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"Comparison of fully non-stationary artificial accelerograms generation methods in non-linear dynamic analyses"

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INTRODUCTION

- **Non-linear dynamic analysis:** powerful tool to assess the performance of earthquake-resistant structural systems provided that key sources of non-linearity are accurately represented

- **Response:** highly sensitive to the way of modelling the **dynamic action**

- **Problem:** choice of proper sets of acceleration records to be used as input motion in Structural dynamic analyses

INTRODUCTION

- **Literature:** different procedures for **selection of real accelerograms** applying large acceleration scale factors leading to unrealistic inputs

- The selection of a **set** of real accelerograms **compatible with the code requirements** applying large acceleration scale factors leads to unrealistic inputs

Alternative:
Artificial accelerograms

AIMS OF THE PAPER

- **Develop a new method** for generating fully non-stationary artificial accelerograms having the same characteristics of the target motion

- Investigate the **effect of different generation procedures** on the non-linear seismic response of structures

Proposed wavelet based method (*CWT*)

Evolutionary Power Spectral density (*EPSD*) function method (Muscolino et al. 2021)

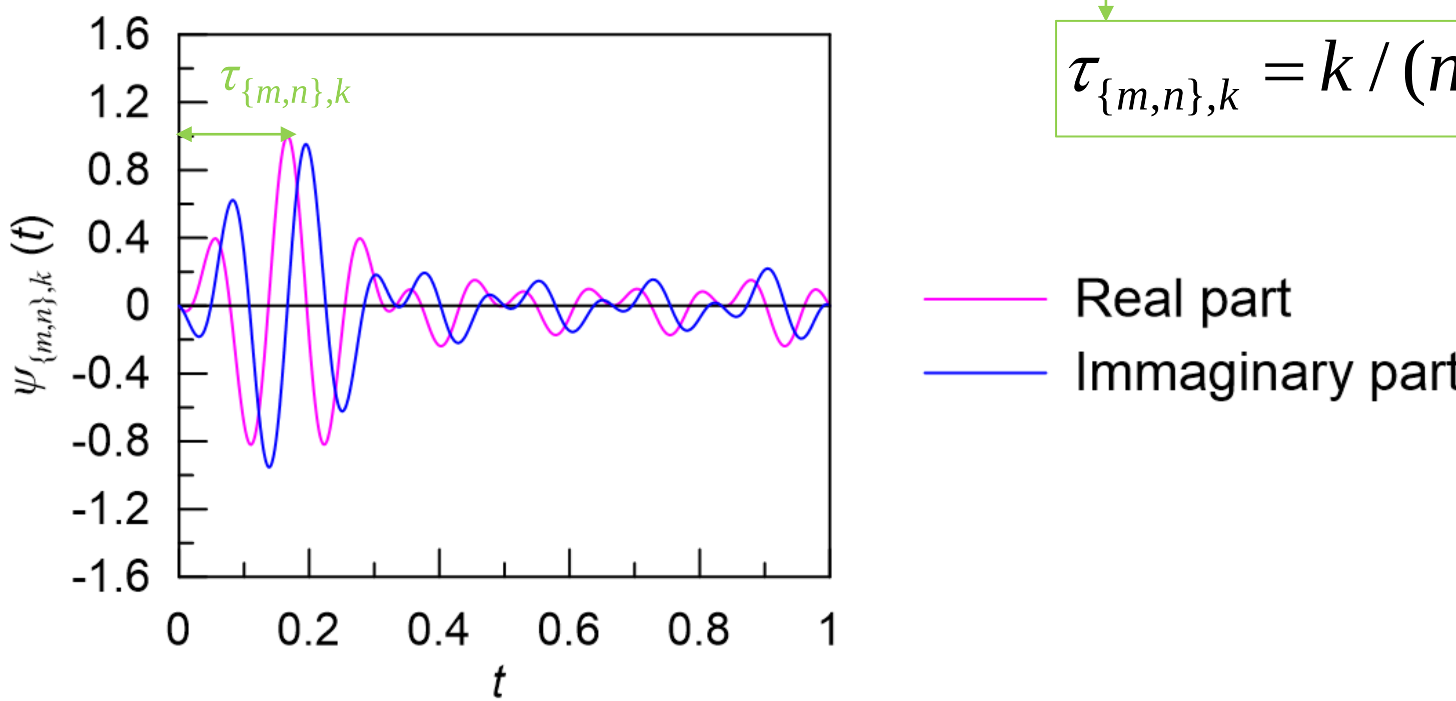
WAVELET BASED METHOD

Mother wavelet: circular harmonic wavelet

$$\omega_m = 2\pi m$$
$$\omega_n = 2\pi n$$

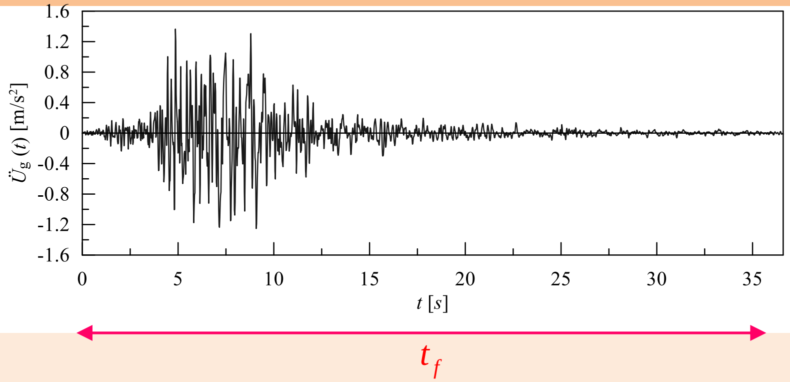
$$\psi_{\{m,n\},k}(t) = \frac{1}{n-m}\sum_{j=m}^{n-1}\exp\left[\mathrm{i}\,\omega_j\left(t-\tau_{\{m,n\},k}\right)\right]$$

$$\tau_{\{m,n\},k} = k/(n-m)$$



STEPS: WAVELET BASED METHOD

1) Select a Target signal $\ddot{U}_g(t)$



2) Evaluate the Harmonic Wavelets Transforms Coefficients

$$a_{\{m,n\},k} = (n-m)\sum_{\ell=0}^{N} \ddot{U}_g\left(t_\ell\right) \cdot \frac{\Delta t}{t_f} \cdot \overline{\psi}_{\{m,n\},k}\left(\frac{t_\ell}{t_f}\right)$$

R.V.

$[0, 2\pi]$

3) Apply the proposed **randomization formula**

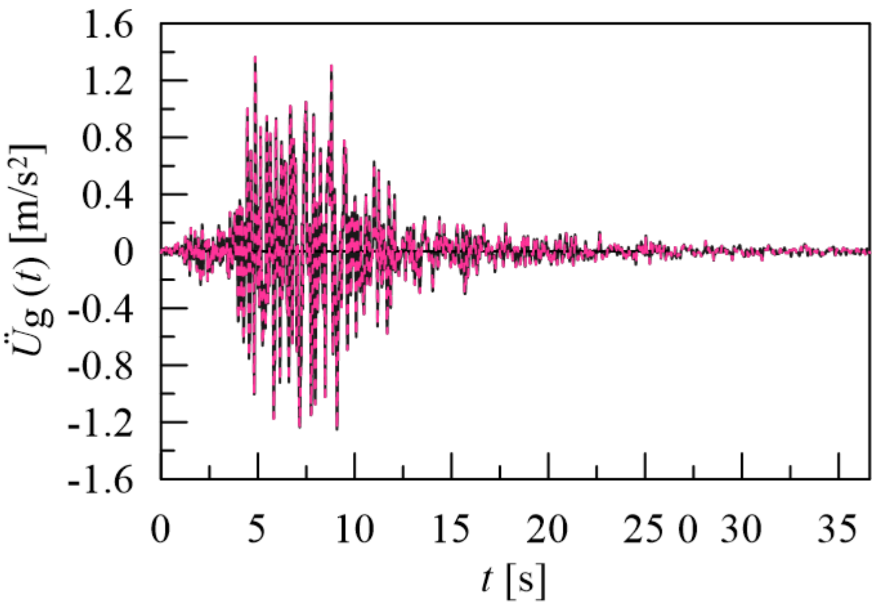
$$f^{(i)}(t) = 2\sum_{\{m,n\}} \sum_{k=0}^{n-m-1} \sum_{j=m}^{n-1} \frac{\left|a_{\{m,n\},k}\right|}{n-m} \times \cos\left[\mathrm{i}\, 2\pi\, j\left(\frac{t}{t_f} - \tau_{\{m,n\},k}\right) + \theta_{\{m,n\},k} + \phi^{(i)}_{\{m,n\},k}\right]$$

