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Towards a multiphase finite element code for the evaluation of the volcanic risk

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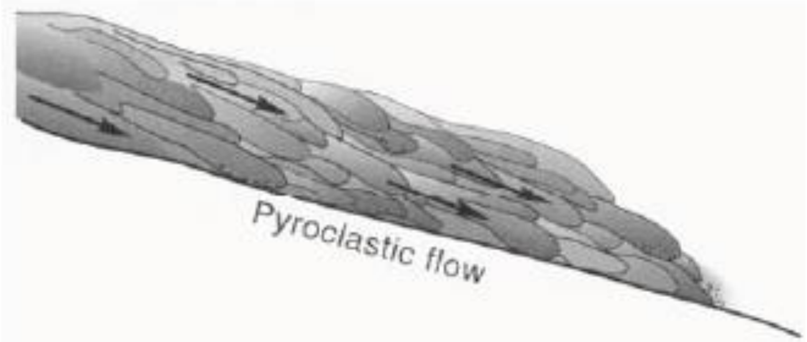
Outline

- Characterization of pyroclastic density currents
- Current evaluation strategies for volcanic risk
- Numerical formulation and numerical simulations
- Results
- Conclusions

Pyroclastic density current

Pyroclastic density currents (**PDC**) are flows of **gas** mixed with **solid particulate** originated from **explosive volcanic events** and flowing along the sides of the volcano.



PDC are among the most dangerous consequences of a volcanic eruptions, since they generate flows at **high pressure** and **high temperature**.

Here we focus on the Vesuvius area: besides the famous catastrophic **event of 79 AD**, teh Vesuvius has been interested by several eruption in the last two millennia, among which the most important in **1631** and **1906**

Volcanic risk

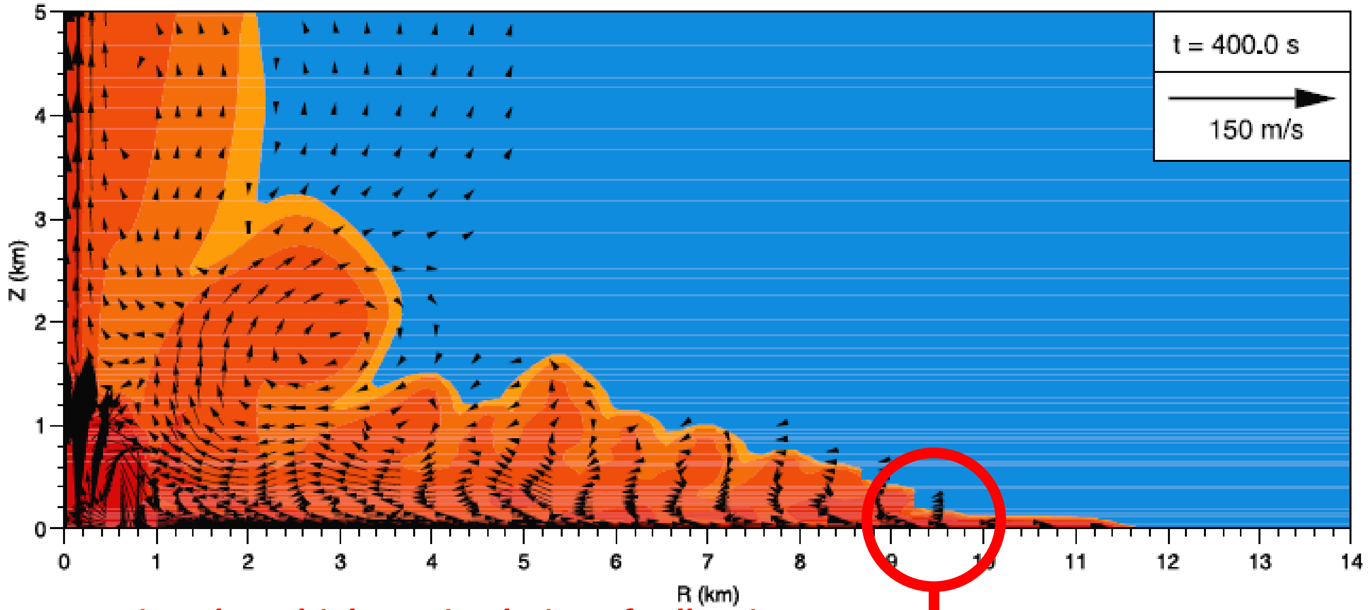
The evaluation of the volcanic risk associated to pyroclastic flows is computed in terms of a reference eruptive event (**Sub-Plinian of type I**), defined by the **Italian Department of Civil Protection**.

