REC2021

Sensitivity of an hysteretic material model for random vibration analysis of base-isolated rigid-block historical artifacts

Salvatore Sessa, D. Pellecchia, N. Vaiana and L. Rosati

Dept. of Structures for Engineering and Architecture
University of Naples Federico II

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Work Outline

1. Purposes of this work
2. Review of TELM
3. The FORM algorithm
4. The Algebraic material
5. Numerical application
6. Conclusions and future work

Purposes of this work

- We present the sensitivity of a uniaxial constitutive model recently proposed by Vaiana et al. (2019) and belonging to a wider phenomenological class (Vaiana et al. 2018).
  - N. Vaiana, S. Sessa, F. Marmo, and L. Rosati, "An accurate and computationally efficient uniaxial phenomenological model for steel and fiber reinforced elastomeric bearings," *Composite Structures*, vol. 211, pp. 196–212, 2019.
  - N. Vaiana, S. Sessa, F. Marmo, and L. Rosati, "A class of uniaxial phenomenological models for simulating hysteretic phenomena in rate-independent mechanical systems and materials," *Nonlinear Dynamics*, vol. 93, no. 3, pp. 1647–1669, 2018.
- Sensitivity is needed to use such a material in the Tail–Equivalent linearization – TELM (Fujimura et al. 2007).
- TELM is a stochastic strategy particularly suitable for determining first-excursion probability of isolated systems, including historical artifacts.

TELM references

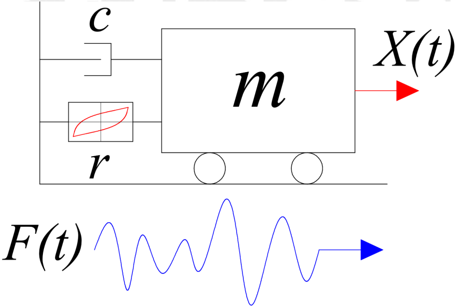
TELM was proposed by Fujimura and Der Kiureghian (2007) and determines a non-parametric linear system whose tail probability is equal to the first-order approximation of the tail probability of a nonlinear system.

A brief overview:

- K. Fujimura and A. Der Kiureghian, "Tail-equivalent linearization method for nonlinear random vibration," *Probabilistic Engineering Mechanics*, vol. 22, no. 1, pp. 63–76, 2007.
- A. Der Kiureghian and K. Fujimura, "Nonlinear stochastic dynamic analysis for performance-based earthquake engineering," *Earthquake Engineering and Structural Dynamics*, vol. 38, no. 5, pp. 719–738, 2009.
- L. Garré and A. Der Kiureghian, "Tail-equivalent linearization method in frequency domain and application to marine structures," *Marine Structures*, vol. 23, no. 3, pp. 322–338, 2010.
- M. Broccardo, U. Alibrandi, Z. Wang, and L. Garrè, "The tail equivalent linearization method for nonlinear stochastic processes, genesis and developments," *Springer Ser. Rel. Eng.*, vol. 0, pp. 109–142, 2017.

Advantages of TELM

- Definition of a non-parametric linearized system: TELS;
- Invariance with respect excitation scale and duration in time;
- Based on Reliability Analysis Methods;
- Convergence ensured by basic hypotheses;
- Independent on DOFs;
- Non-Gaussian, non-symmetric and non-zero-mean distributions.



- The random excitation is defined in terms of a discrete set of standard-normal random variables $\boldsymbol{\upsilon}$ and a deterministic array $\mathbf{s}\,(t)$:

$$F\,(t) = \sum_{i=1}^{n} s_i\,(t)\,\upsilon_i = \mathbf{s}^T\,(t)\,\boldsymbol{\upsilon}$$

where $\mathbf{s}\,(t)$ depends on the excitation's covariance;

