Topology Optimization with Polymorphic Uncertainties using Artificial Neural Networks

Steffen Freitag, Simon Peters, Philipp Edler, Günther Meschke

Research Group on Computational Intelligence and Structural Reliability
Institute for Structural Mechanics
Ruhr University Bochum

Optimization in Structural Mechanics

Objectives

- minimization of self weight (required material)
- maximizing the stiffness (minimizing the deformations)
- balancing the stresses of all structural members

Constraints

- load bearing capacity (strength of materials)
- serviceability (tolerated displacements)

Optimization approaches

- Sizing

- Shape Optimization

- Topology Optimization

Design variables

- dimensions (height, width)
- domain boundaries (shape functions)
- material distribution (densities)

Topology Optimization – SIMP

Problem formulation

$$\min_{\rho}: \quad c(\boldsymbol{\rho}) = \boldsymbol{u}^T \boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{u} = \sum_{e=1}^{N} E_e(\rho_e)\, \boldsymbol{u}_e^T \boldsymbol{k}_0 \boldsymbol{u}_e$$

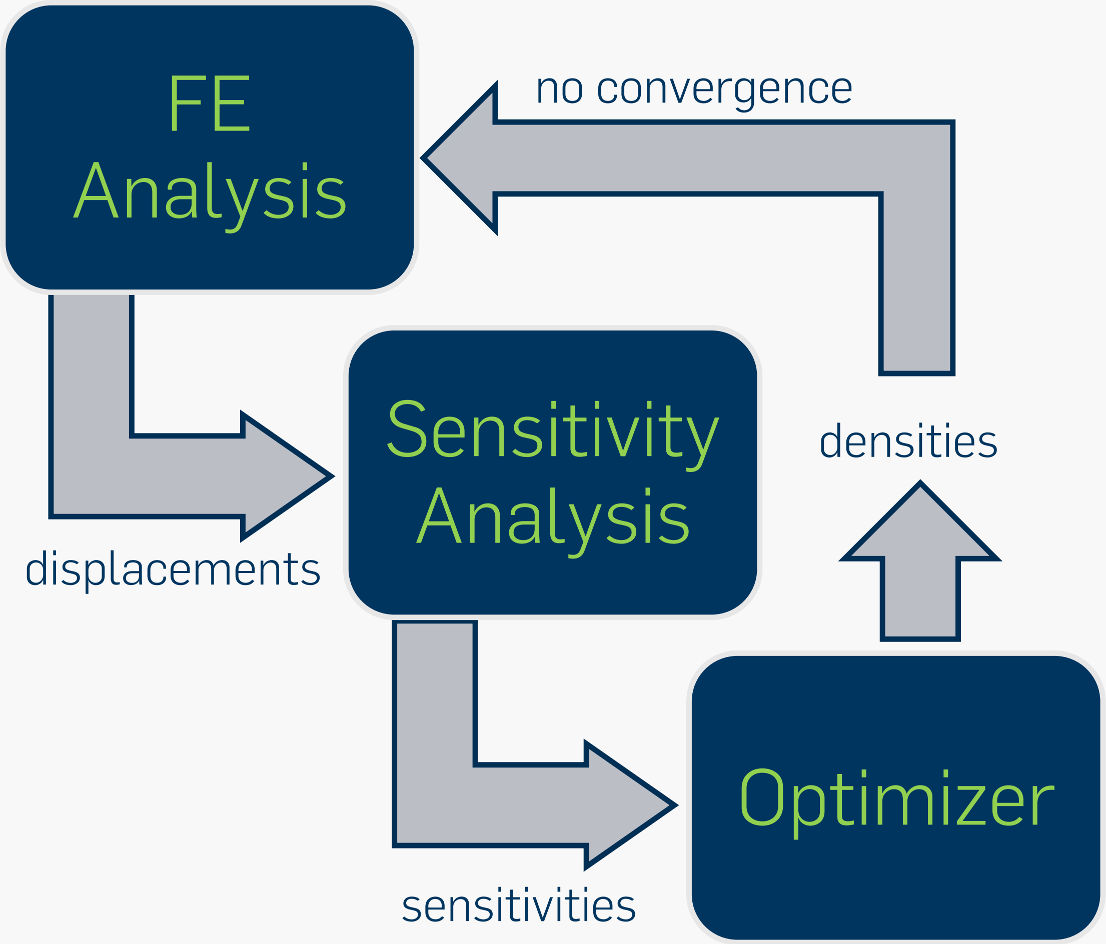
$$s.t \begin{cases} V_{req} = \sum_{e=1}^{N} V_e \cdot \rho_e \\ \boldsymbol{K}(\rho)\boldsymbol{u} = \boldsymbol{F} \\ \rho_e \in [0,1]\ \forall e = 1,2,\dots,N \end{cases}$$