From the reference scenario, numerical simulation are adopted to quantify the hazard at different distances from the Vesuvius, and to define the risk areas (**red areas**)

On the basis on these data, and accounting for the presence of buildings and their typologies, for the road network and other peculiarities of the areas, it is possible to quantify the risk (**impact analysis**).

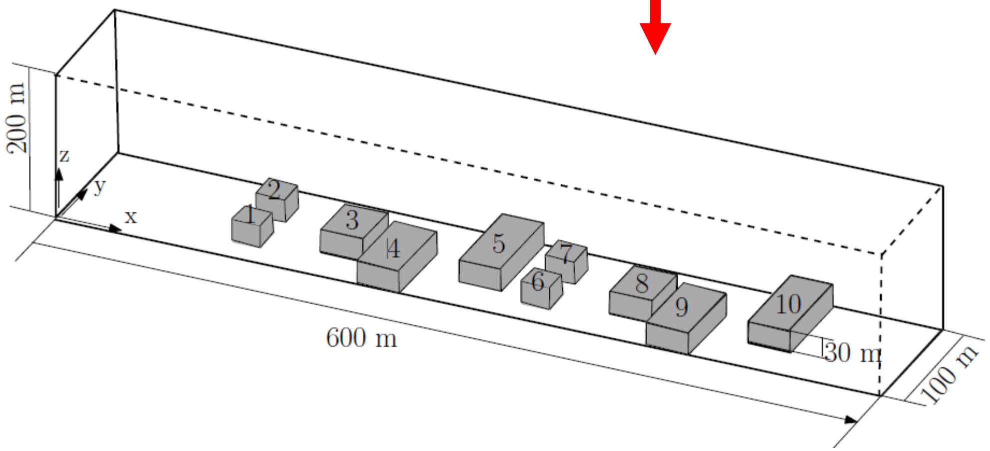
A change of scale

Current numerical simulation are run at the scale of the volcano. The presence of building is accounted for through a roughness at the ground level.



Neri et al, *Multiphase simulation of collapsing volcanic columns and pyroclastic flows*, 2003

Our scope is to investigate what happens at the scale of an **urban settlement**: the influence of the **height** of the buildings, their **distancing**, their **reciprocal position**. We start from existing large scale simulation and study the modification of the flow at the scale of the buildings



Numerical formulation

Gas phase

$$\frac{\partial}{\partial t}(\varepsilon_g\rho_g) + \nabla\cdot(\varepsilon_g\rho_g\boldsymbol{u}_g) = 0$$

$$\frac{\partial}{\partial t}(\varepsilon_g\rho_g\boldsymbol{u}_g) + \nabla\cdot(\varepsilon_g\rho_g\boldsymbol{u}_g\otimes\boldsymbol{u}_g + \varepsilon_g p_g) = D\,(u_g - u_s) + \varepsilon_g\rho_g\boldsymbol{g} + VISCOUS\ TERMS$$

$$\frac{\partial}{\partial t}(\varepsilon_g\rho_g e_g) + \nabla\cdot\left((\varepsilon_g\rho_g e_g + \varepsilon_g p_g)\boldsymbol{u}_g\right) = D\,(u_g - u_s)u_s - Q\,(T_g - T_s) + \varepsilon_g\rho_g\boldsymbol{g}\cdot\boldsymbol{u}_g + VISCOUS\ TERMS$$

Solid phase

$$\frac{\partial}{\partial t}(\varepsilon_s\rho_s) + \nabla\cdot(\varepsilon_s\rho_s\boldsymbol{u}_s) = 0$$

$$\frac{\partial}{\partial t}(\varepsilon_s\rho_s\boldsymbol{u}_s) + \nabla\cdot(\varepsilon_s\rho_s\boldsymbol{u}_s\otimes\boldsymbol{u}_s) = -D\,(u_g - u_s) + \varepsilon_s\rho_s\boldsymbol{g} + VISCOUS\ TERMS$$

$$\frac{\partial}{\partial t}(\varepsilon_s\rho_s e_s) + \nabla\cdot(\varepsilon_s\rho_s e_s\boldsymbol{u}_s) = -D\,(u_g - u_s)u_s + Q\,(T_g - T_s) + \varepsilon_s\rho_s\boldsymbol{g}\cdot\boldsymbol{u}_s + VISCOUS\ TERMS$$

