tests, e.g., the tensile test
  - Mechanical forces and responses, e.g., stress, strain, displacements
  - Uncertainty in measurements
    - Interval valued sets of data
  - Constructing an INN by employing the extremal approach
    - Train the network with interval-valued data

Computational Framework

- INN constitutive model
  - The trained network represents the relationship between interval stress and strain data
  - Providing a description of the material behavior
  - Can be integrated into any IFEM procedure

Computational Framework

- Integrated into FEM endpoint combination analysis
  - Element stiffness matrices
  - Exact enclosure
  - Validation
- Implementation
  - MATLAB
  - On top of the Neural Network toolbox
  - Interval analysis using INTLAB

Algorithm 1 Integrated Framework

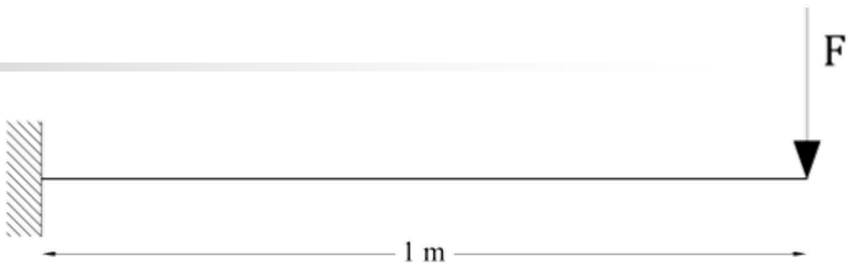
Input: Problem definition (including geometry, loading , boundary conditions), the trained INN constitutive model
Output: System response (nodal displacements, internal forces, reaction forces, stresses, and strains)

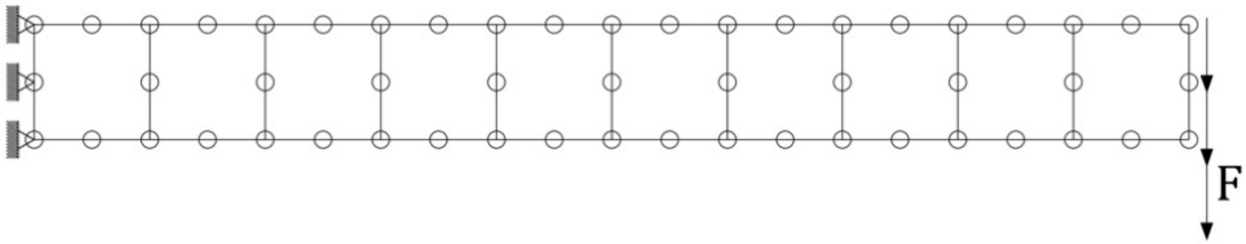
for ($step = 1$ **to** $load\_steps$) **do**
  Generating all possible endpoints combinations for loads and material properties
  for ($i \in load\_combinations$) **do**
    Forming the global vector of equivalent nodal load for the current load step
    for $\boxed{(j \in material\_properties\_combinations \text{ (obtained by INN predictions)})}$ **do**
      for ($k = 1$ **to** $number\_of\_elements$) **do**
        Forming the element i stiffness matrix
      end for
      Assembling the global stiffness matrix
      Imposing boundary conditions
      Solving the system of equations and obtaining primary unknowns (nodal displacements)
      Calculating secondary variables (stress and strains)
      Calculating principal stresses
      Storing the calculated values for the current combination of loads and material properties
    end for
  end for
  Report the minimum and maximum of the calculated values as the lower bound and the upper bound of the system response in the current load step
  Predicting the interval-valued material properties for the next load step by inputting the vector of interval-valued principal stresses into the trained interval neural network
end for

return The interval-valued results of the system response for the last load step

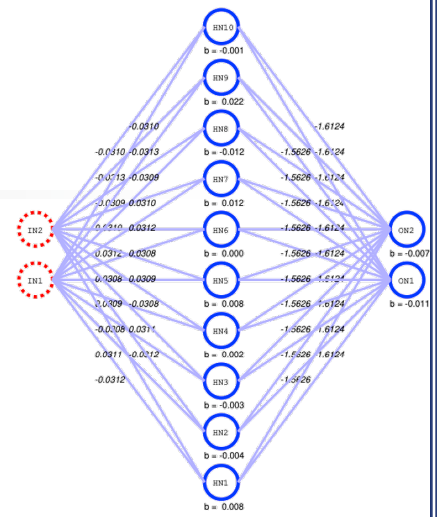
Example 1

- Cantilever beam
  - Linear elastic
    - Homogeneous
    - Inhomogeneous
  - L = 1 m, A = 0.01 × 0.01 m^2, F = 100 KN/m^2

