Institute for Structural Analysis
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Quantification of Data and Production Uncertainties for Tire Design Parameters

9th International Workshop on Reliable Engineering Computing (REC2021)
May 17th – 20th, 2021

Outline

Tire design procedure

Scope of selected uncertainty modeling methods

Uncertainty modeling for tire design parameters
- Data analysis
- Sensitivity analysis
- Uncertainty modeling

Conclusion and outlook

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Tire design procedure

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Tire design procedure
tire performances

tire performances

- rolling resistance
- dry, wet braking
- noise
- wear
- cornering stiffness
- …

https://www.continental-reifen.de/autoreifen

tire design procedure as multi-objective optimization

$$\min_{\underline{x}_i \in X} \left(f_{\mathrm{rr}}(\underline{x}_i), f_{\mathrm{br}}(\underline{x}_i), f_{\mathrm{no}}(\underline{x}_i), f_{\mathrm{we}}(\underline{x}_i), \boldsymbol{f_{\mathrm{cs}}(\underline{x}_i)}, \dots \right)$$

with design and a priori parameters

$$\underline{x} = [\underline{x}_{\mathrm{d}} \quad \underline{x}_{\mathrm{p}}]^{\mathrm{T}} \in X \subseteq \mathbb{R}^{n_{\mathrm{x}}}$$

Tire design procedure
design parameters for performance cornering stiffness

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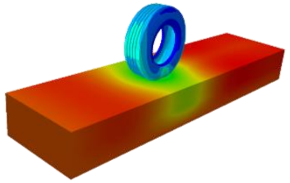
- FEM simulation model $f(\underline{x}) : \underline{x} \in \mathbb{R}^{n_\mathrm{x}} \mapsto \underline{z} \in \mathbb{R}^{n_\mathrm{z}}$ with $n_\mathrm{x} = 31$ input and $n_\mathrm{z} = 2$ output parameters
- input parameters: 24 continuous
  4 discrete
  [3 categorical]
- input parameters sorted grouped into
  - general tire dimensions
  - tire parts A-F and
  - service



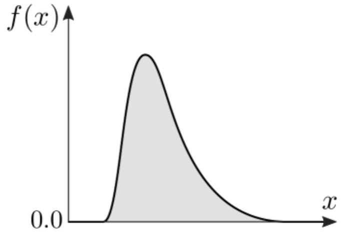
https://www.continental-reifen.de/autoreifen/reifen/premiumcontact6

	general tire dimensions	tire parts A	B	C	D	E	F	service
geometry	$\{x^\mathrm{g}_{0,i}, i \in 5\}$	$\{x^\mathrm{g}_{\mathrm{A},i}, i \in 4\}$	$\{x^\mathrm{g}_{\mathrm{B},i}, i \in 2\}$	$\{x^\mathrm{g}_{\mathrm{C},i}, i \in 3\}$	$\{x^\mathrm{g}_{\mathrm{D},i}, i \in 2\}$	$\{x^\mathrm{g}_{\mathrm{E},i}, i \in 2\}$		
compound		$\{x^\mathrm{c}_{\mathrm{A},i}, i \in 2\}$	$\{x^\mathrm{c}_{\mathrm{B},i}, i \in 2\}$	$\{x^\mathrm{c}_\mathrm{C}\}$	$\{x^\mathrm{c}_\mathrm{D}\}$	$\{x^\mathrm{c}_{\mathrm{E},i}, i \in 2\}$	$\{x^\mathrm{c}_\mathrm{F}\}$	
misc.								$\{x_\mathrm{S}\}$

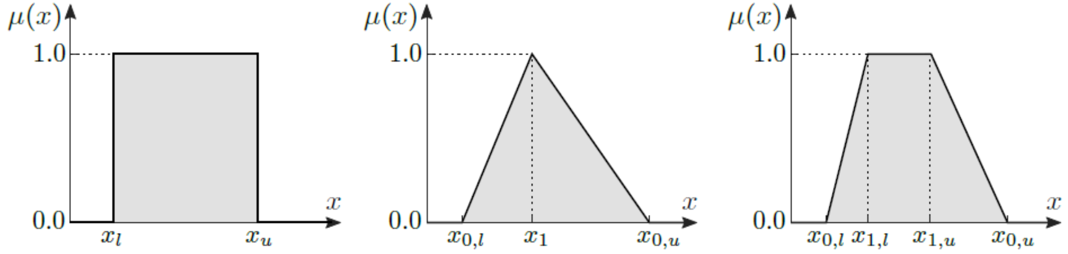
Outline

Uncertainty modeling methods
aleatory, epistemic and polymorphic uncertainty models