Sensitivities

$$\frac{\partial c}{\partial \rho_e} = \sum_{i=1}^{M} -p\rho_e^{p-1}(E_0 - E_{min})\boldsymbol{u}_{e,i}^T \boldsymbol{k_0} \boldsymbol{u}_{e,i}$$

Modified SIMP

$$E_e(\rho_e) = E_{min} + \rho_e^p (E_0 - E_{min})$$



Topology Optimization – SIMP

Problem formulation

$$\min_{\rho}: \quad c(\boldsymbol{\rho}) = \boldsymbol{u}^T \boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{u} = \sum_{e=1}^{N} E_e(\rho_e)\, \boldsymbol{u}_e^T \boldsymbol{k}_0 \boldsymbol{u}_e$$

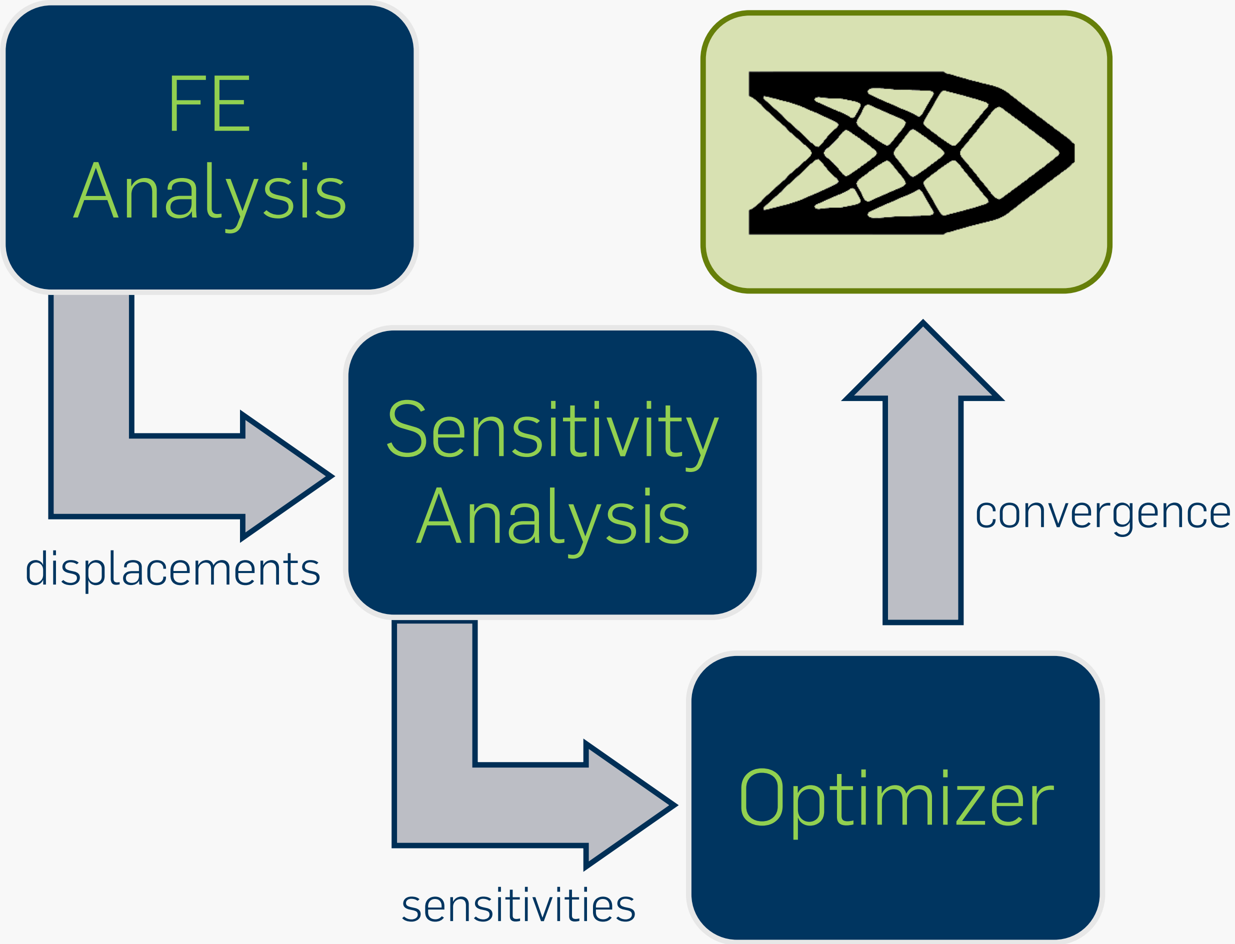
$$s.t \begin{cases} V_{req} = \sum_{e=1}^{N} V_e \cdot \rho_e \\ \boldsymbol{K}(\rho)\boldsymbol{u} = \boldsymbol{F} \\ \rho_e \in [0,1]\ \forall e = 1,2,\dots,N \end{cases}$$

Sensitivities

$$\frac{\partial c}{\partial \rho_e} = \sum_{i=1}^{M} -p\rho_e^{p-1}(E_0 - E_{min})\boldsymbol{u}_{e,i}^T \boldsymbol{k}_0 \boldsymbol{u}_{e,i}$$