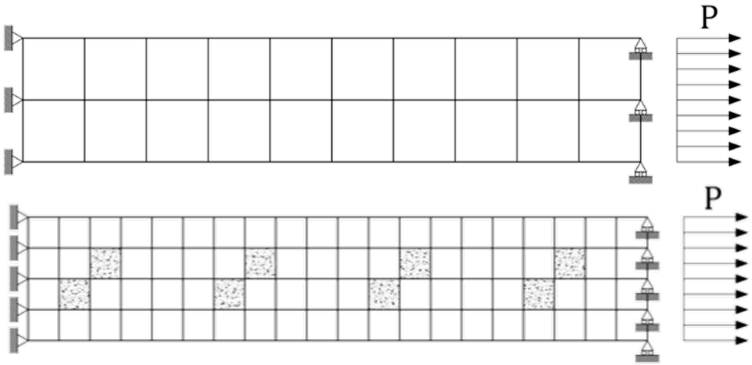


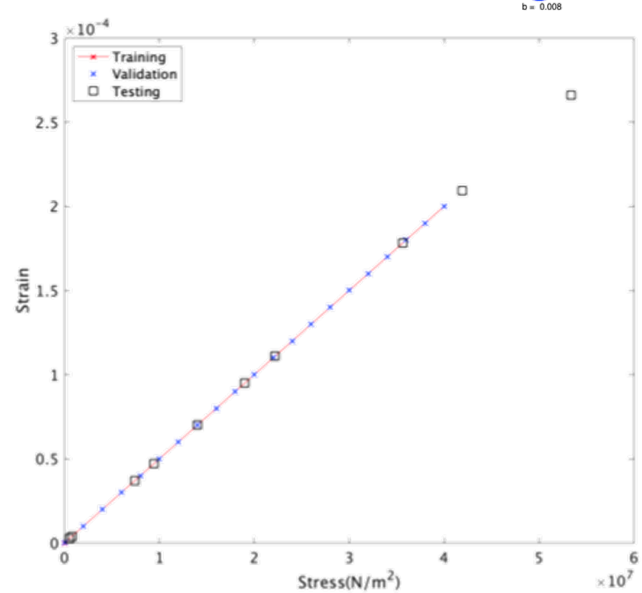


Example 1



- Synthetic measurement data
  - Simulating a uniaxial tensile test
    - Interval tensile load
    - Homogeneous, Inhomogeneous