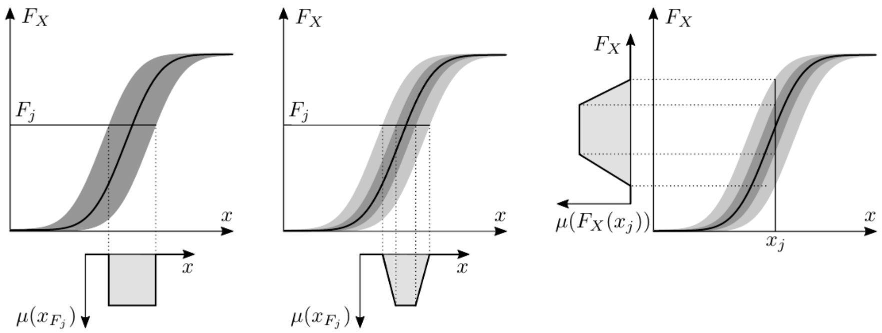
aleatory uncertainty modeling
- production variation, variation during usage
- *probabilistic modeling:* defined function types and empirical distribution
- linear correlation to model parameter interactions



epistemic uncertainty modeling
- imprecise data, incomplete data
- *possibilistic modeling:* fuzzy and interval variables



polymorphic uncertainty modeling
- combined presence of aleatory and epistemic uncertainties
- *imprecise probabilities*
  combining probabilistic and possibilistic models



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Data analysis
available data geometry parameters

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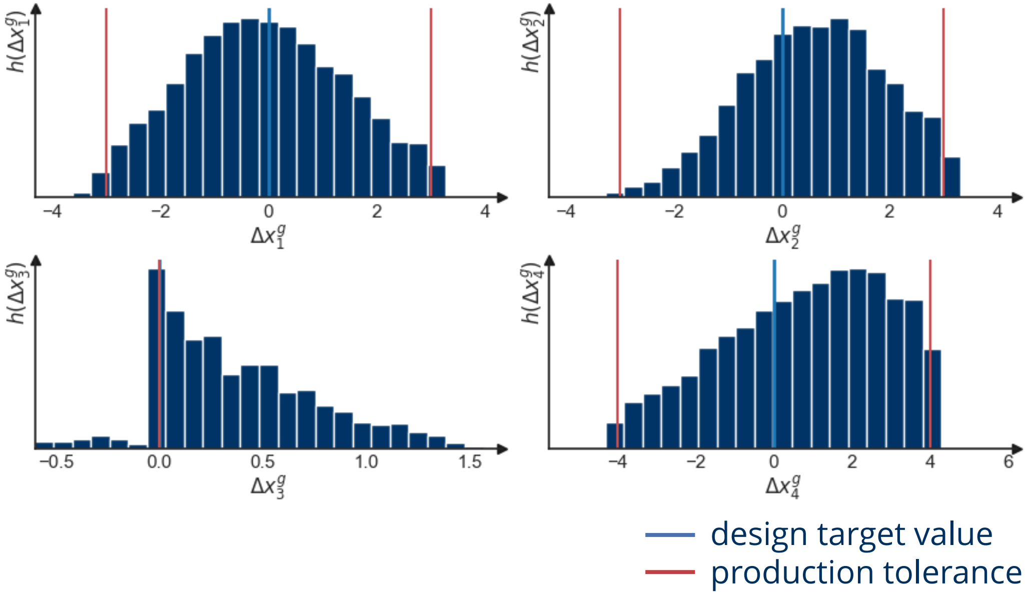
$x^{\mathrm{g}}_{k,i} = \langle \Delta x, x_{\mathrm{d}}, \mathrm{Prod}, \mathrm{Plant}, \mathrm{Date} \rangle,\ k \in n^{\mathrm{g}} = 7$

- Δx deviation of produced tire to specified design
- x_{d} specified design



→ almost exclusively skewed, non-normal distributions

association of samples for different parameters to same tire possible

$\underline{x}^{\mathrm{g}}_{i} = \langle \Delta \underline{x}, \underline{x}_{\mathrm{d}}, \mathrm{Prod}, \mathrm{Plant}, \mathrm{Date} \rangle$