$$\theta_{\{m,n\},k} = \arg\left\{a_{\{m,n\},k}\right\}$$

STEPS: WAVELET BASED METHOD

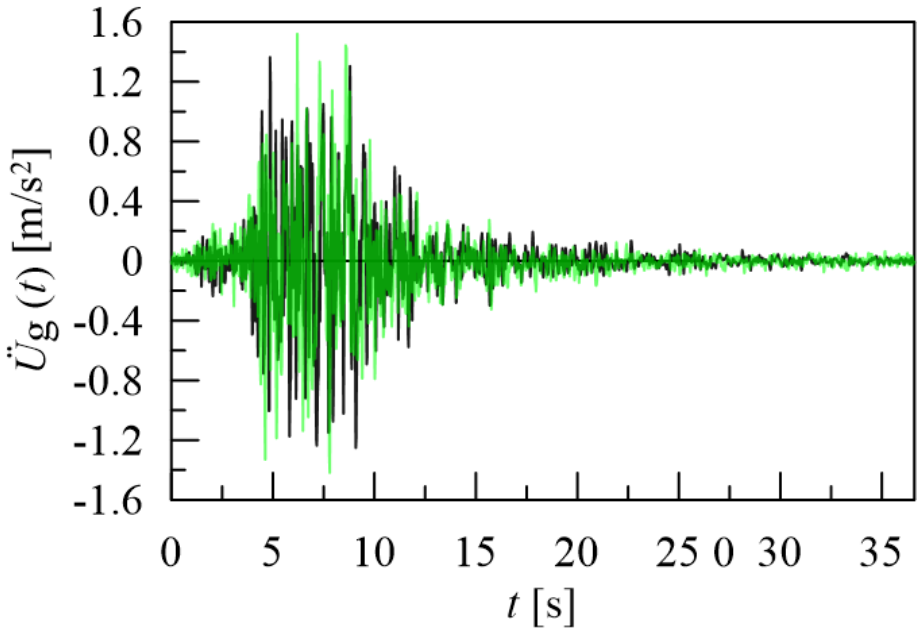
$$f^{(i)}(t) = 2\sum_{\{m,n\}}\sum_{k=0}^{n-m-1}\sum_{j=m}^{n-1}\frac{\left|a_{\{m,n\},k}\right|}{n-m}\times\cos\left[\mathrm{i}\,2\pi\, j\left(\frac{t}{t_f}-\tau_{\{m,n\},k}\right)+\theta_{\{m,n\},k}+\phi^{(i)}_{\{m,n\},k}\right]$$

