Numerical Simulation of a 3D Concrete Printing Process under Polymorphic Uncertainty

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MPA life cycle material engineering.

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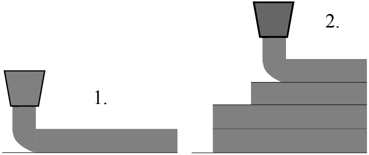
Additive manufacturing – 3D Concrete Printing (3DCP)

- **Extrusion** techniques: layer-by-layer process
- Different **materials**: polymer, metal, concrete, ...
- **Automatic** production enables a fully digitized production process
- Material and cost **efficiency**
- **Application**:
  - rapid prototyping
  - customized production
  - complex geometries and shapes
  - ...



Uncertainties in 3DCP

Geometric derivations

- Lateral position
- Layer thickness

Gosselin et al., 2016

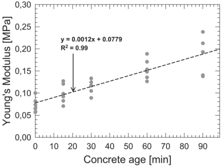
Stability failure

- Local material strength-based
- Global "buckling"

Buswell et al., 2018; Le, Austin, Lim, Buswell, Law, et al., 2012

Material behavior

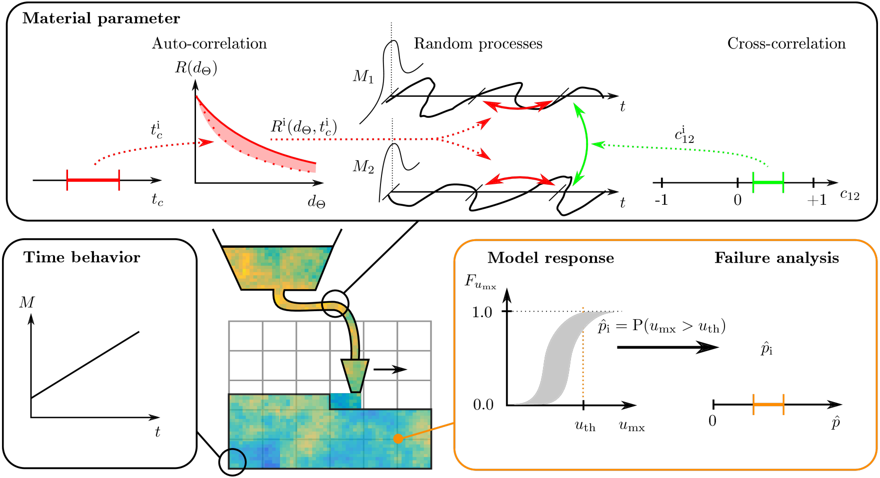
- Layer bonding time
- Temp. evolution of material parameters



Under-filling influenced by

- Printing process parameters (nozzle velocity, pumping rate ...)
- Material parameters (rheology)

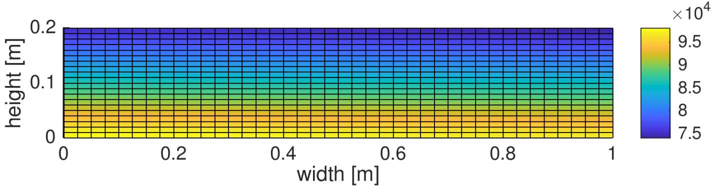
Principle scheme



Printed wall 2D model

2D plane-stress FE model of printed wall

- Size: 1 m × 0.2 m
- Elements: 40×20 (Q8)
- Load: self-weight with const. $\rho = 2020\,\mathrm{kg/m^3}$ (Wolfs et al., 2018)
- Material: linear-elastic, parameters scaled via pseudo-density $\tilde{\rho}$
  - time-dependent Young's modulus: $E(\boldsymbol{r}, t) = \tilde{\rho}(\boldsymbol{r}) E_0(\boldsymbol{r}) + E_1 \cdot t$
  - time-dependent yield stress: $\sigma_y(\boldsymbol{r}, t) = \tilde{\rho}(\boldsymbol{r}) \sigma_{y,0}(\boldsymbol{r}) + \sigma_{y,1} \cdot t$
- Process parameter:
  - const. printing velocity $v_\mathrm{p} = 1\,\mathrm{m/min}$, layer height 10 mm
  - pseudo-density $\tilde{\rho}$ from non-constant material flow rate $Q_\mathrm{m}(t)$: $\frac{Q_\mathrm{m}(t)}{Q_{\mathrm{m},0}} \rightarrow \tilde{\rho}$, $Q_\mathrm{m} \in [0, Q_{\mathrm{m},0}]$