Kinetic and thermal equilibrium

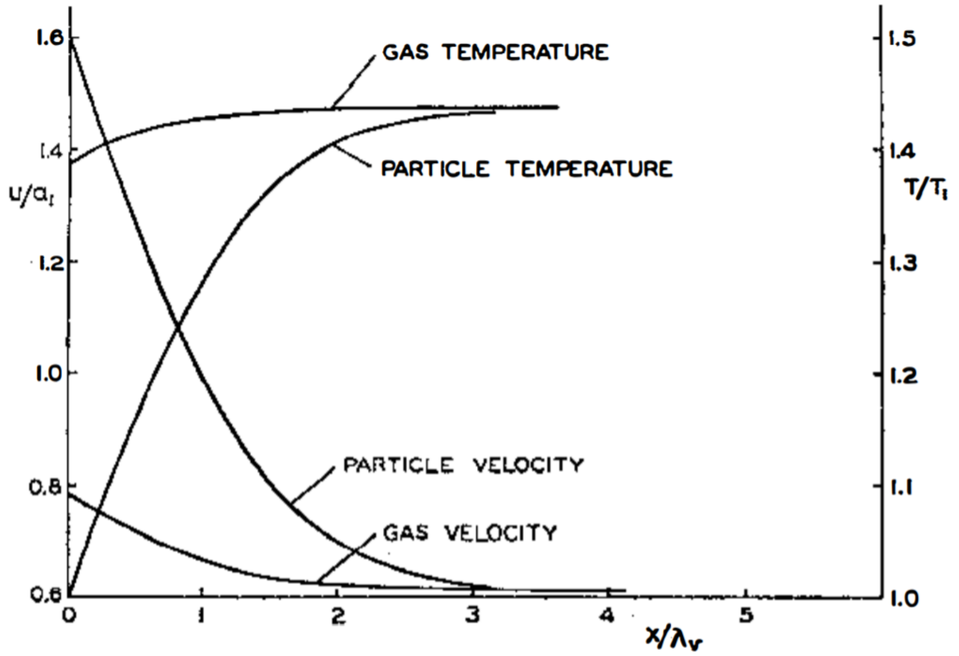
$D\left(u_g - u_s\right)$

Inter-phase dissipation terms

$Q\left(T_g - T_s\right)$

Characteristic dissipation times $\tau_v = \frac{m}{6\pi\sigma\mu}$ and $\tau_T = \frac{m\,C_p}{6\pi\sigma k}$

Depending on particle geometry and flow properties



Marble, *Dynamics of dusty gases*, 1970

Table 1. Employed Properties of the Six Solid Particulate Phases[a]

Phase	d_k, µm	ρ_k, kg m^{-3}	c_k (Equation (20)), Pa s	τ_V, s	τ_T, s
1	32	2100	0.5	$6.5 \times 10^{-3}/2.6 \times 10^{-3}$	$8.6 \times 10^{-3}/2.9 \times 10^{-3}$
2	95	2100	0.5	$5.7 \times 10^{-2}/2.3 \times 10^{-2}$	$7.6 \times 10^{-2}/2.6 \times 10^{-2}$
3	190	1800	1.0	$2.0 \times 10^{-1}/7.7 \times 10^{-2}$	$2.6 \times 10^{-1}/8.7 \times 10^{-1}$
4	375	1100	1.0	$4.7 \times 10^{-1}/1.8 \times 10^{-1}$	$6.2 \times 10^{-1}/2.1 \times 10^{-1}$
5	750	1100	2.0	$1.9/7.4 \times 10^{-1}$	$2.5/8.3 \times 10^{-1}$
6	1500	1100	2.0	$7.5/2.9$	$9.9/3.3$

Neri et al, *Multiphase simulation of collapsing volcanic columns and pyroclastic flows*, 2003