Modified SIMP

$$E_e(\rho_e) = E_{min} + \rho_e^p (E_0 - E_{min})$$

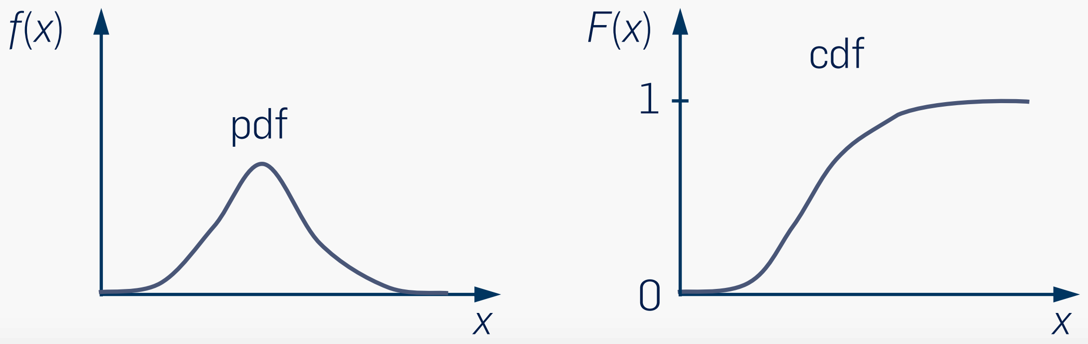


Polymorphic Uncertainty Quantification

Aleatory uncertainty

- randomness / variability
- not reducible
- objective assessment

stochastic distributions (random variable)

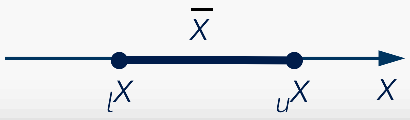


Epistemic uncertainty

- lack of knowledge
- reducible
- subjective / objective assessment

ranges (interval)

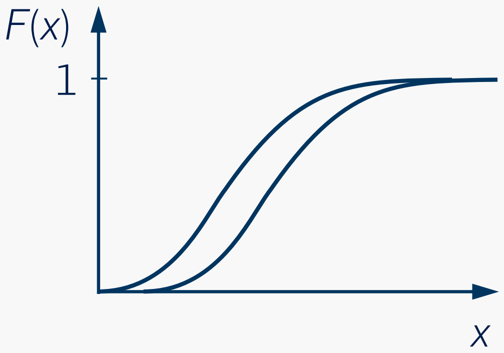
parameter bounds	accepted tolerance
[10, 12]	± 10 %



Polymorphic Uncertainty Quantification

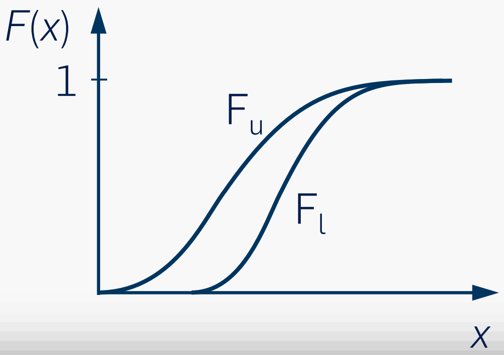
probability box (p-box)

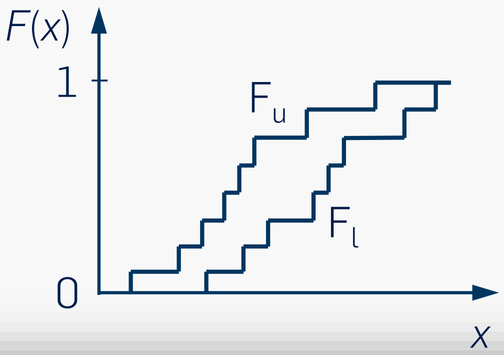
- random variable with interval parameters (parametric p-box)



e.g. Normal Distribution
with interval mean value

- lower and upper cumulative distribution functions (free p-box)