TELM overview

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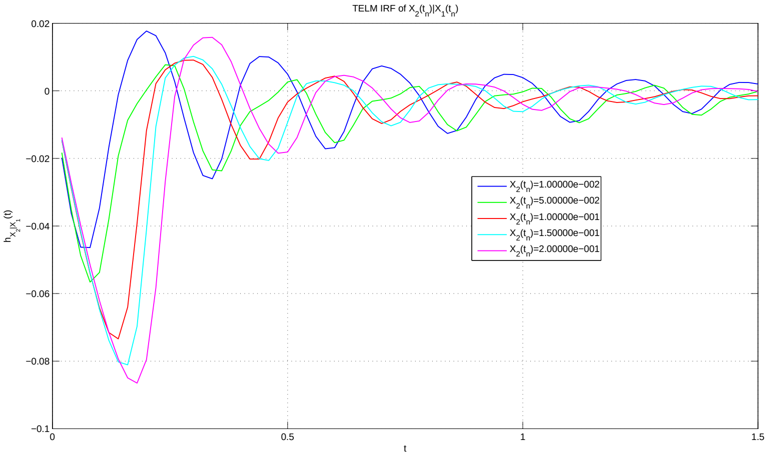
where $\mathbf{s}(t)$ depends on the excitation's covariance;

- Nonlinear response: $\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{R}(\mathbf{y}, \dot{\mathbf{y}}) = \mathbf{p}\mathbf{s}(t) \cdot \boldsymbol{\upsilon}$
  $X(t_n, \boldsymbol{\upsilon}) \sim \mathbf{y}(t_n), \dot{\mathbf{y}}(t_n), \ddot{\mathbf{y}}(t_n)$
- Limit State Function:

$$\Theta(x, t_n, \boldsymbol{\upsilon}) = x - X(t_n, \boldsymbol{\upsilon})$$

- Given x and t_n, TELM performs a FORM method and gets: $\boldsymbol{\upsilon}^*_{NL} = \arg\min\left[\boldsymbol{\upsilon}, x - X\left(t_n, \boldsymbol{\upsilon}\right) = \Theta\left(\boldsymbol{\upsilon}\right) = 0\right]$
- the response of a linear system is: $X\left(t, \boldsymbol{\upsilon}\right) = \int_0^t h\left(t-\tau\right) \sum_{i=1}^n s_i\left(\tau\right) u_i \, d\tau = \mathbf{a}\left(t\right) \cdot \boldsymbol{\upsilon}$
- $a_i\left(\tau\right) = \int_0^t h\left(t-\tau\right) s_i\left(\tau\right) d\tau;$
- Geometric considerations lead to: $\mathbf{a}\left(x, t_n, \boldsymbol{\upsilon}\right) = \frac{x}{\|\boldsymbol{\upsilon}^*(x,t_n)\|} \frac{\boldsymbol{\upsilon}^*(x,t_n)^T}{\|\boldsymbol{\upsilon}^*(x,t_n)\|}$
- which is related to the IRF of the linearized system: $\sum_{j=1}^n h\left(t_n - t_j\right) s_i\left(t_j\right) \Delta t = a_i\left(t_n\right)$
- applying the procedure at each threshold, the Tail Equivalent Linearized System (TELS) becomes a *collection* of IRFs, one for each threshold.

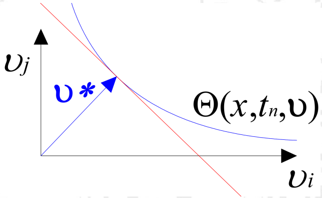
TELM outline



The FORM algorithm

the FORM algorithm is defined as:

$$\boldsymbol{\upsilon}^{\star}\left(x,\, t_n\right) = \arg\,\min\left\{\|\boldsymbol{\upsilon}\| \,\left|\Theta\left(x,\, t_n,\, \boldsymbol{\upsilon}\right) = 0\right.\right\}$$



Hence, it is a *constrained optimization problem* aiming to minimize $\|\boldsymbol{\upsilon}\|$ with nonlinear constraint $\Theta\left(x,\, t_n,\, \boldsymbol{\upsilon}\right)$

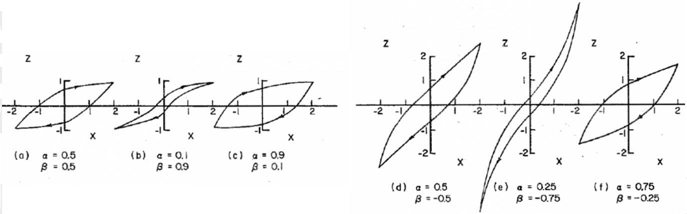
Most of the optimization algorithms addressing such a problem require the computation of the response gradient!

The response gradient

The gradient of the response is an n–dimensional vector:

$$[\nabla_{\upsilon} X(t_n, \boldsymbol{\upsilon})]_i = \frac{\partial X(t_n, \boldsymbol{\upsilon})}{\partial \upsilon_i}$$

It cannot be computed by finite differences but requires the application of the Direct Differentiation Method (DDM)[1]



DDM implementations are available for a limited set of materials in fact most of the applications of TELM are based on the Bouc-Wen model.