→ correlation information

$\rho(\Delta x^{\mathrm{g}}_{k}, \Delta x^{\mathrm{g}}_{l}),\ k, l \in n^{\mathrm{g}}, k \neq l$

	x^{g}_1	x^{g}_2	x^{g}_3	x^{g}_4	x^{g}_5	x^{g}_6	x^{g}_7
x^{g}_1	1	-0.18	-0.29	-0.017	0.033	-0.044	0.093
x^{g}_2	-0.18	1	0.35	-0.082	0.039	-0.02	0.026
x^{g}_3	-0.29	0.35	1	0.024	0.014	-0.013	-0.027
x^{g}_4	-0.017	-0.082	0.024	1	0.3	0.037	0.024
x^{g}_5	0.033	0.039	0.014	0.3	1	0.023	0.081
x^{g}_6	-0.044	-0.02	-0.013	0.037	0.023	1	0.023
x^{g}_7	0.093	0.026	-0.027	0.024	0.081	0.023	1

PEARSON correlation coefficient

Data analysis
available data compound parameters

$$x^{\mathrm{c}}_{k,i} = \langle G'_1, G'_{10}, G'_{100}, \mathrm{Part}, \mathrm{Date} \rangle, \ k \in n^{\mathrm{c}} = 47$$

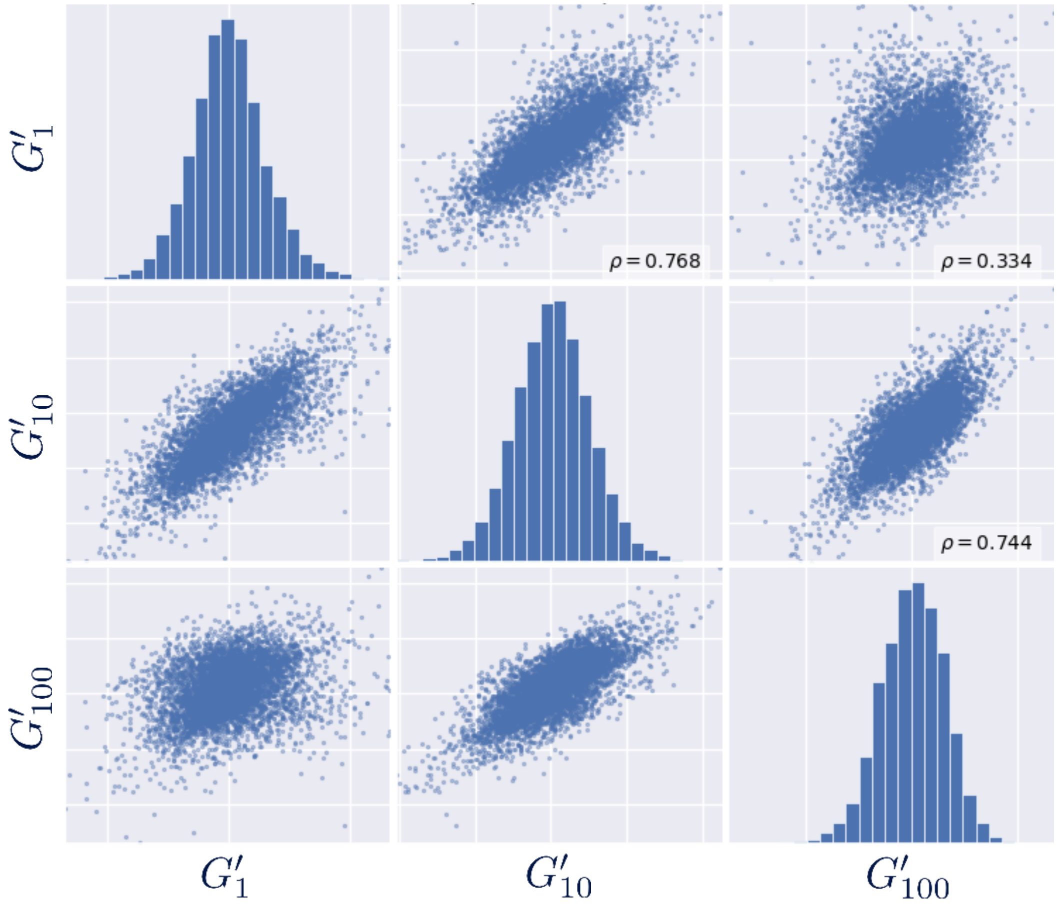
G'_{ε_j} storage shear modulus for strains $\varepsilon_j \in \{1, 10, 100\}\%$