Computing with random variables and intervals

inputs: random variables X intervals $\bar{x}$

outputs: p-boxes $\bar{Z}$ probability intervals $\bar{P}_f$

- Monte Carlo simulation and interval analysis

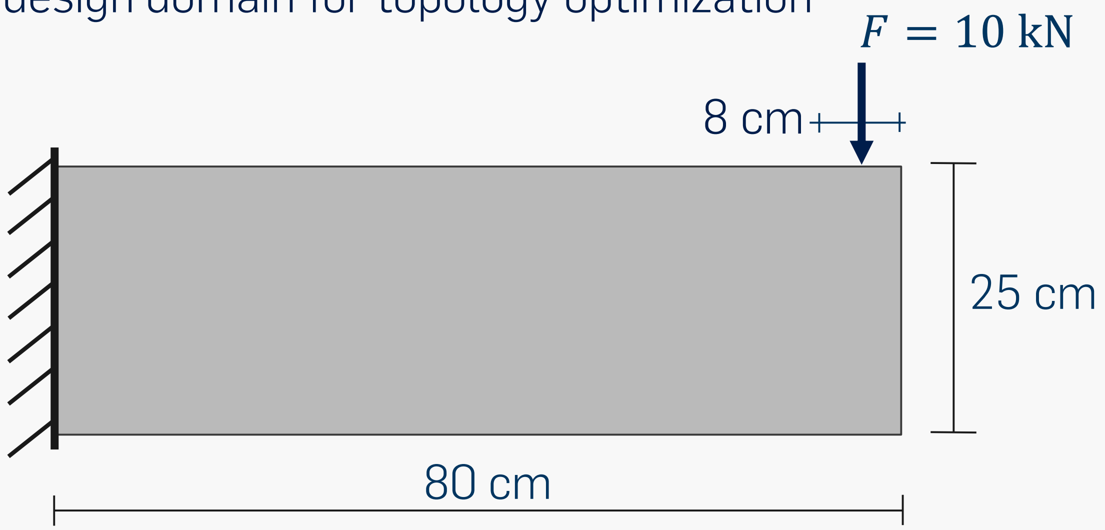
interval analysis

stochastic analysis

deterministic simulation

Weight Minimization of a Cantilever Beam

- design domain for topology optimization



- plane stress, constant width $b = 1$ cm
- finite element model with 25 x 80 elements
- material (steel S235)

  $E = 21{,}000$ kN/cm^2, $\nu = 0.3$

  $f_y = 23.5$ kN/cm^2

- objective function

  $$\min: \ \bar{z}(d_1, \bar{d}_2) = V(A_f, \bar{b}) = 80\ cm \cdot 25\ cm \cdot A_f \cdot \bar{b}$$

- design variables (to be optimized)

  $d_1 = A_f$ area fraction (deterministic design variable)

  $\bar{d}_2 = \bar{b}$ width (interval design variable, fixed radius 1 mm, midpoint to be optimized)

- uncertain a priori parameters (cannot be optimized)

  $A_1 = F$ load (normal, $\mu_F = 10$ kN, $\sigma_F = 2$ kN)

  $A_2 = E$ modulus of elasticity (lognormal, $\mu_E = 21{,}000$ kN/cm^2, $\sigma_E = 1050$ kN/cm^2)

  $A_3 = f_y$ yield strength (lognormal, $\mu_{fy} = 23.5$ kN/cm^2, $\sigma_{fy} = 1.175$ kN/cm^2)

- constraints (reliability-based)

  $g_1 = P\{\sigma_V > f_y\} - 7 \cdot 10^{-5} \leq 0$ load bearing capacity

  $g_2 = P\{w > 3\ \mathrm{mm}\} - 6.7 \cdot 10^{-2} \leq 0$ serviceability

Weight Minimization of a Cantilever Beam

Artificial neural network surrogate models

- constraints evaluation

$$A_f \rightarrow \boxed{1-5-1} \rightarrow \sigma_{V,ANN} \qquad \sigma_V = \frac{P}{10\ \mathrm{kN}} \cdot \frac{1\ \mathrm{cm}}{b} \cdot \sigma_{V,ANN}$$

$$A_f \rightarrow \boxed{1-5-1} \rightarrow w_{ANN} \qquad w = \frac{P}{10\ \mathrm{kN}} \cdot \frac{1\ \mathrm{cm}}{b} \cdot \frac{21{,}000\ \frac{\mathrm{kN}}{\mathrm{cm}^2}}{E} \cdot w_{ANN}$$

- shortest distance to constraints

$$\left.\begin{matrix} A_f \\ b \end{matrix}\right\} \rightarrow \boxed{2-10-1} \rightarrow s_{d1}\{g_1 = 0\}$$

$$\left.\begin{matrix} A_f \\ b \end{matrix}\right\} \rightarrow \boxed{2-4-1} \rightarrow s_{d2}\{g_2 = 0\}$$

- topology prediction

$$A_f \rightarrow \boxed{1-1000-1000-1000-2000} \rightarrow \rho_e$$

Particle swarm optimization

- swarm with 5 particles
- partial derivatives of s_{d1} and s_{d2} to move particles from unfeasible design region onto the constraint limit state

Topology Optimization with Polymorphic Uncertain Parameters

Computational scheme for reliability-based Topology Optimization

Global Reliability-based Optimization (Particle Swarm Optimization)