[1] T. Haukaas, A. Der Kiureghian, "Strategies for finding the design point in non-linear finite element reliability analysis," *Prob. Eng. Mech.*, 21(2):133-147, 2006.

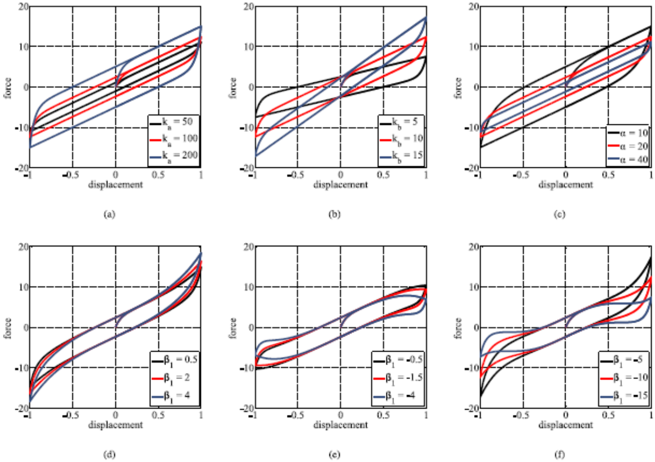
The Bouc-Wen model

A few drawbacks of the Bouc-Wen model:

- It is a differential model requiring iterative algorithms to get the response.
- Its parameters have no physical sense.
- Parameter identification may be very hard and often requires the use of complex identification strategies (e.g., Kalman filter, neural networks etc.).

The Algebraic Material

A phenomenological hysteretic material, capable of reproducing Bouc-Wen-like loops, has been recently proposed by Vaiana et al. (2019):



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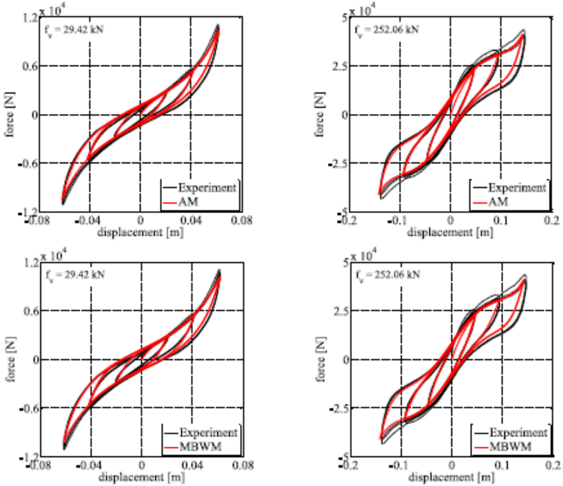
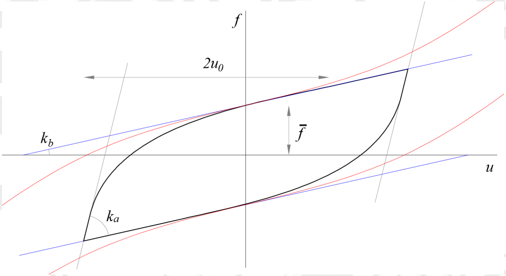


The Algebraic Material

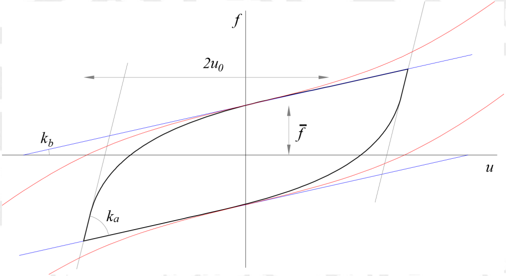
The main benefits of the Algebraic materials:

- Its parameters are physically related to the hysteresis loop;
- Parameter identification is straightforward
    - S. Sessa, N. Vaiana, M. Paradiso, and L. Rosati, "An inverse identification strategy for the mechanical parameters of a phenomenological hysteretic constitutive model," *Mechanical Systems and Signal Processing*, vol. 139, p. 106622, 2020.
- The model provides the response with a closed-form solution and does not require iterations;
- It has been implemented in OpenSees
    - N. Vaiana, R. Capuano, S. Sessa, F. Marmo, and L. Rosati, "Nonlinear dynamic analysis of seismically base-isolated structures by a novel opensees hysteretic material model," *Applied Sciences (Switzerland)*, vol. 11, no. 3, pp. 1–900., 2021.
- It has been used in seismic isolation of museum statues
    - D. Pellecchia, S. Sessa, N. Vaiana, and L. Rosati, "Comparative assessment on the rocking response of seismically base-isolated rigid blocks," Procedia Structural Integrity, vol. 29, pp. 95–102, 2020.