$$\phi^{(i)}_{\{m,n\},k} = 0$$



Original signal

$$\phi^{(i)}_{\{m,n\},k} \neq 0$$



i-th sample

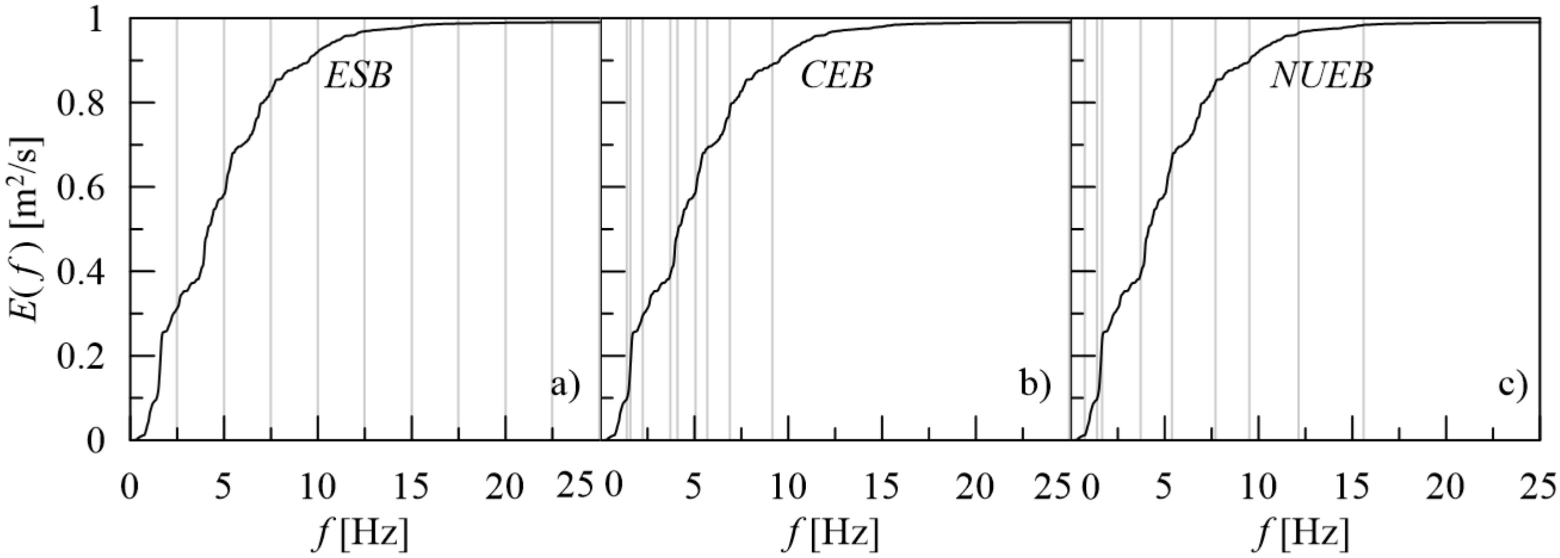
FREQUENCY DOMAIN SUBDIVISION

1) equal-spaced bandwidths (*ESB*)

2) constant energy bandwidths (*CEB*)

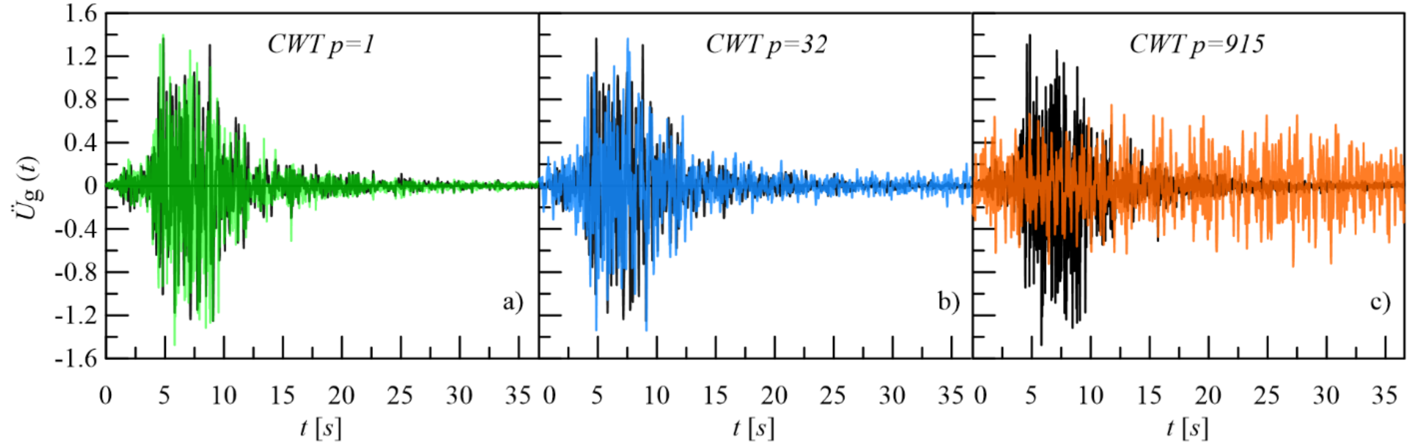
3) non-uniform energy bandwidths (*NUEB*)

$$E(f) = \int_0^{fc} \left(| \mathscr{F} [\ddot{U}_g(t)] | \right)^2 \mathrm{d}f$$