Constraint evaluation g_1 and g_2

interval analysis
(worst case) ${}_{u}P_f$

stochastic reliability analysis
(MCS) P_f

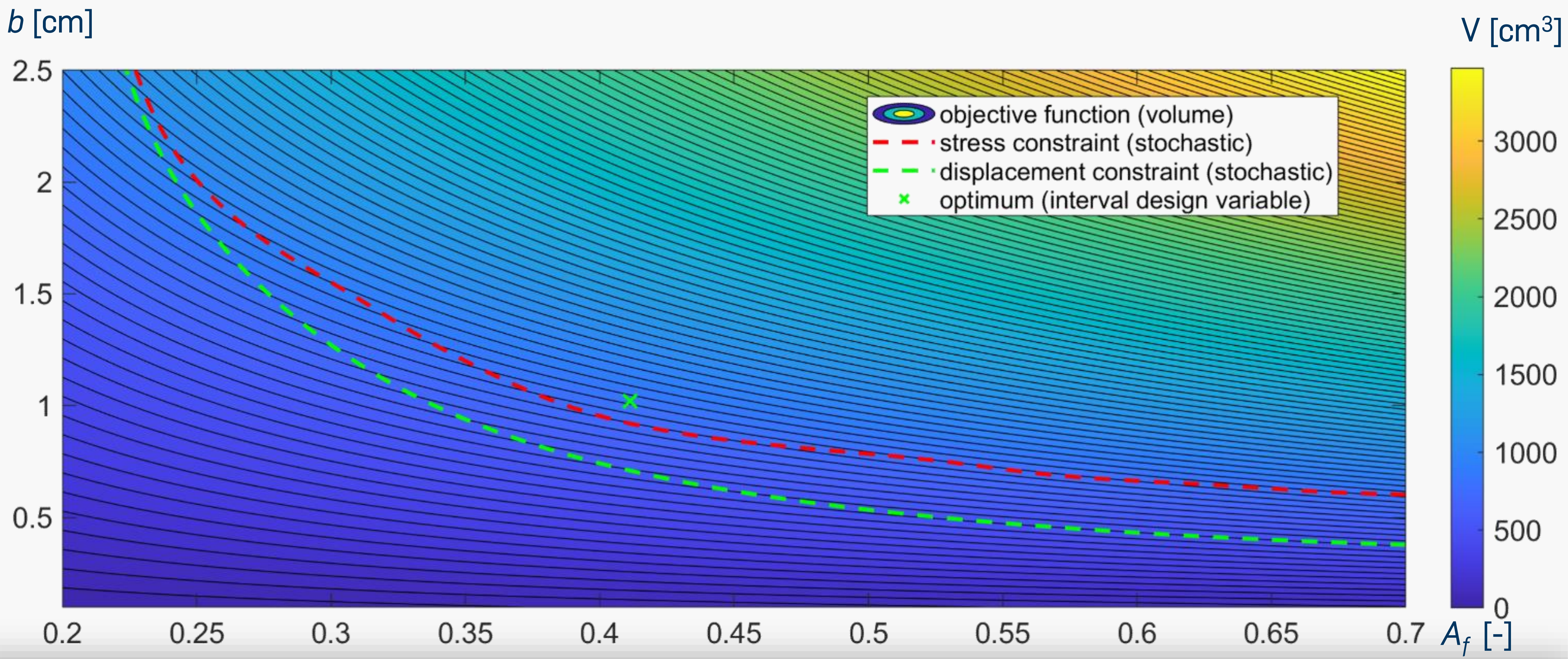
Objective function evaluation min: ${}_{m}V$

interval analysis
(midpoint) ${}_{m}V$

Topology Optimization
(ANNs trained based on FE simulation considering interval load position)