Parameter set: k_a, k_b, α, β_1, β_2;
auxiliary parameters:

$$u_0 = \frac{1}{2}\left[\left(\frac{k_a - k_b}{\delta}\right)^{\frac{1}{\alpha}} - 1\right]; \quad \bar{f} = \frac{k_a - k_b}{2}\left[\frac{(1+2u_0)^{1-\alpha} - 1}{1-\alpha}\right]$$



closed-form response:

$$f\left(u_t, u_c,\, s_t\right) = \beta_1 u_t^3 + \beta_2 u_t^5 + k_b u_t + \left(k_a - k_b\right)\left[\frac{\left(1 + s_t u_t - s_t u_j + 2u_0\right)^{1-\alpha}}{s_t\left(1-\alpha\right)} - \frac{\left(1+2u_0\right)^{1-\alpha} - 1}{1-\alpha}\right] + s_t \bar{f}$$

with:

$$u_j = u_c + s_t\left(1+2u_0\right) - s_t\left\{\frac{s_t\left(1-\alpha\right)}{k_a - k_b}\left[f_c - \beta_1 u_c^3 - \beta_2 u_c^5 - k_b u_c - s_t \bar{f} + \left(k_a - k_b\right)\frac{\left(1+2u_0\right)^{1-\alpha}}{s_t\left(1-\alpha\right)}\right]\right\}^{\frac{1}{1-\alpha}}$$

$$\frac{\partial u_j}{\partial f_c} = 1 - \frac{s_t^2}{k_a - k_b}\left\{\frac{s_t\,(1-\alpha)}{k_a - k_b}\left[f_c - \beta_1 u_c^3 - \beta_2 u_c^5 - k_b u_c - s_t \bar{f} + (k_a - k_b)\,\frac{(1+2u_0)^{1-\alpha}}{s_t\,(1-\alpha)}\right]\right\}^{\frac{\alpha}{1-\alpha}}$$

$$\frac{\partial u_j}{\partial u_c} = \frac{\partial u_j}{\partial f_c}\left[-3\beta_1 u_c^2 - 5\beta_2 u_c^4 - k_b\right]$$

$$\frac{\partial f\,(u_t, u_c,\, s_t)}{\partial f_c} = -\frac{\partial u_j}{\partial f_c}\,(k_a - k_b)\left[1 + s_t u_t - s_t u_j + 2u_0\right]^{-\alpha}$$

$$\frac{\partial f\,(u_t, u_c,\, s_t)}{\partial u_c} = -\frac{\partial u_j}{\partial u_c}\,(k_a - k_b)\left[1 + s_t u_t - s_t u_j + 2u_0\right]^{-\alpha}$$

$$\frac{\partial f\,(u_t, u_c,\, s_t)}{\partial u_t} = 3\beta_1 u_t^2 + 5\beta_2 u_t^4 + k_b + (k_a - k_b)\left[1 + s_t u_t - stu_j + 2u_0\right]^{-\alpha}$$

Sensitivity of the Algebraic Material

HystereticPoly.cpp* | J2PlateFiber.cpp | TclModelBuilderSectionCommand.cpp | MP_Constraint.cpp | DamageUM.h | DamageUM.cpp