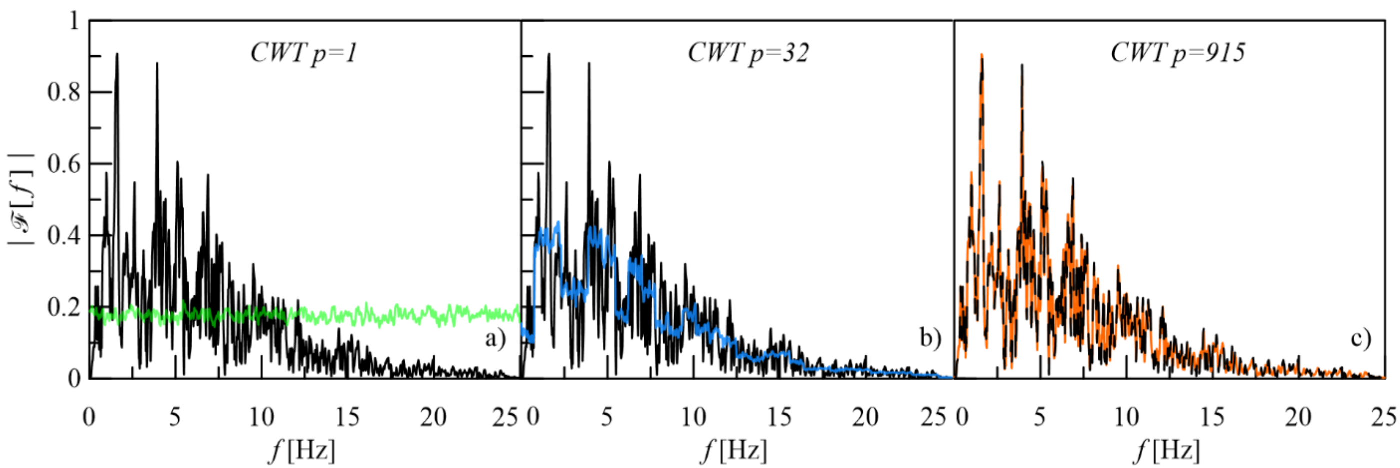


THREE *ESB* SCHEMES

Time histories

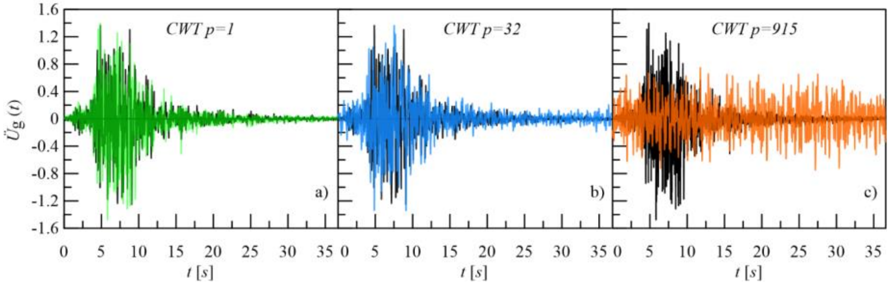


Fourier spectra

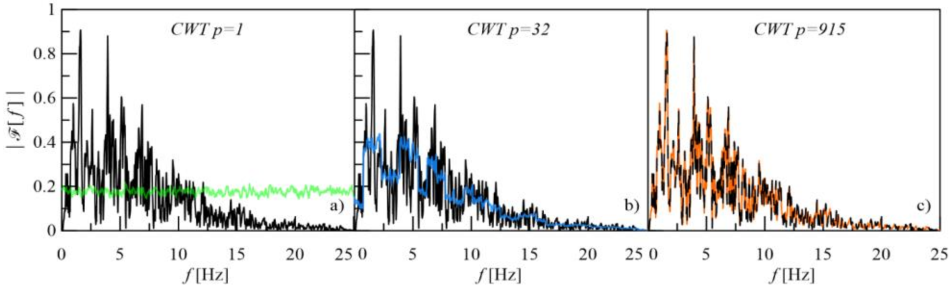


THREE *ESB* SCHEMES

Time Domain

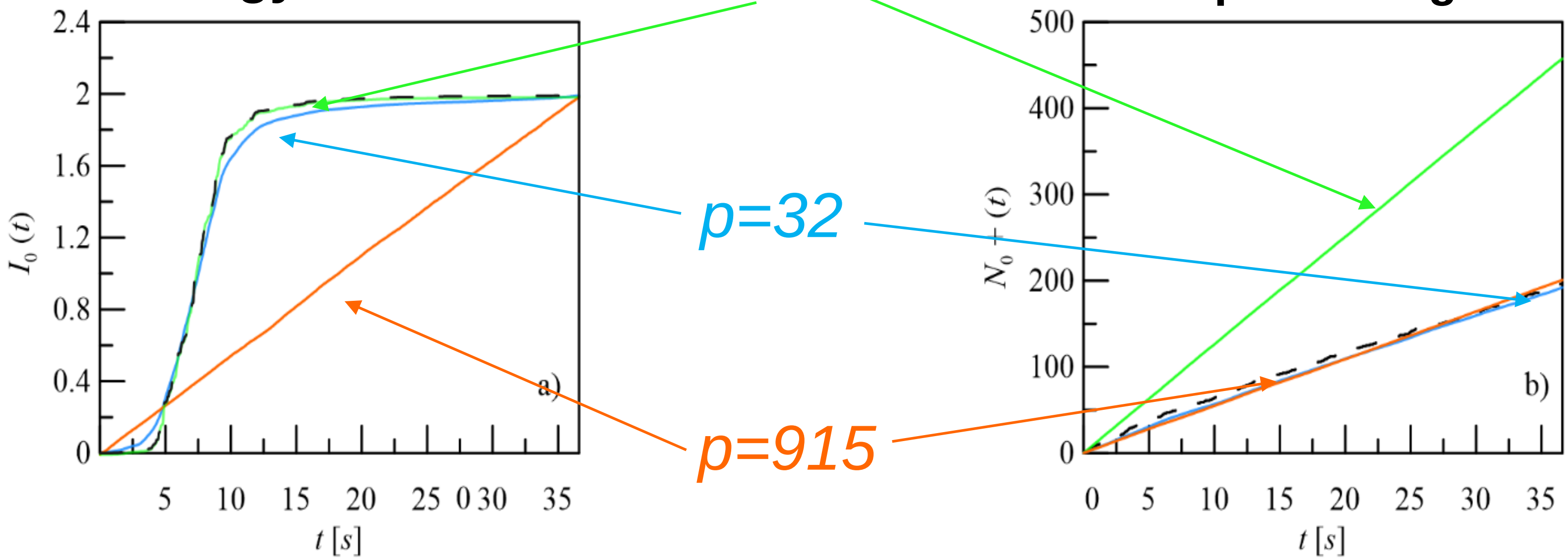


Frequency Domain



Cumulative Energy functions

Cumulative Zero-level up-crossing functions

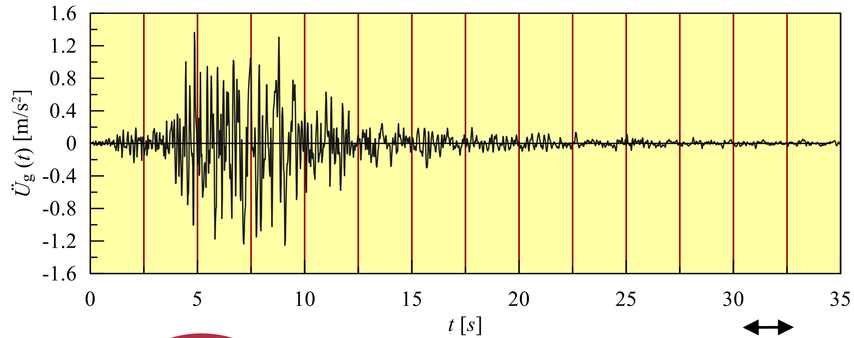


EPSD METHOD

(Muscolino G., Genovese F., Biondi G. and Cascone E., Soil Dynamics 2021)

Four steps procedure:

1) Subdivide a Target signal in many time intervals



ΔT_k

$$F_0(t) = \sum_{k=1}^{n} F_{0,k}(t) = \sum_{k=1}^{n} a(t)\, X_k(t)\, \mathbb{W}(t_{k-1}, t_k)$$

Fully non stationary process

Uniformly modulated random process

Modulating function

Stationary subprocess

$$G_{X_k}(\omega) = \beta_k \left(\frac{\omega^2}{\omega^2 + \omega_{H,k}^2}\right)\left(\frac{\omega_{L,k}^4}{\omega^4 + \omega_{L,k}^4}\right)\frac{\rho_k}{\pi}\left(\frac{1}{\rho_k^2 + (\omega + \Omega_k)^2} + \frac{1}{\rho_k^2 + (\omega - \Omega_k)^2}\right); \quad k = 1, \ldots, n$$