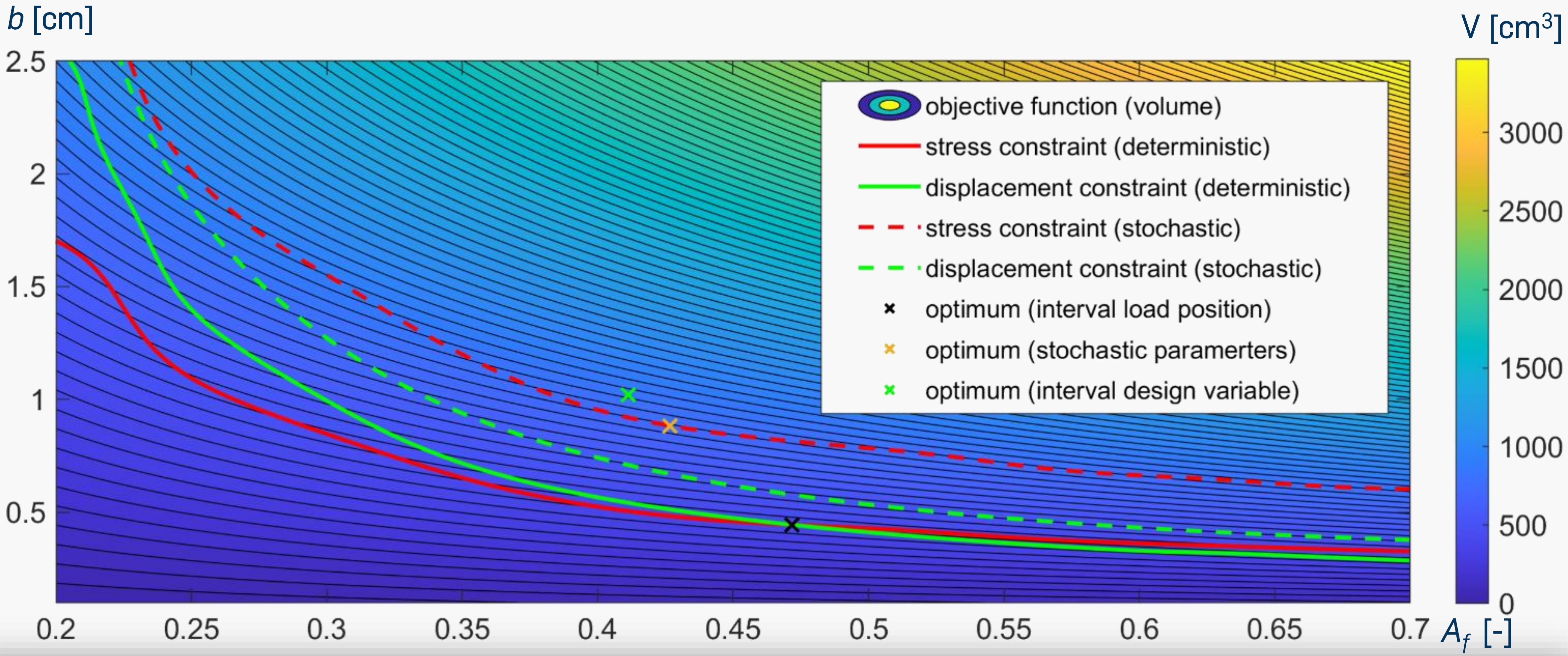
Weight Minimization of a Cantilever Beam

Results with two design variables



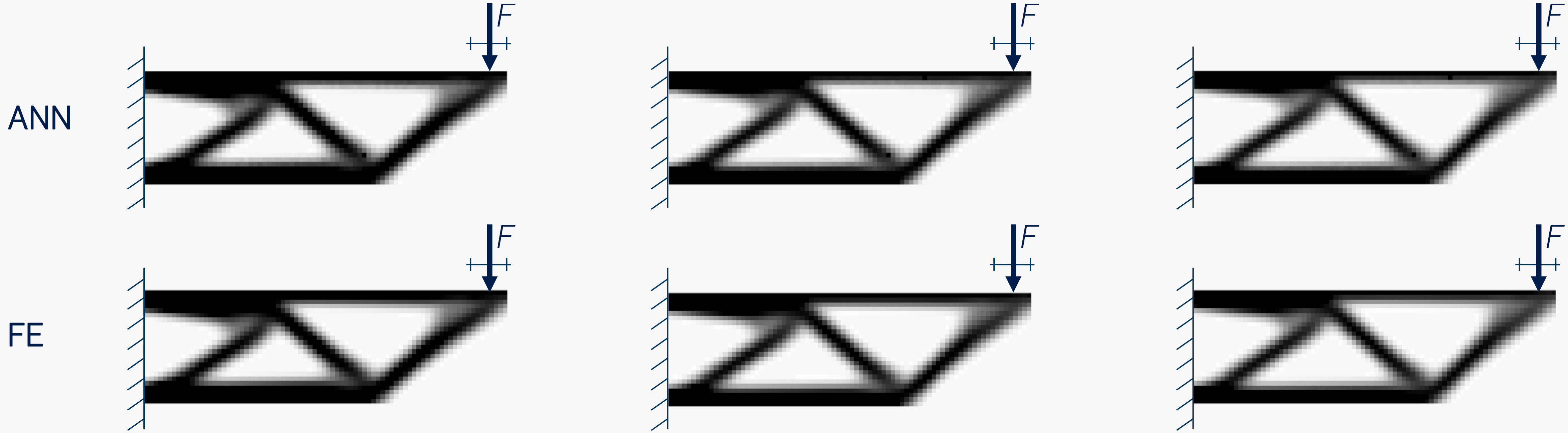
Weight Minimization of a Cantilever Beam