Part associated tire part (body or tread)

- values for G'_{ε_j} statistically normalized

$$G'_{k,i,\varepsilon_j} = \frac{G'_{k,i,\varepsilon_j,\mathrm{orig}}}{\overline{G}'_{k,\varepsilon_j,\mathrm{orig}}}$$

- almost symmetrical distributions
- positive (linear) correlation of variation for different strains



Data analysis
aleatory and epistemic uncertainty modeling

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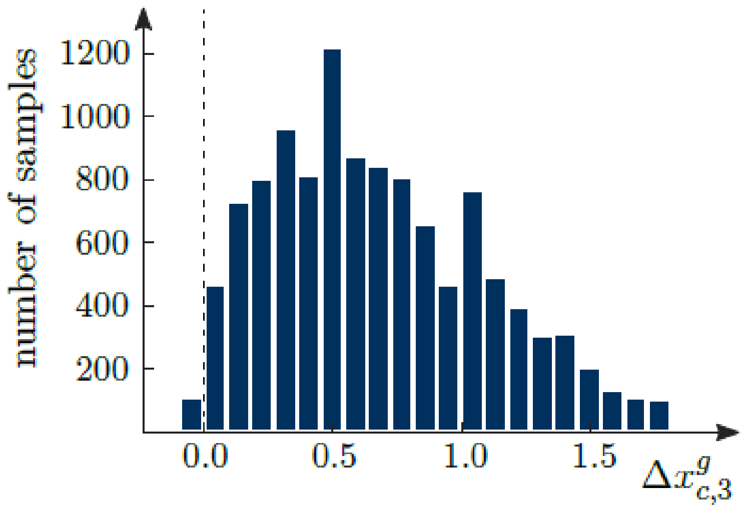
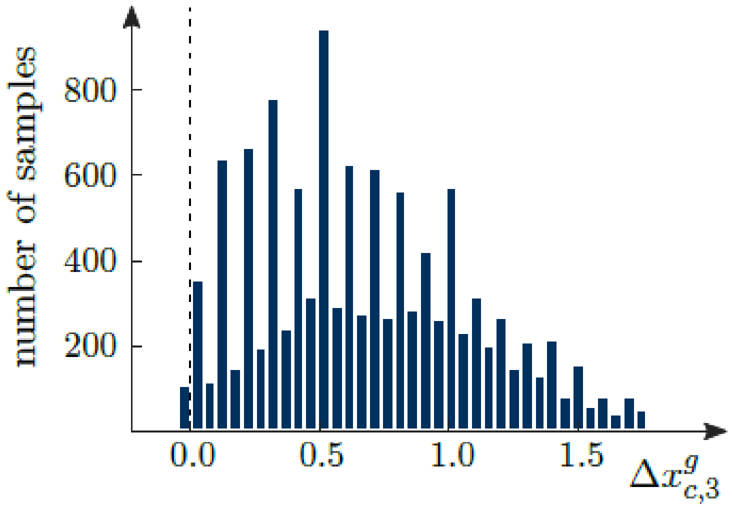
example geometry parameter data $\Delta x^{\mathrm{g}}_{\mathrm{c},3}$

- observable production variation ➡ aleatory uncertainty non-negligible, stochastic variable
- number of samples $n(\Delta x^{\mathrm{g}}_{\mathrm{c},3}) = 11\,706$ ➡ large number of samples available, epistemic uncertainty caused by incompleteness is disregarded
- limited accuracy of measurement observed ➡ epistemic uncertainty caused by imprecision, fuzzy/interval variable



increase in no. of bins



Uncertainty modeling methods
aleatory and epistemic uncertainty models

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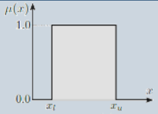
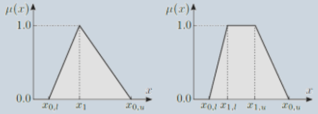
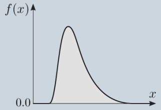
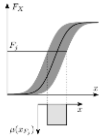
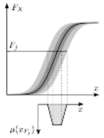
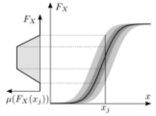
data			uncertainty model	
quantity	precision	assessment		
few	imprecise	no	interval 	basic uncertainty models
few	imprecise	yes	fuzzyness 	
very many	precise	–	randomness 	
many	imprecise	no	probability-box 	polymorphic uncertainty models
many	imprecise	yes	fuzzy random variable 	
some	precise	–	fuzzy-probability based randomness 	