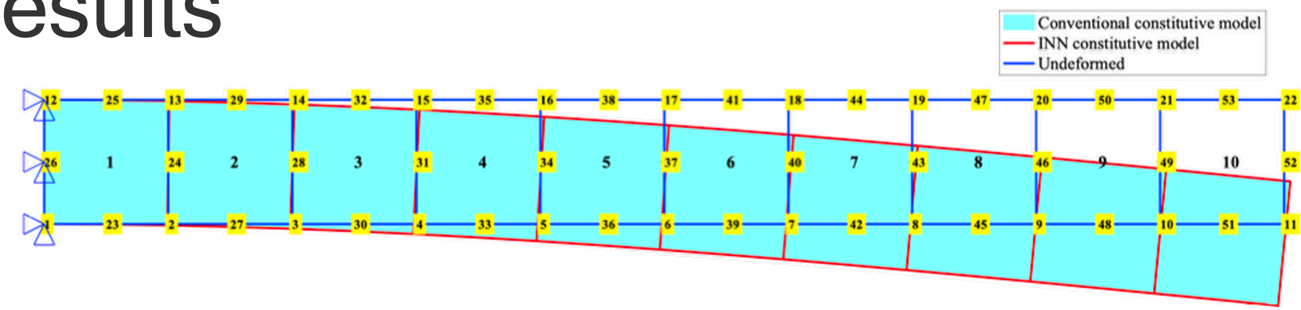


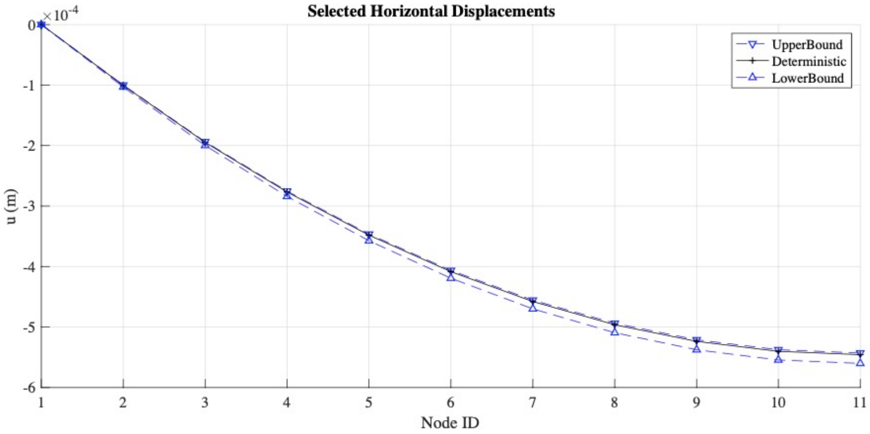


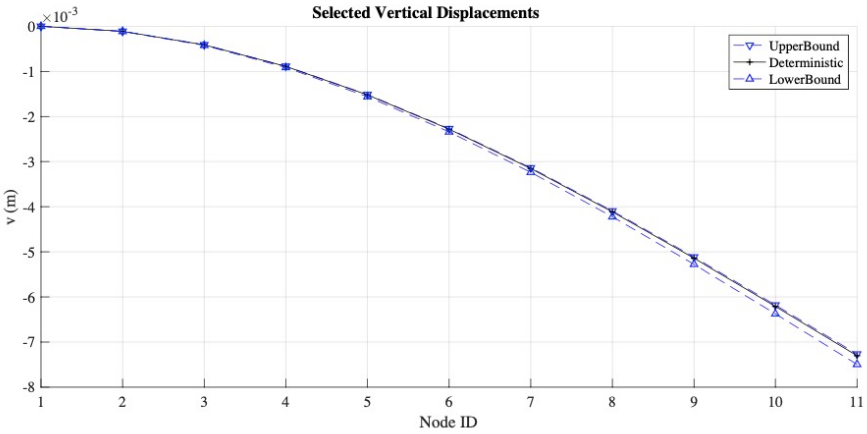
- Training INN
  - Learning tress-strain relationship