Equilibrium formulation

$t > \tau_v$

$t > \tau_T$

$\boldsymbol{u}_g = \boldsymbol{u}_s = \boldsymbol{u}$

$T_g = T_s = T$

Velocities and temperatures of the two phases coincide

Dusty gas

$$\frac{\partial}{\partial t}\rho + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

$$\frac{\partial}{\partial t}(\rho \boldsymbol{u}) + \nabla \cdot [\rho \boldsymbol{u} \otimes \boldsymbol{u} + \varepsilon_g p_g] = \rho \boldsymbol{g} + VISCOUS\ TERMS$$

$$\frac{\partial}{\partial t}(\rho e) + \nabla \cdot [(\rho e + \varepsilon_g p_g)\boldsymbol{u}] = \rho \boldsymbol{g} \cdot \boldsymbol{u} + VISCOUS\ TERMS$$

$$\rho = \varepsilon_g \rho_g + \varepsilon_s \rho_s$$

$$e = \frac{1}{2}\boldsymbol{u} \cdot \boldsymbol{u} + (\varepsilon_g \rho_g c_v + \varepsilon_s \rho_s c_s)T$$

$$p_g = \rho_g R T$$

Solid phase transport

$$\frac{\partial}{\partial t}(\varepsilon_s \rho_s) + \nabla \cdot (\varepsilon_s \rho_s \boldsymbol{u}) = 0$$

Conservative variable formulation

$$\frac{\partial}{\partial t}\boldsymbol{U} + \boxed{\frac{\partial}{\partial x}\boldsymbol{F}(\boldsymbol{U})} = \boxed{\frac{\partial}{\partial x}\boldsymbol{G}(\boldsymbol{U})} + \boldsymbol{S}(\boldsymbol{U})$$

Vector form

$$\boldsymbol{U} = \begin{bmatrix} \rho \\ P_s = \varepsilon_s \rho_s \\ \boldsymbol{M} = \rho \boldsymbol{u} \\ E_{TOT} \end{bmatrix}$$

Convective term

Diffusive term

Vector of conservative variables

ADVECTION DIFFUSION PROBLEM

Finite Element formulation

$$\left(V, \frac{\partial}{\partial t}\boldsymbol{U}\right) + \left(V, \frac{\partial}{\partial x}\boldsymbol{F}(\boldsymbol{U})\right) = \left(\frac{\partial}{\partial x}\boldsymbol{V}, \boldsymbol{G}(\boldsymbol{U})\right) + \left(\boldsymbol{V}, \boldsymbol{S}(\boldsymbol{U})\right)$$

Weak form

Variational Multiscale (VMS) stabilization

The finite element discretization of **the advection diffusion equation** suffers from numerical instability, due to the fact that Finite Elements cannot represent phoenomena **(such as turbolence)** at lower scales than the discretization size

$$\boldsymbol{U} = \boldsymbol{U}^h + \widetilde{\boldsymbol{U}} \quad , \qquad \boldsymbol{V} = \boldsymbol{V}^h + \widetilde{\boldsymbol{V}}$$

After some algebra and approximation

$$\left(V, \frac{\partial}{\partial t} \boldsymbol{U}\right) + \left(V, \frac{\partial}{\partial x} \boldsymbol{F}(\boldsymbol{U})\right) + \text{STABILIZING TERM} = \left(\frac{\partial}{\partial x} \boldsymbol{V}, \boldsymbol{G}(\boldsymbol{U})\right) + \left(\boldsymbol{V}, \boldsymbol{S}(\boldsymbol{U})\right)$$