[Pannier et al., MSSP 2013]

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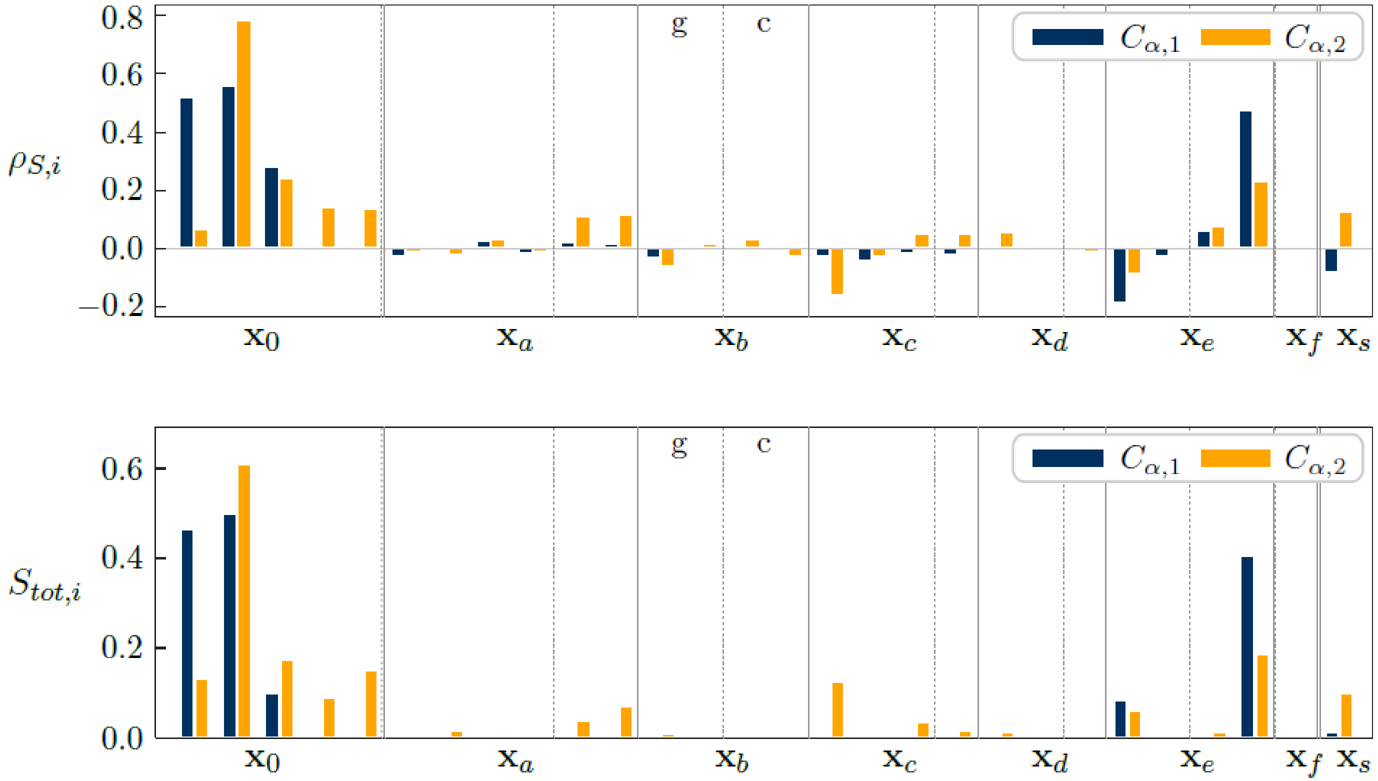
Sensitivity analysis
resulting sensitivities for tire design parameters

applied sensitivity measures

- coefficient of correlation
  - PEARSON ρ_{P} (linear)
  - SPEARMAN ρ_{S} (rank)
  - $\rightarrow$ almost identical results (max. deviation $\Delta(\rho_{\mathrm{P},k}, \rho_{\mathrm{S},k}) = 2\%$)
- standardized regression coefficient
  - linear model $\hat{z} = b_0 + \sum_{j=1}^{n_{\mathrm{x}}} b_j x_j$
- SOBOL indices
  - total index S_{tot}