Example 1

- Results

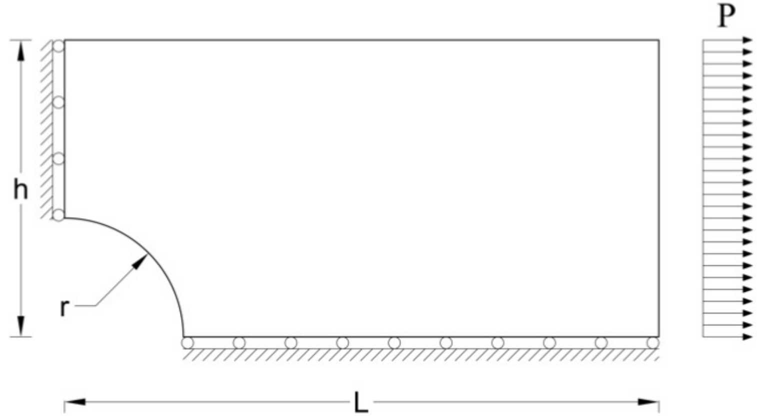


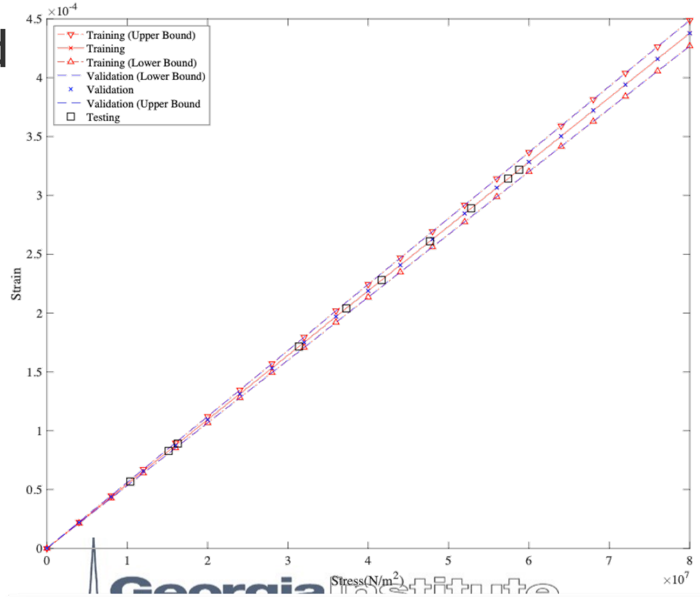




Example 2

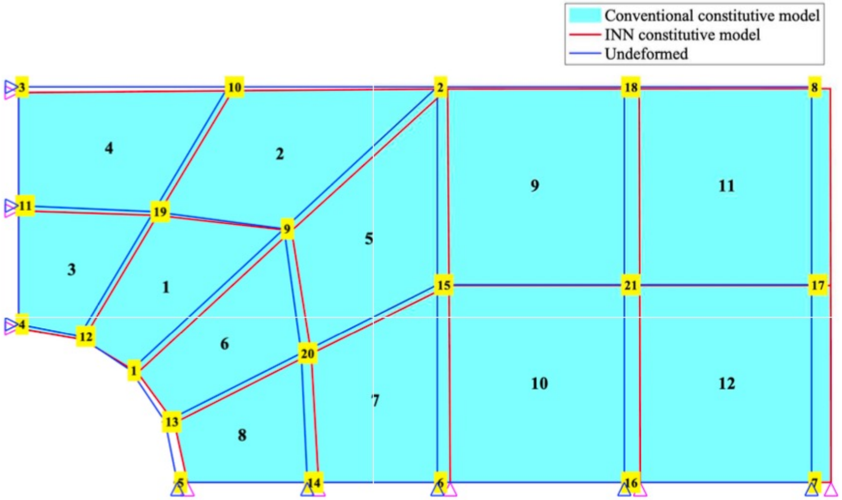
- Rectangular plate with a circular hole
  - h = 0.05 m, L = 0.1 m, r = 0.02 m, and t = 0.005 m
  - P = [19.5, 20.5] N/mm^2
- Same inhomogeneous material
  - 5% of uncertainty in the applied load

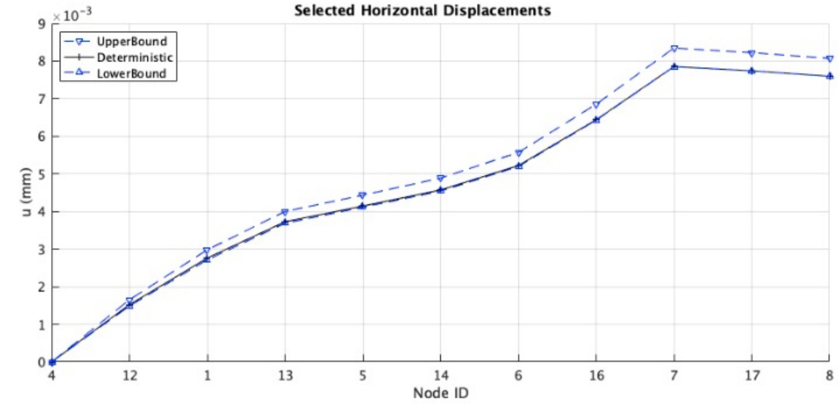


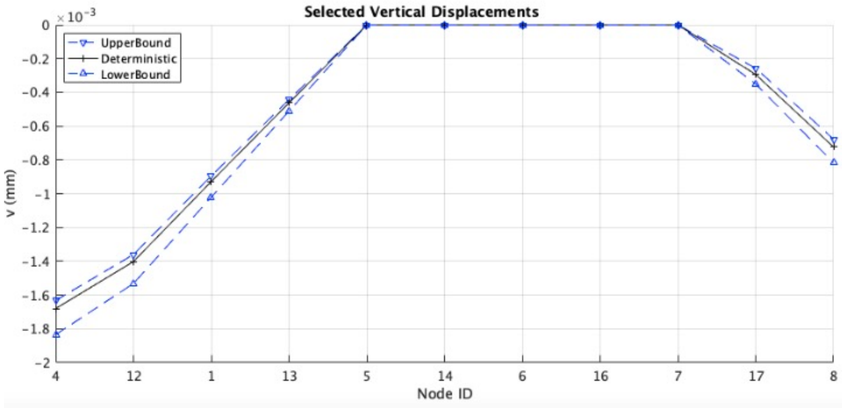


Example 2

- Displacements







Conclusion

- Interval neural networks
- INN-based constitutive material model
  - Uncertainty in the experimental measurements
  - Inhomogeneous materials
- Numerical experiments

Future Studies

- More advanced integrations of interval-valued data in neural networks
- Incorporating the proposed approach in IFEM computational framework
- Using real-world experimental data

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Thank you!

- Questions?