material | HystereticPoly | getStressSensitivity(int gradIndex, bool conditional)

```cpp
    // GET SENSITIVITY
    // This is the Tstress derivative with respect to Tstrain
    double Delta1 = st*(1 - a) / (k1 - k2);
    double Delta3 = (k1 - k2)*pow(1 + 2 * uo, 1 - a) / st / (1 - a);
    double Delta2 = Cstress - c*pow(Cstrain, 3) - d*pow(Cstrain, 5) - k2*Cstrain - st*Fbar + Delta3;
    uj = Cstrain + st*(1 + 2 * uo) - st*pow(Delta1*Delta2, 1 / (1 - a));
    double Delta4 = pow(1 + st*Tstrain - st*uj + 2 * uo, 1 - a) / st / (1 - a);
    double Delta5 = (pow(1 + 2 * uo, 1 - a) ) / (1 - a);
    Tstress = c*pow(Tstrain, 3) + d*pow(Tstrain, 5) + k2*Tstrain + (k1 - k2)*(Delta4 - Delta5) + st*Fbar;

    double Duo = 0.5*uo / a * (tol / (k1 - k2) * ((Dk1 - Dk2) / tol - (k1 - k2) / pow(tol, 2)*Dtol) - Da / a* log((k1 - k2) / tol));
    double DFbar = 0.5*(Dk1 - Dk2) * (pow(1 + 2 * uo, 1 - a) - 1) / (1 - a) +
        0.5*(k1 - k2) * (pow(1 + 2 * uo, 1 - a) / (1 - a) * (2*Duo*(1-a)/(1+2*uo) - Da*log(1+2*uo) ) + Da*(pow(1+2*uo,1-a)-1)/pow(1-a,2) );

    double DD1 = (st*Da * (k1 - k2) + st*(1 - a)*(Dk1 - Dk2)) / pow(k1 - k2, 2);
    double DD3 =     (Dk1 - Dk2)*pow(1 + 2 * uo, 1 - a) / st / (1 - a) + (k1 - k2)*pow(1 + 2 * uo, 1 - a) / pow(st, 2) / pow(1 - a, 2)*Da +
                     Delta3 * (2 * Duo*(1 - a) / (1 + 2 * uo) - Da*log(1 + 2 * uo));

    double DD2 = Dfc - Dc*pow(Cstrain, 3) - Dd * pow(Cstrain, 5) - Dk2*Cstrain - (3 * c*pow(Cstrain, 2) + 5 * d*pow(Cstrain, 4) + k2)*Duc - st*

    double Duj = Duc + 2 * st*Duo - st*pow(Delta1*Delta2, 1 / (1 - a)) * (Da*log(Delta1*Delta2) / pow(1 - a, 2) + (DD1*Delta2 + Delta1*DD2) / (

    double DD4 = pow(1 + st*Tstrain - st*uj + 2 * uo, 1 - a) / pow(st, 2) / pow(1 - a, 2)*Da + Delta4 * (1 - a)*(st*Dut - st*Duj + 2 * Duo) / (
    double DD5 = (pow(1 + 2 * uo, 1 - a)) / pow(1 - a, 2)*Da + pow(1 + 2 * uo, 1 - a) / (1 - a)*(2 * Duo*(1 - a) / (1 + 2 * uo) - Da*log(1 + 2


    gradient = Dc*pow(Tstrain, 3) + Dd*pow(Tstrain, 5) + Dk2*Tstrain + (3 * c*pow(Tstrain, 2) + 5 * d*pow(Tstrain, 4) + k2)*Dut +
        st*DFbar + (Dk1 - Dk2)*(Delta4 - Delta5) + (k1 - k2)*(DD4 - DD5);


    return gradient;
}
```

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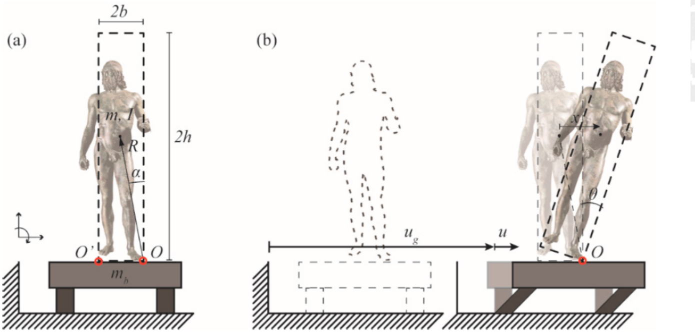
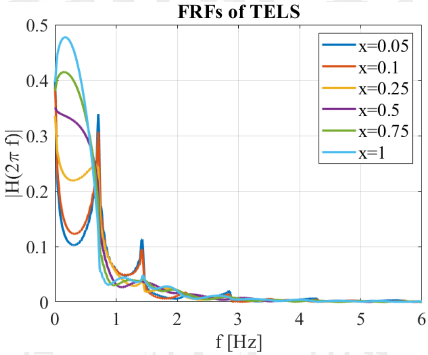


The analyzed model consists in the Riace Bronze A with basement isolated by elastomeric bearings (see De Canio 2012[2] and Pellecchia et al. 2020).