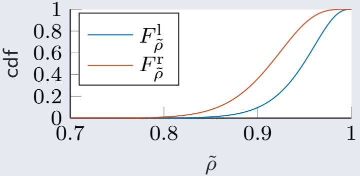
Uncertainty model

Uncertain material parameters of pumped concrete

- Young's Modulus E_0 and yield stress $\sigma_{y,0}$
  cross-correlated interval probability-based random process (ip-rp) (Schietzold et al., 2019)
  - lognormal **random process** $E_{\mathrm{p}}^{\mathrm{iprp}}(t) \rightarrow$ **random field** $E_0^{\mathrm{iprf}}(\boldsymbol{r})$
  - lognormal **random process** $\sigma_{y,\mathrm{p}}^{\mathrm{iprp}}(t) \rightarrow$ **random field** $\sigma_{y,0}^{\mathrm{iprf}}(\boldsymbol{r})$
  - **interval** auto-correlation time $t_c^{\mathrm{i}} = [7.5,\ 75]$ s
  - **interval** cross-correlation coefficient $c^{\mathrm{i}} = [0.7,\ 0.9]$

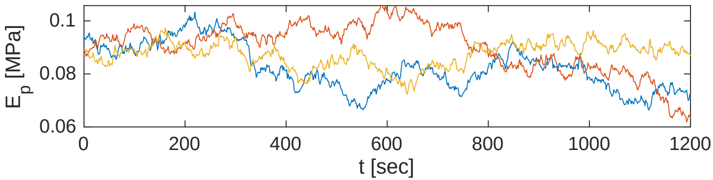
Uncertain process parameter

- varying material flow rate $Q_{\mathrm{m}}(t)$ is described via a uncertain pseudo-density $\tilde{\rho} \in [0,\ 1]$
- pseudo-density $\tilde{\rho}$: interval-probability based random variable (ip-r) (Schietzold et al., 2019)
  - beta distributed **random variable** $\tilde{\rho}^{\mathrm{ipr}} \sim \mathcal{B}(\alpha, \beta^{\mathrm{i}})$ with shape parameters α and β^{i}
  - deterministic $\alpha = 50$
  - **interval** variable $\beta^{\mathrm{i}} = [3,\ 5]$

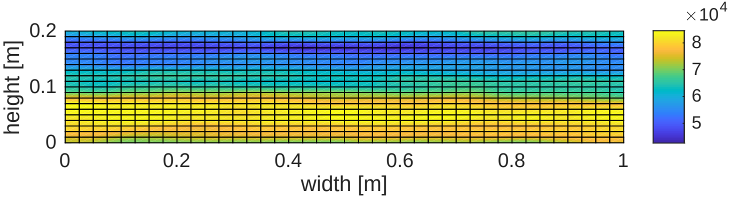


Random process mapped on random field

Random process $E_{\mathrm{p}}^{\mathrm{r}}(t)$ mapped on FE model parameter $E_0^{\mathrm{r}}(\boldsymbol{r})$ for time-dependent Young's modulus: $E(\boldsymbol{r},t) = E_0(\boldsymbol{r}) + E_1 \cdot t$



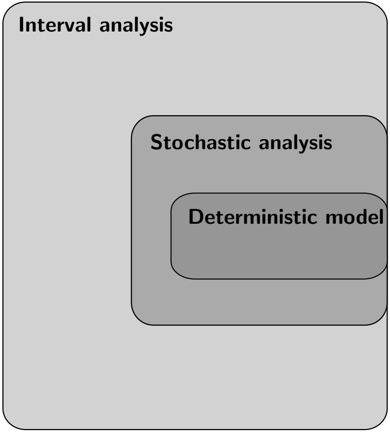
Samples of random process $E_{\mathrm{p}}^{\mathrm{r}}(t)$