Implementation

KRATOS MULTI-PHYSICS

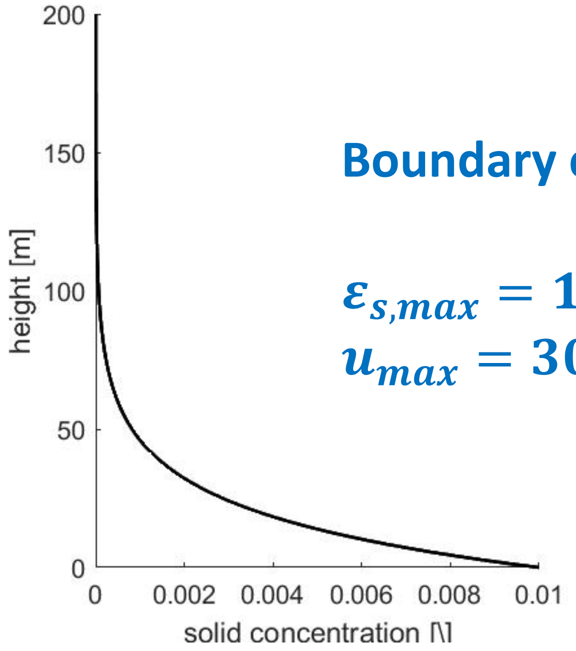
Numerical example

Initial conditions

Stratified air density and air pressure
Absence of solid particulate

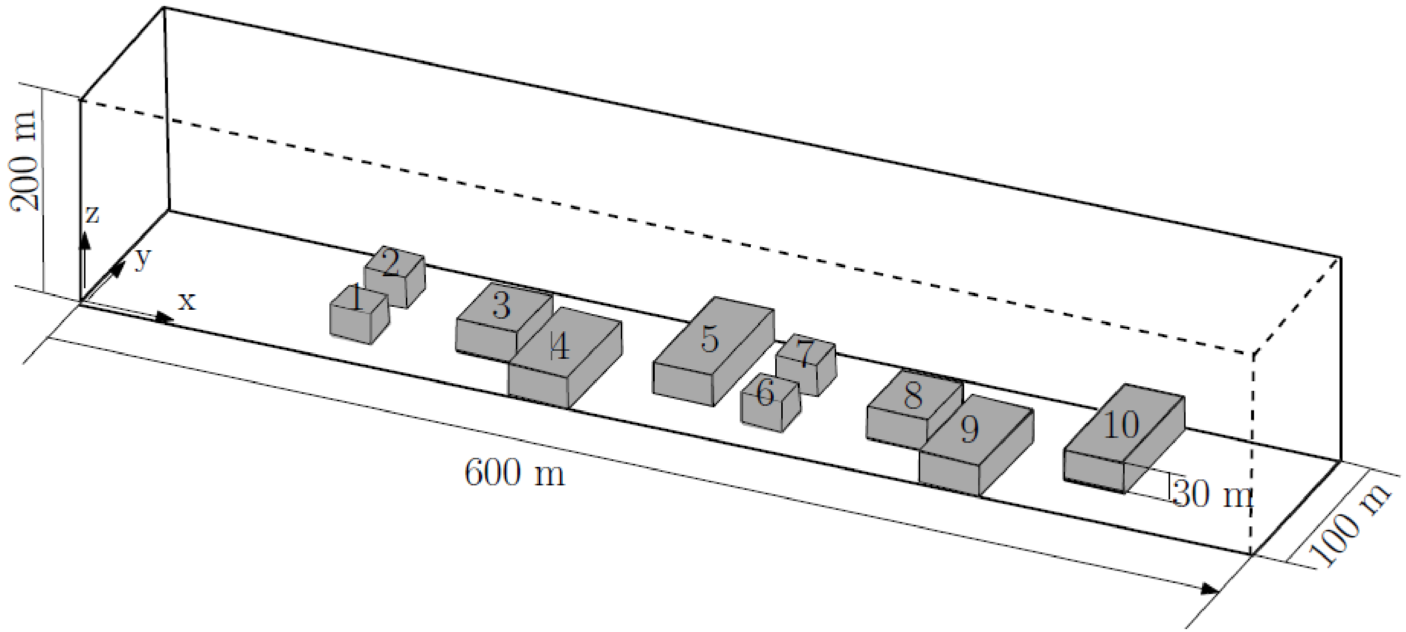
10 buildings randomly distributed on a rectangular surface