The response of interest is the horizontal displacement of the basement.

[2]G. De Canio, "Marble devices for the base isolation of the two Bronzes of Riace: a proposal for the David of Michelangelo," Proc. WCEE, 2012.

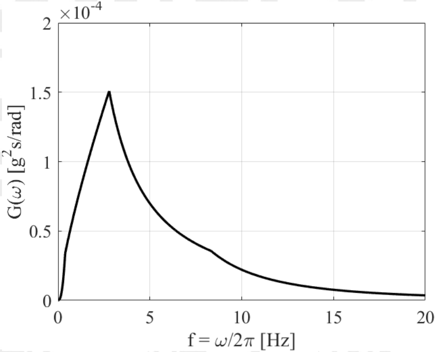
Tail-Equivalent Linearization System



The TELS is defined by means of Frequency Response Functions determined as the Fourier Transforms of the IRFs:

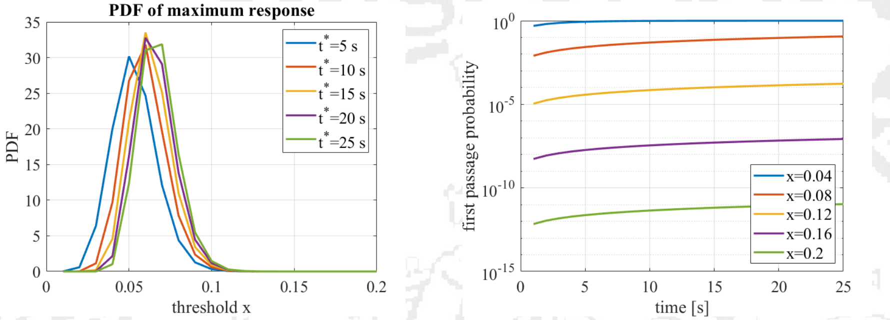
$$H^{\star}\left(x,\, t_n,\, \omega\right) = \int_0^{\infty} h^{\star}\left(x,\, t_n,\, \tau\right) \exp\left(-i\omega\tau\right)\, d\tau$$

Tail-Equivalent Linearization System

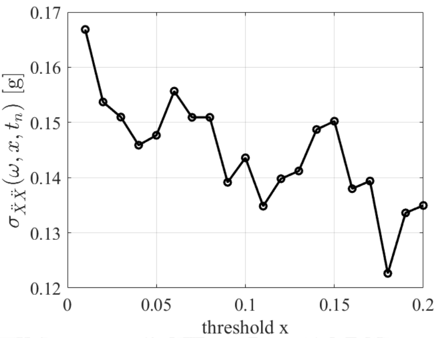


First-passage probability analysis is then performed by adopting the Power Spectral Density proposed in

- G. Barone, F. Iacono, G. Navarra, and A. Palmeri, "A novel analytical model of power spectral density function coherent with earthquake response spectra," pp. 394–406, UNCECOMP 2015.



Response statistics computed by the Algebraic material are indistinguishable from those computed by adopting a Bouc Wen model calibrated on the same hysteresis loop.



(Square root of the) Mean-square acceleration at the base of the Bronze vs the maximum horizontal displacement of the basement.

Conclusions

- The Algebraic material is capable of reproducing several typologies of hysteretic loops by means of a closed-form analytical expression and does not require iterative procedures to determine the response.
- Computation of closed-form response sensitivity permits the use of the A.M. within the framework of the Tail-Equivalent Linearization and the Direct Differentiation Method.
- Numerical applications concerning the random vibration analysis of the base-isolated Bronze of Riace A show the effectiveness of the implementation and of the whole procedure since differences between the outcomes and the ones relevant to a Bouc-Wen model calibrated on the same hysteresis loop are negligible.
- Compared to the Bouc-Wen model, the Algebraic material requires a smaller computational burden and its parameters are directly related to hysteresis loop physical properties.

Future work

- Implementation of asymmetric hysteresis
    - N. Vaiana, S. Sessa, and L. Rosati, “A generalized class of uniaxial rate-independent models for simulating asymmetric mechanical hysteresis phenomena,” *Mechanical Systems and Signal Processing*, vol. 146, 2021.
- Extension to multi-dimensional responses;
- Implementation of degradation;
- Design optimization of base-isolation system.

Thank You for your attention

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