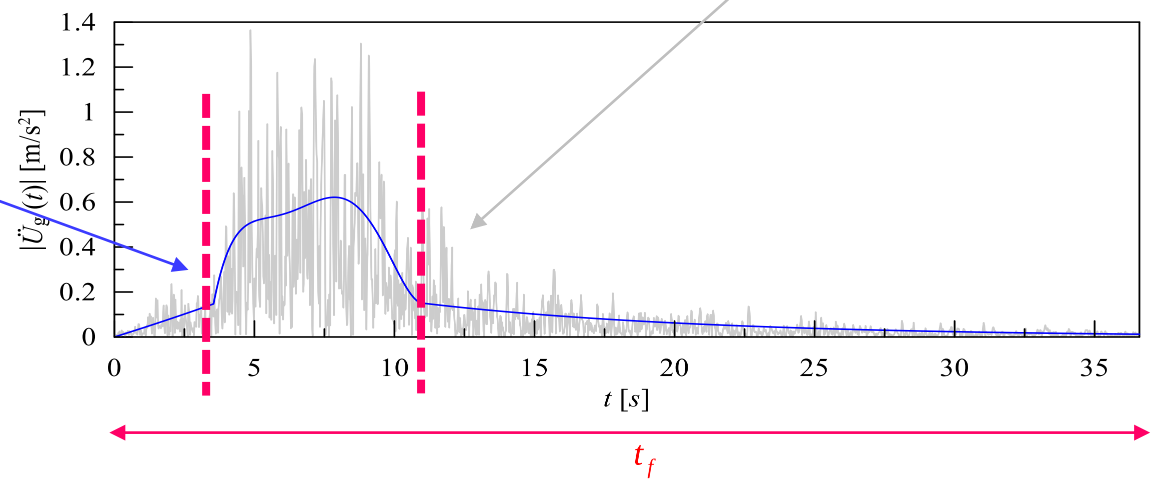
EPSD METHOD

(Muscolino G., Genovese F., Biondi G. and Cascone E., Soil Dynamics 2021)

2) Estimate modulating function

$$a(t)=\sum_{j=1}^{2}\bar{a}_j(t)\,\mathbb{W}(t_{j-1},t_j)+a(t_2)\exp\left[\frac{t-t_2}{t_f-t_2}\ln\left(\frac{|\ddot{U}_g(t_f)|}{a(t_2)}\right)\right]\mathbb{W}(t_2,t_f)$$

$a(t)$



Optimization problem

$$\begin{cases} \text{find } \alpha_1,\ \alpha_2, \cdots \alpha_p; \\ \text{minimising} \quad \int_{t_{j-1}}^{t_j-\Delta t}\left[E_{\ddot{U}_g}(t)-\psi_j(t)\right]^2 \mathrm{d}t; \qquad j=1,2 \\ \text{such that} \quad \bar{a}_j(t)\geq 0, \quad \bar{a}_j(t)=\sum_{i=1}^{p}\alpha_i\,(t-t_{j-1})^i, \quad t_{j-1}\leq t<t_j; \end{cases}$$

$$E_{\ddot{U}_g}(t)=\int_0^t \ddot{U}_g^2(\tau)\,\mathrm{d}\tau=\sum_{j=1}^{m\leq n_a}\int_{t_{j-1}}^{t_j}\ddot{U}_g^2(\tau)\,\mathrm{d}\tau$$

$$\mathrm{E}\left\langle E_{F_0}(t)\right\rangle=\int_0^t \mathrm{E}\left\langle F_0^2(\tau)\right\rangle \mathrm{d}\tau=\sum_{j=1}^{m\leq n_a}\int_{t_{j-1}}^{t_j}a^2(\tau)\,\mathrm{d}\tau=\sum_{j=1}^{m\leq n_a}\psi_j(t)=\sum_{j=1}^{m\leq n_a}\int_{t_{j-1}}^{t}\left[a(t_{j-1})+\bar{a}_j(\tau)\right]^2\mathrm{d}\tau$$

EPSD METHOD

(Muscolino G., Genovese F., Biondi G. and Cascone E., Soil Dynamics 2021)

3) Fully non-stationary samples

$$\boxed{F_0^{(i)}(t) = a(t)\sqrt{2\Delta\omega}\left[\sum_{k=1}^{n}\sum_{r=1}^{m_N}\mathbb{W}(t_{k-1},t_k)\sin\left(r\,\Delta\omega t + \theta_r^{(i)}\right)\sqrt{G_{X_k}\left(r\,\Delta\omega\right)}\right]}$$