Inlet at $x = 0$

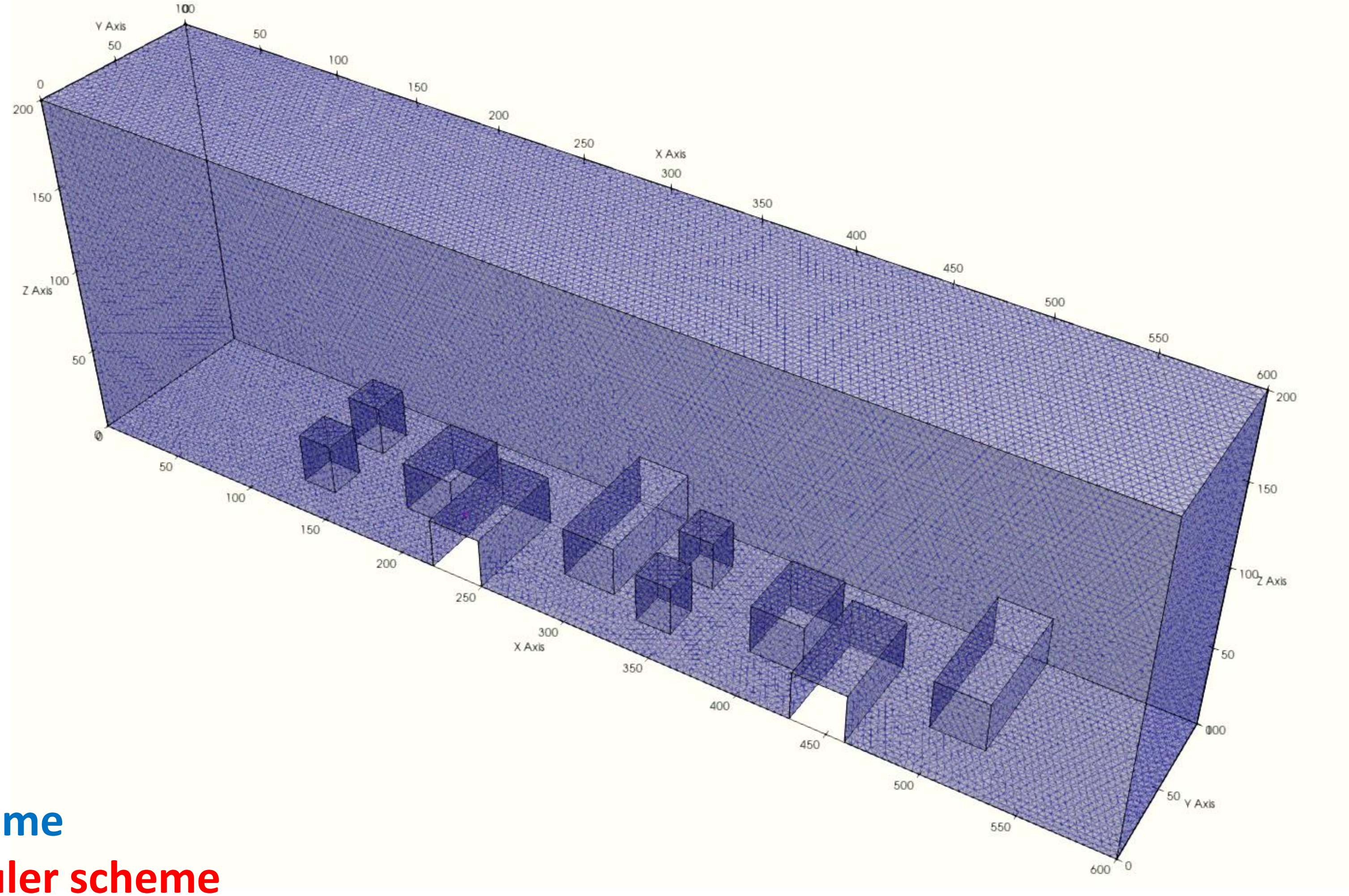


Boundary conditions:

$$\varepsilon_{s,max} = 10^{-2}$$
$$u_{max} = 30 \text{ m/s}$$



Discretization



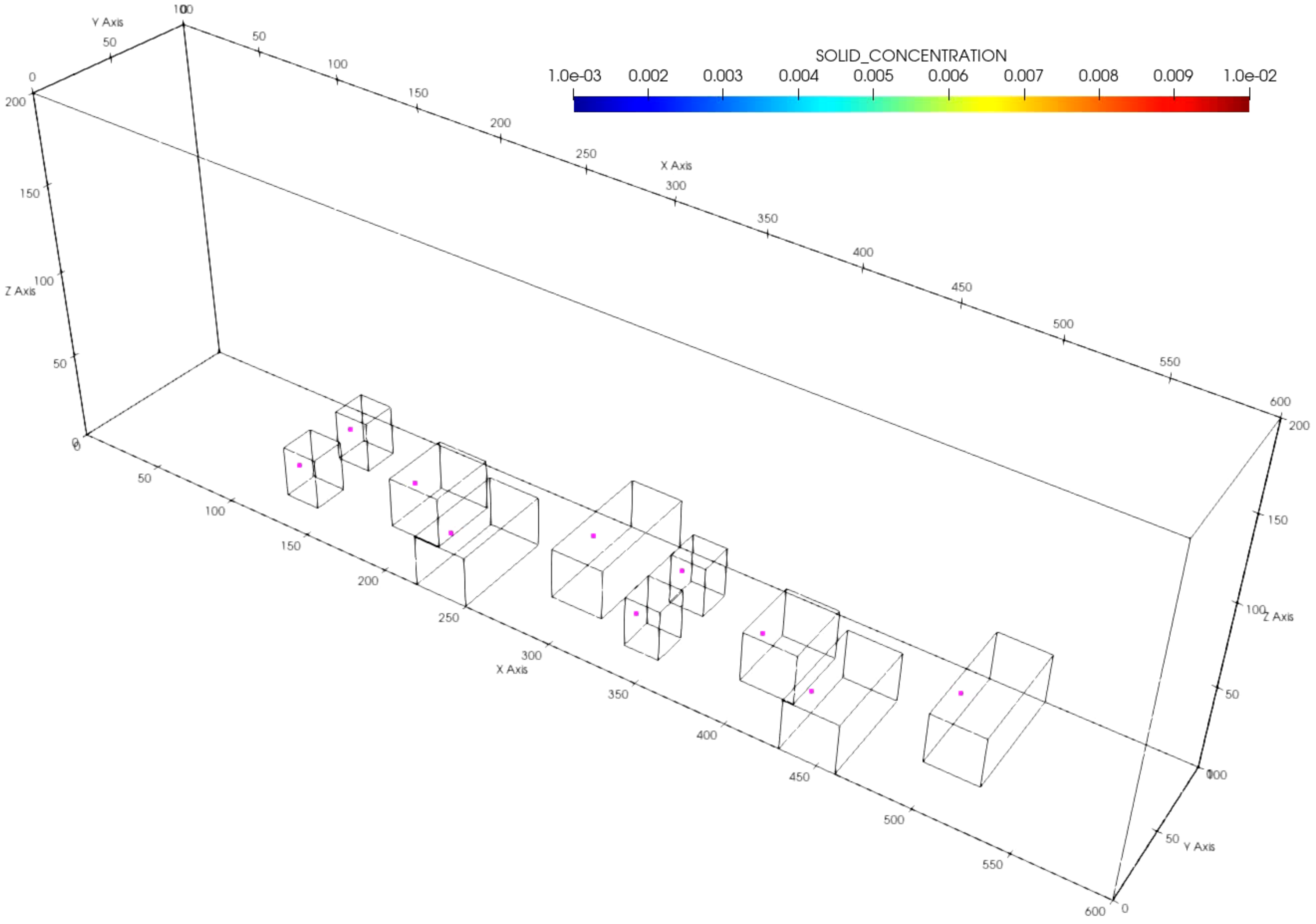
Finite Element Discretization

326654 nodes
1840787 tetrahedral elements
Average element size = 5 m

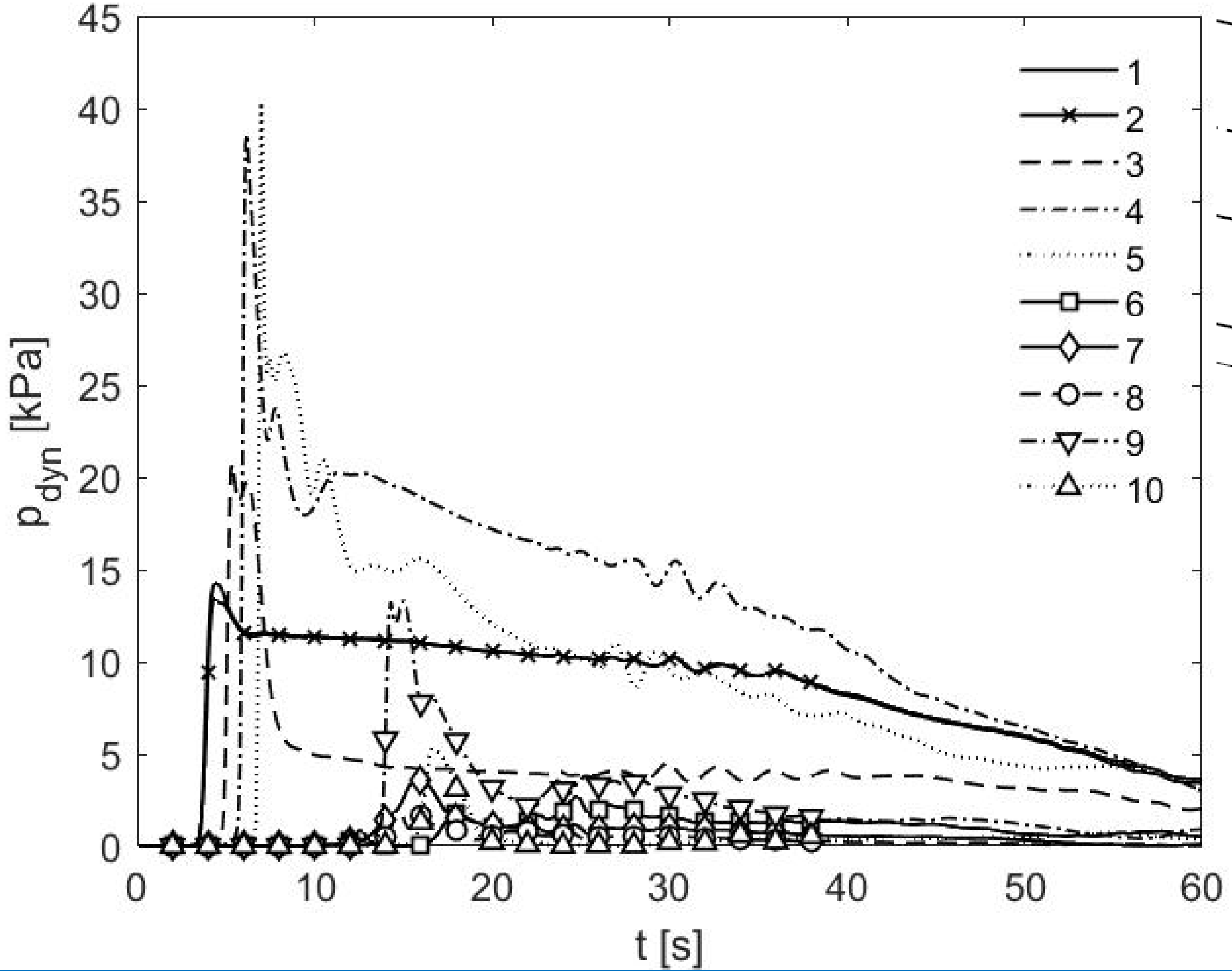
Time discretization
$\Delta t = 2.5\ 10^{-4}$ s