Results with two design variables



Weight Minimization of a Cantilever Beam

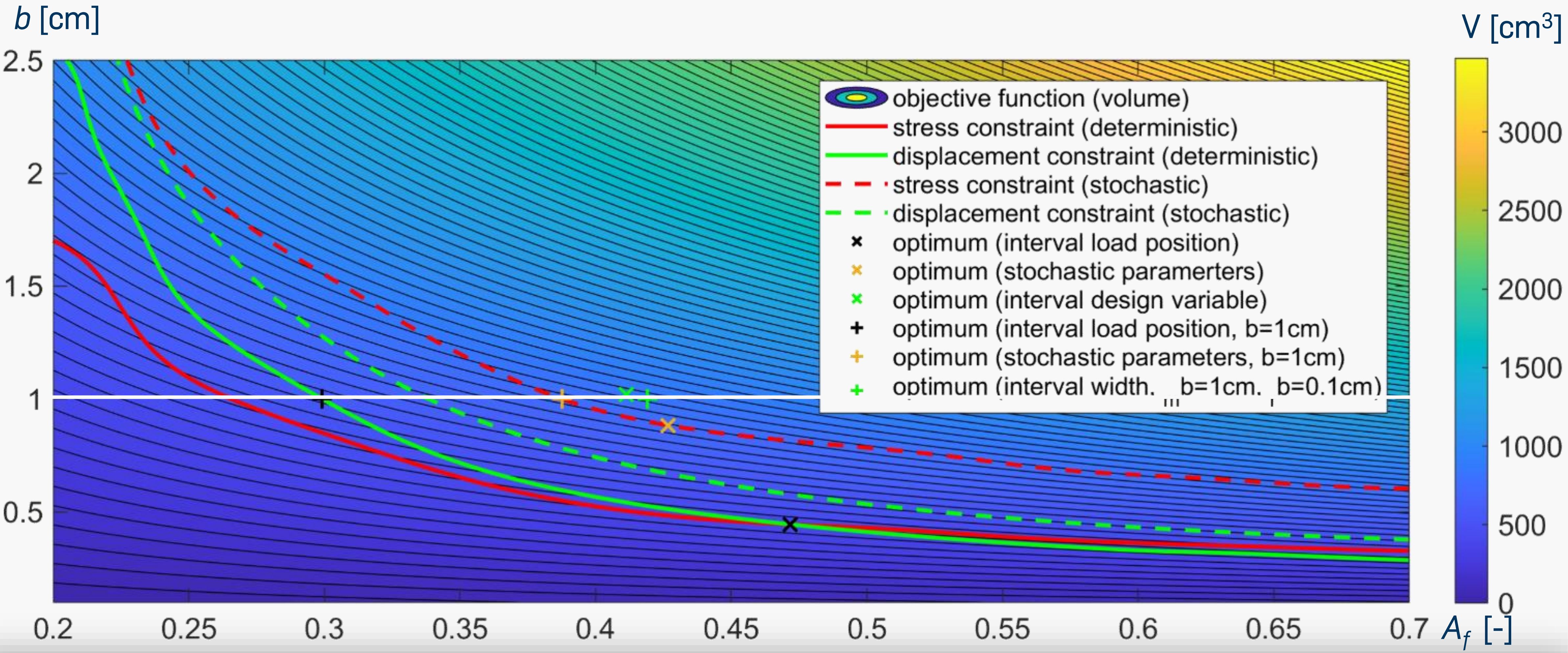
Optima with two design variables



	interval load position	stochastic a priori parameters P, E, f_y	interval design variable $\bar{b}$
$d_1 = A_f$ [-]	0.47	0.43	0.41
$d_2 = b$ [cm]	0.44	0.88	[0.92, 1.12]
$Z = V$ [cm^3]	419	753	[757, 921]