results for different sensitivity measures strongly coincide

Sensitivity analysis
resulting sensitivities for tire design parameters

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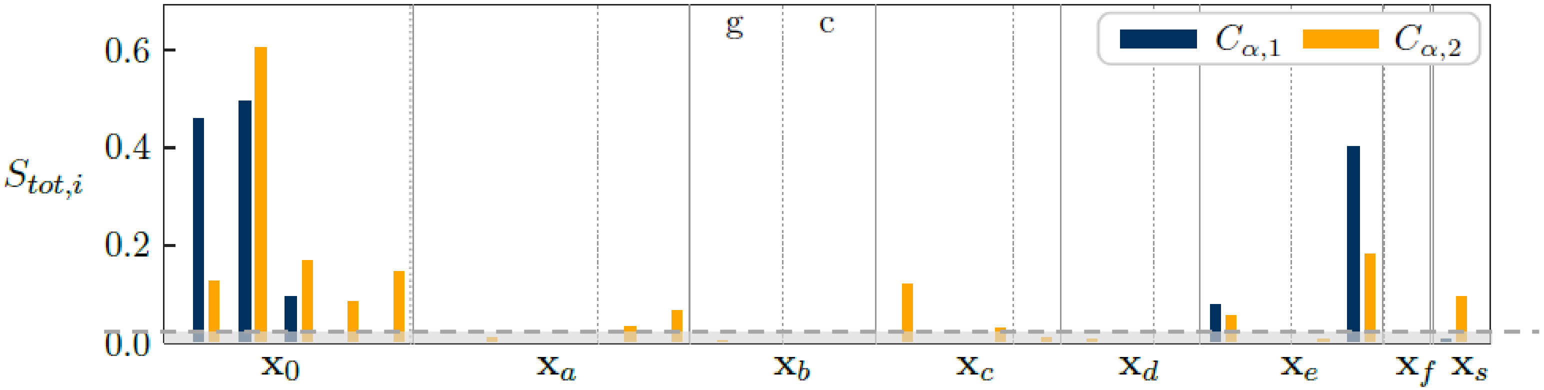

	general tire dimensions	tire parts A	B	C	D	E	F	service
none to low		$\{x^{c}_{A,i}, i \in 2\}$	$\{x^{g}_{B,i}, i \in 2\}$ $\{x^{c}_{B,i}, i \in 2\}$	$\{x^{g}_{C,3}\}$ $\{x^{c}_{C}\}$	$\{x^{g}_{D,i}, i \in 2\}$ $\{x^{c}_{D}\}$	$\{x^{g}_{E,2}\}$ $\{x^{c}_{E,1}\}$	$\{x^{c}_{F}\}$	
medium to high	$\{x^{g}_{0,i}, i \in 5\}$	$\{x^{g}_{A,i}, i \in 4\}$		$\{x^{g}_{C,1}, x^{g}_{C,2}\}$		$\{x^{g}_{E,1}\}$ $\{x^{c}_{E,2}\}$		$\{x_{S}\}$

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resulting sensitivities for tire design parameters