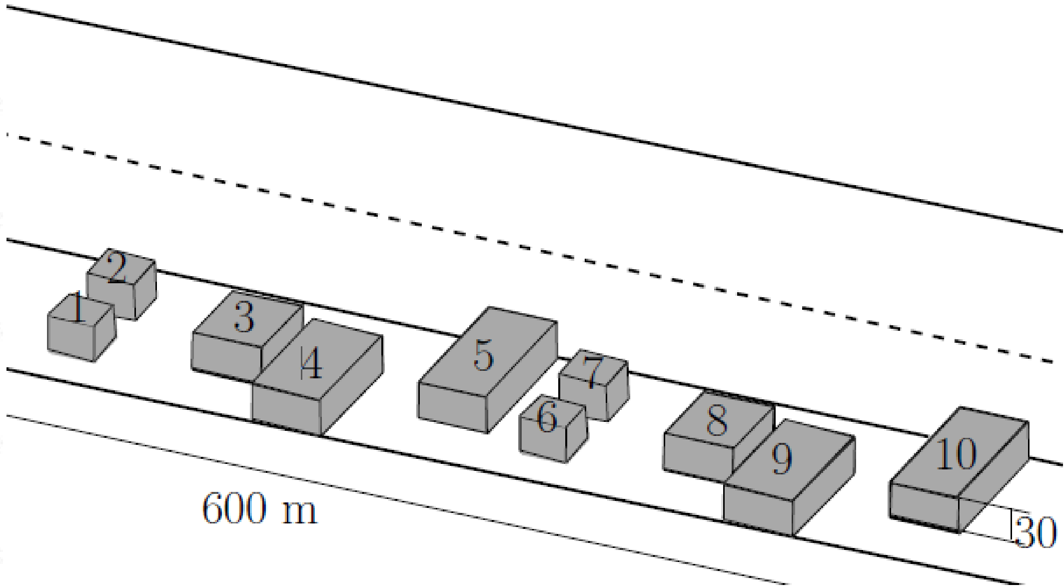
Time scheme
Explicit Euler scheme

Numerical simulation



Results





Peak of dynamic pressure due to impulsive input

Evident shielding effect on the second set of buildings

Conclusions

For a more assessment of volcanic risk, it is necessary to consider numerical simulations

Numerical simulation at the scale of the volcano cannot give precise information about the input on buildings

The simulation at the scale of the urban settlement evidences the influence of the urban texture on the hazard deriving from volcanic phoenomena.

The numerical simulations via stabilized FEM confirm the validity of the numerical approach for the quantification of volcanic risk.
The numerical code is efficient and able to run in a reasonable time also on desktop workstations.

THANK YOU!

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