Weight Minimization of a Cantilever Beam

Results with one design variable

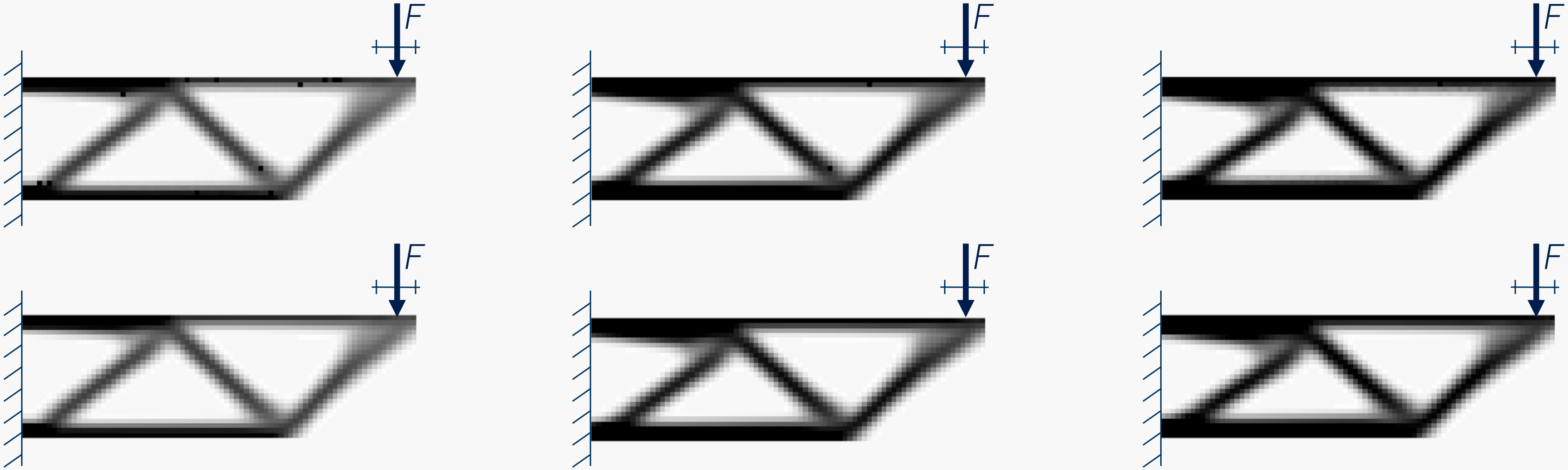


Weight Minimization of a Cantilever Beam

Optima with one design variable

ANN

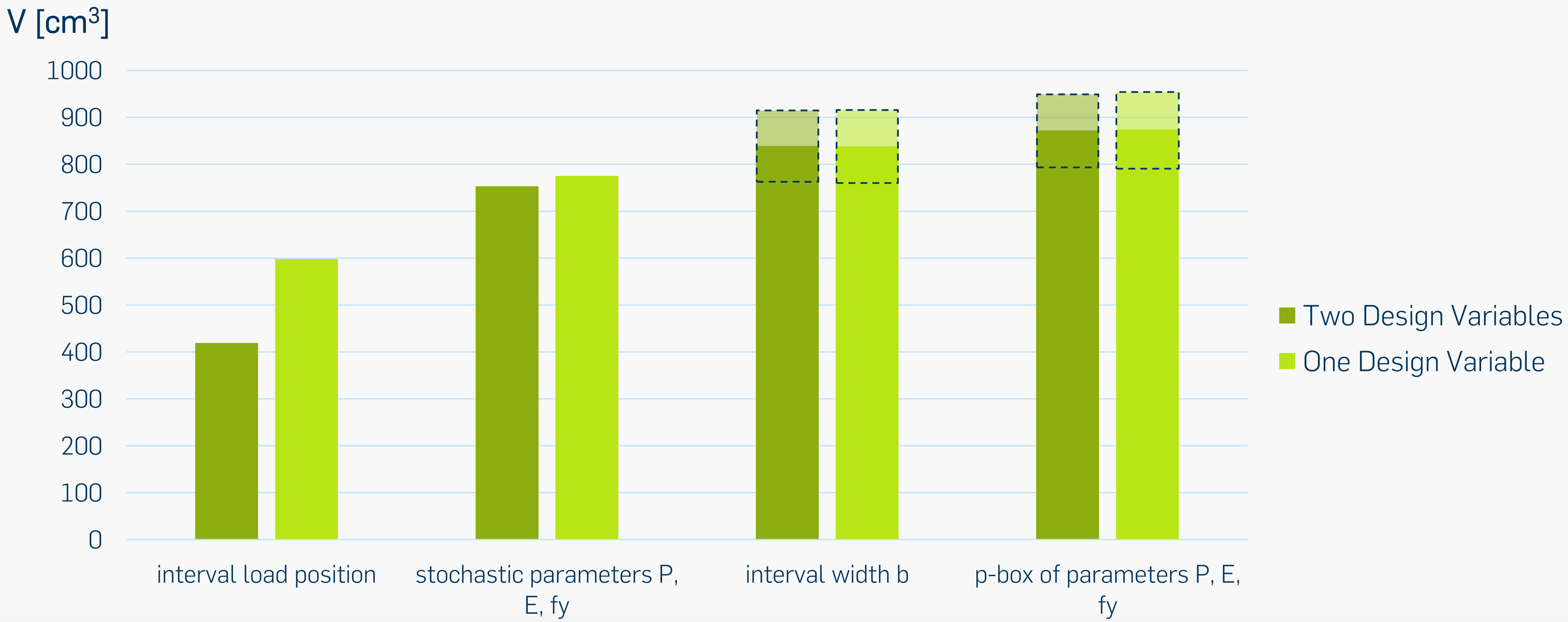
FE



	interval load position	stochastic a priori parameters *P*, *E*, f_y	interval width *b*
$d_1 = A_f$ [-]	0.30	0.39	0.42
$Z = V$ [cm^3]	598	775	[755, 922]

Weight Minimization of a Cantilever Beam

Comparison of the optimized material volumes



Conclusion & Outlook

Conclusion

- Reliability-based Topology Optimization considering polymorphic uncertainties
- Probabilistic stress and displacement constraints
- Artificial neural networks to approximate constraints and to predict material densities
- Weight minimization of a cantilever beam structure

Outlook

- Influence of FE discretization, ANN training errors and minimization residuals
- Extension to 3D structures
- Smoothing and multi-scale techniques to design producible sub-structures
- Using artificial neural networks within the Topology Optimization

Acknowledgements

- Thanks to Prof. Günther Meschke, Simon Peters, Philipp Edler



- Financial support is provided within Subproject 6 of the DFG-Priority Program (SPP 1886) "Polymorphic Uncertainty Modeling for the Numerical Design of Structures"

Gefördert durch DFG Deutsche Forschungsgemeinschaft

SPP 1886