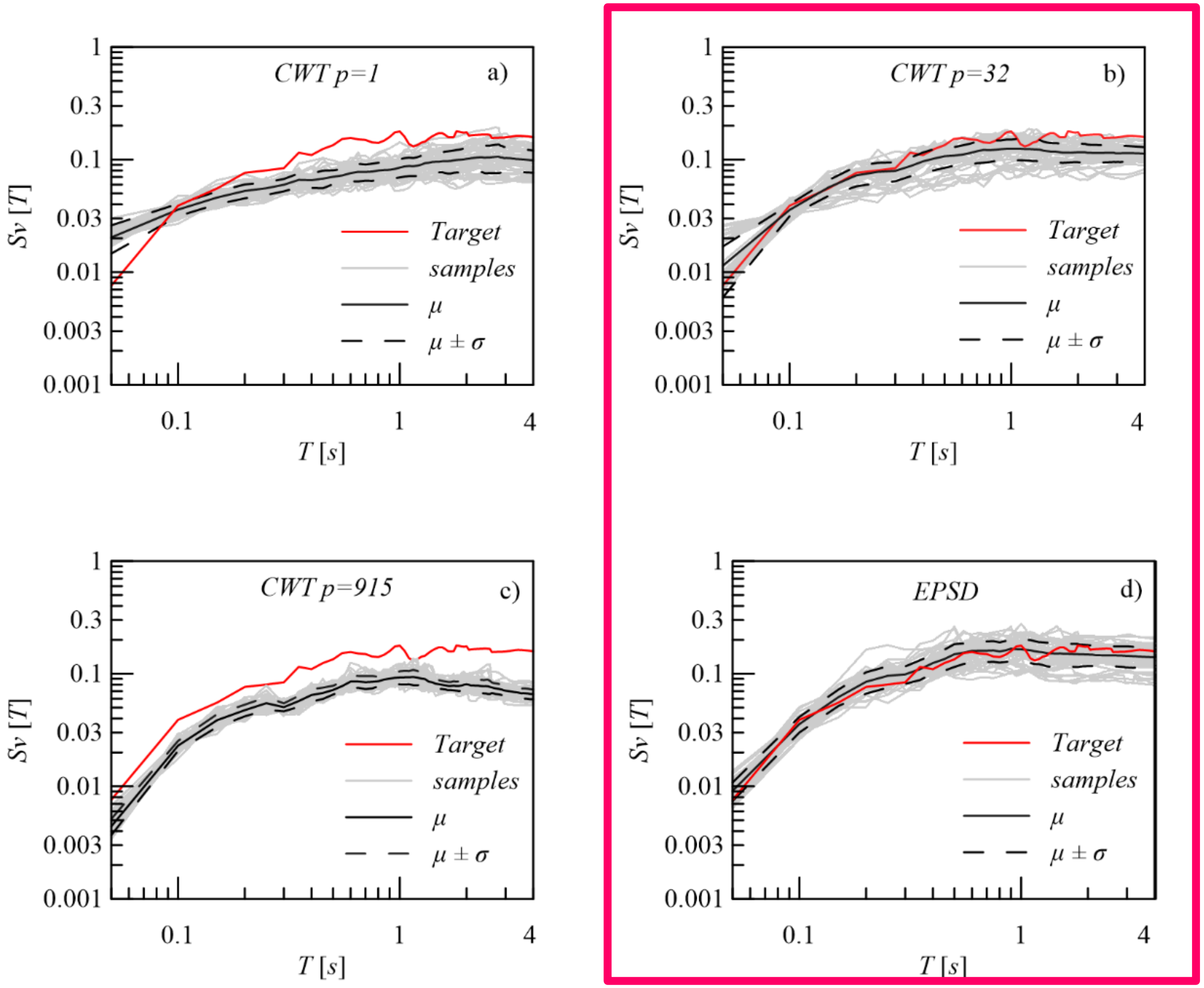
4) Fully non-stationary samples spectrum compatible artificial accelerograms (*j*-th iteration)

$$\bar{F}_0^{(i,j)}(t) = a(t)\sqrt{2\Delta\omega}\left[\sum_{k=1}^{n}\sum_{r=1}^{m_N}\mathbb{W}(t_{k-1},t_k)\sin\left(r\,\Delta\omega t + \theta_r^{(i)}\right)\sqrt{\bar{G}_{X_k}^{(j)}(r\,\Delta\omega)G_{X_k}\left(r\,\Delta\omega\right)}\right]$$

$$\bar{G}_{X_k}^{(j)}(r\,\Delta\omega) = \bar{G}_{X_k}^{(j-1)}(r\,\Delta\omega)\,\frac{\boxed{S_e^{(T)}(r\,\Delta\omega,\zeta_0)}^2}{\boxed{\bar{S}_e^{(j-1)}(r\,\Delta\omega,\zeta_0)}^2}$$

COMPARISON

CONSTANT-DUCTILITY RESPONSE SPECTRA



CONCLUSIONS

- Two fully non-stationary artificial accelerograms generation methods have been compared

- The *CWT*-method, allows generating the samples without the need of defying the *EPSD* function

- The *EPSD* function method allows to obtain samples with characteristics closer to the target event than those generated with *CWT* method (*ESB*) but tends to be more complex and requires several iterative steps