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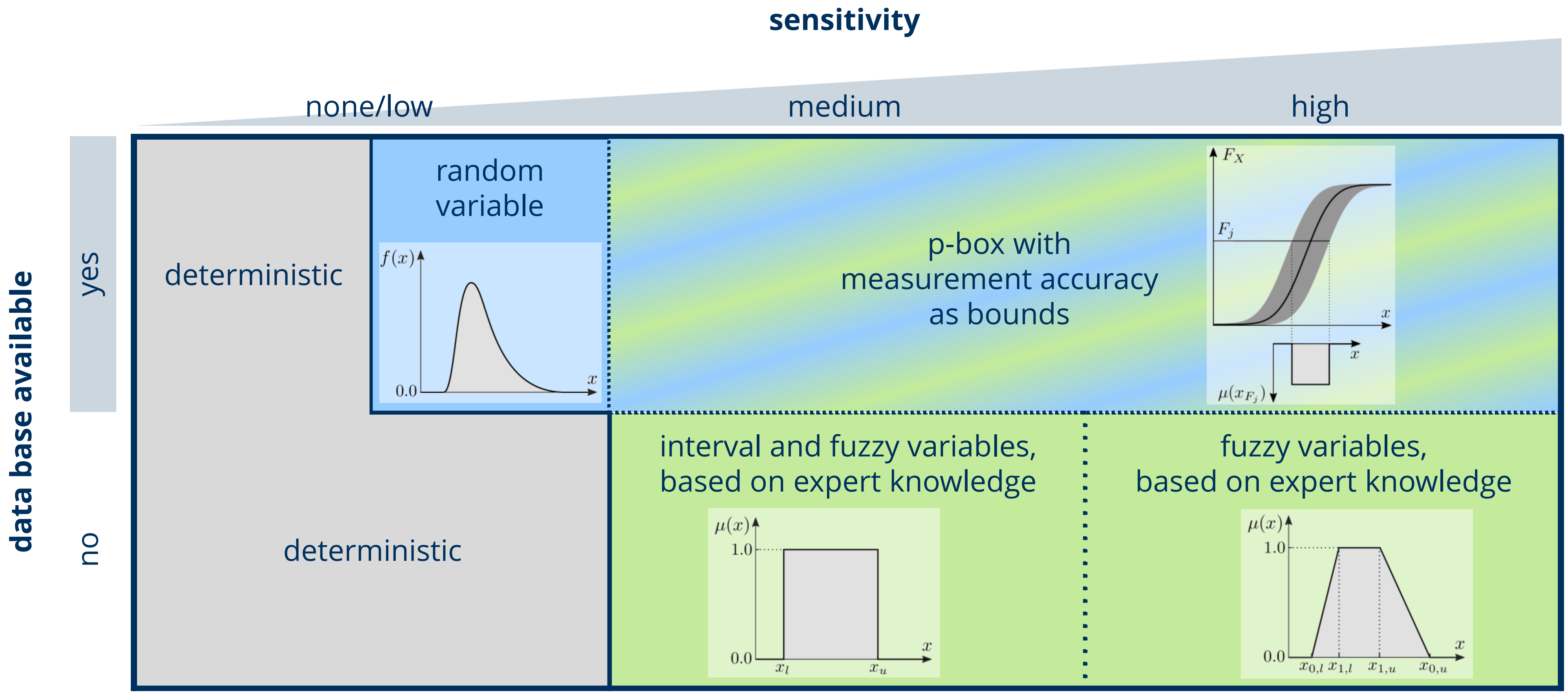

Uncertainty modeling for tire design parameters
uncertainty modeling examples

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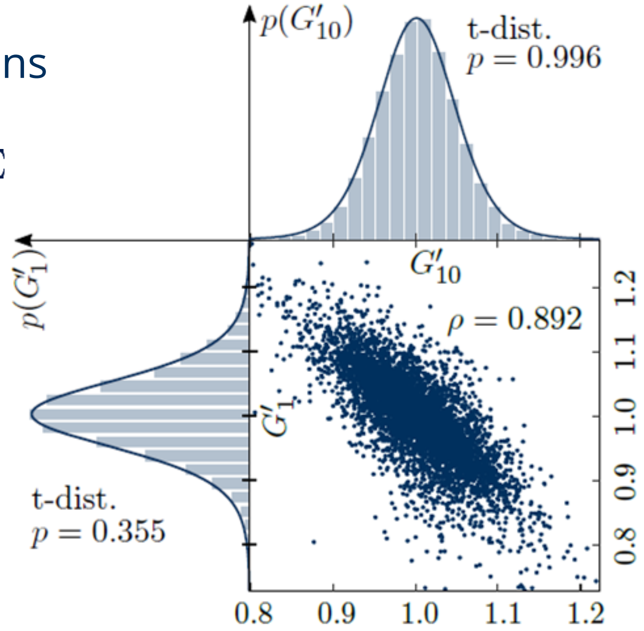
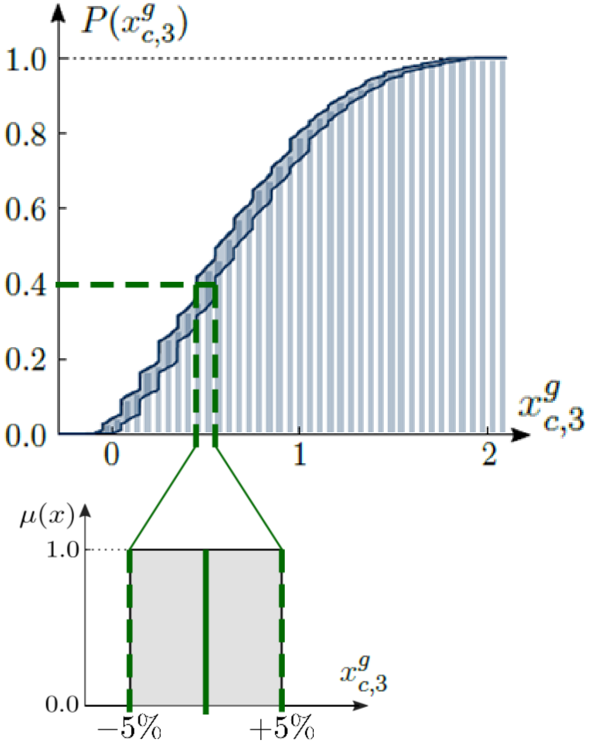
multivariate random variable $t(\underline{\nu}, \Sigma)$