$E(\boldsymbol{r})$ in [Pa] for $t = 1200\,\mathrm{s}$

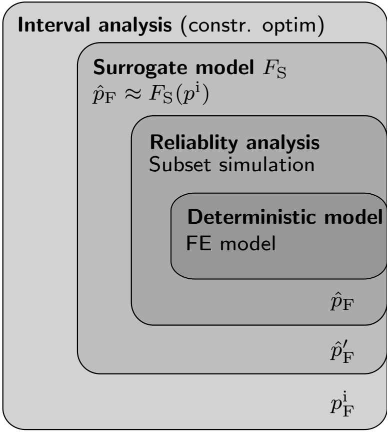
Interval stochastic analysis

- Usage of subset simulation for **failure probability** estimation $\hat{p}_F$
- generation of **surrogate model** F_S approximating $\hat{p}'_F$ in interval space p^i (Schmidt et al., 2017)
- α-level optimization yields failure probability **interval** p^i_F (Möller et al., 2000)



Standard interval stochastic analysis scheme

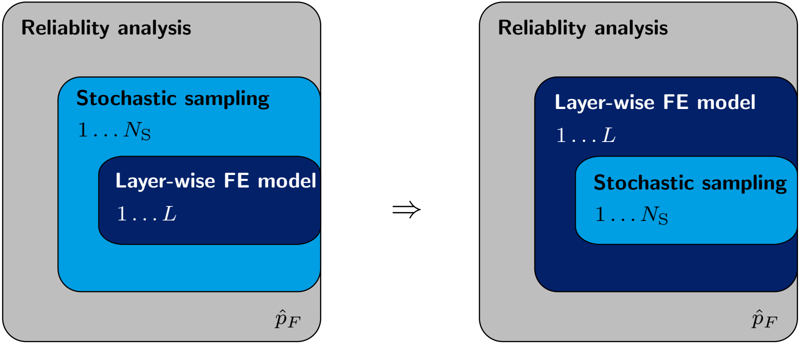
$\Rightarrow$



Interval surrogate-based reliability analysis scheme

Stochastic layer-wise simulation

- **FE model** is solved in a **layer-wise** fashion
- Increase efficiency with an **inner stochastic sampling loop**
  → reuse system matrix assembly and effective solver (preconditioning and initial guess)



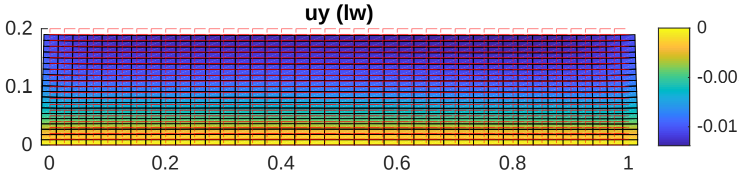
Standard scheme with outer loop stochastic sampling

Layer-wise simulation with inner loop stochastic sampling

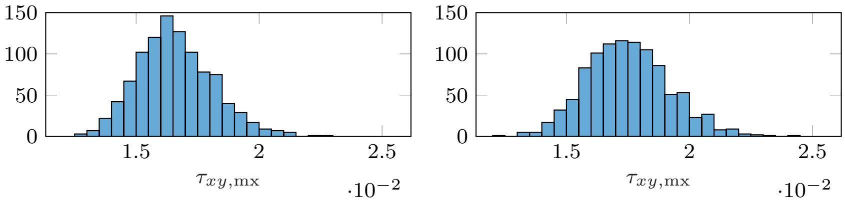
- Result: **Reduction** of computational costs up to **40%**, depending on model geometry and stochastic sample size

Exemplary FE model results

Results for points in the **interval space** $p^{\mathrm{i}} \in \{t_{\mathrm{c}}^{\mathrm{i}},\, c^{\mathrm{i}},\, \beta^{\mathrm{i}}\}$ (auto-correlation time, cross-correlation coefficient, β shape parameter of pseudo-density)