- t-distributions, hyperparameter $\underline{\nu}$ fitted using MLE
- fit evaluation using χ^2-test and KS-test $\rightarrow p > \alpha = 0.1$
- marginal distributions joined through covariance matrix Σ



probability-box

- empirical distribution function $\hat{F}$
- accuracy $\pm 5\%$ of measurement as bound limits



Uncertainty modeling for tire design parameters
design target value dependency

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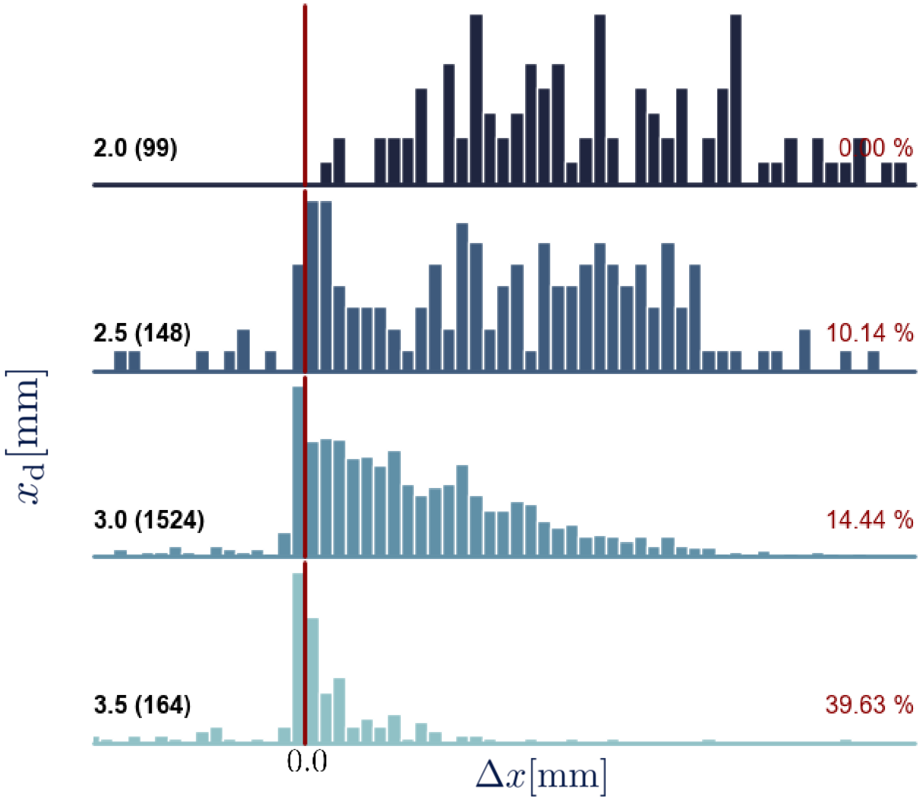
deviation from the design $\Delta(x, x_{\mathrm{d}})$,
dependent on the design target value x_{d}

- production objective:
  actual geometry
  not below design target value
- decrease of (positive) deviation
  with increase in
  design target value
- consideration in uncertainty modeling:
  design target
  dependent affine transformation

$$x_{\mathrm{d}}^{\mathrm{u}}(x_{\mathrm{d}}) = x_{\mathrm{d}} + x_{\mathrm{p_d}}^{\mathrm{u}}(x_{\mathrm{d}})$$



production variation Δx for different target values

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- uncertainty modeling based on
  - available data, experience and expert knowledge
  - design parameters’ sensitivity to select uncertainty model complexity
- for tire design
  - random variables based on production data
  - interval and fuzzy variables based on experience, production variation limits
  - combination of both to p-box

- stochastic and fuzzy analysis utilizing metamodels
- uncertainty reduction measures for performance and robustness

Acknowledgement:

The provision of tire production data and expert knowledge on the subject of tire design as well as the financial provided by Continental AG is highly appreciated.

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Thank you

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