Sample of u_y in [m] for $t_c = 51.57,\ c = 0.84,\ \beta = 3.01$

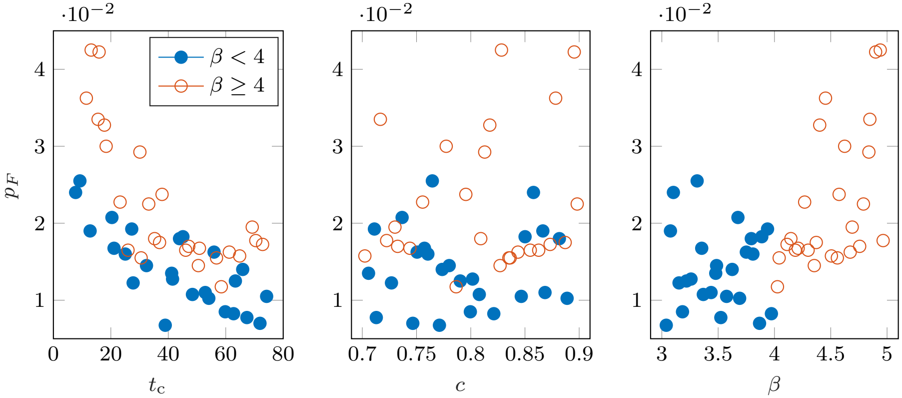


Histograms of maximum shear strain τ_{xy} for two different p^{i}

life cycle
material engineering.

Results reliability analysis

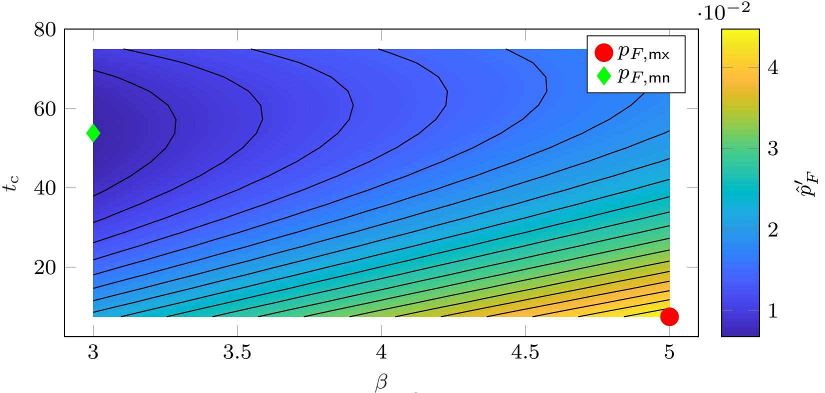
- **Limit state** based on maximum deformation and maximum compressive strength
- **Significant influence** of auto-correlation time t_c and shape parameter β on failure prob. p_F



Failure probability p_F as a function of
auto-correlation time t_c, cross-correlation c and shape parameter β

Results interval analysis

- Generalized Linear Regression Model (GLRM) with polynomial basis (and binomial error term) is used as **surrogate model** $\hat{p}'_F = F_\mathrm{S}(t_\mathrm{c}^\mathrm{i}, c^\mathrm{i}, \beta^\mathrm{i})$
- **Constrained optimization** on surrogate model yields minimum and maximum failure probability $p_{F,\mathrm{mn}}$ and $p_{F,\mathrm{mx}}$
- **Interval failure probability** is defined as $p_F^\mathrm{i} = [p_{F,\mathrm{mn}},\, p_{F,\mathrm{mx}}] = [0.68,\, 4.47] \times 10^{-2}$



Failure probability surrogate $\hat{p}'_F = F_\mathrm{S}(t_\mathrm{c},\, c,\, \beta)$ for $c = 0.8$

Summary and outlook

Results

- 2D **FE model** for concrete printing process
- Polymorphic uncertainty model:
  - **interval probability-based** cross-correlated **random processes** mapped onto spatially varying Young's modulus and yield stress
  - varying material flow governed by interval-probability based **random pseudo-density**
- **Reliability analysis** using efficient **surrogate-based** interval-stochastic analysis scheme
- Promising results using **layer-wise stochastic analysis** scheme

Outlook

- Addition of **global failure modes**, e.g. e.g. cold joints, buckling
- Extend FE model to 3D
- **RBDO** of printing process

Acknowledgment

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“Polymorphic uncertainty modelling for the numerical design of structures”

Many thanks. Questions?

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