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Reliability Analysis of Randomly Excited Structures with Interval Stiffness Parameters via Sensitivity Analysis

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Introduction

- Reliability estimates provided by classical probabilistic methods are very sensitive to the assumed probabilistic distribution of the uncertain parameters.

- Since Ben-Haim (1994) introduced a non-probabilistic concept of reliability, structural safety assessment based on non-traditional uncertainty models has attracted growing attention.

- To the best of the authors' knowledge, very few studies have been devoted to reliability analysis of structures subjected to random excitations and having uncertain parameters modeled using non-probabilistic approaches (see e.g., Do et al. 2014, Muscolino et al. 2015, 2016, Sofi at al. 2020).

Focus: reliability analysis of structures with interval stiffness parameters subjected to stationary Gaussian random excitation.

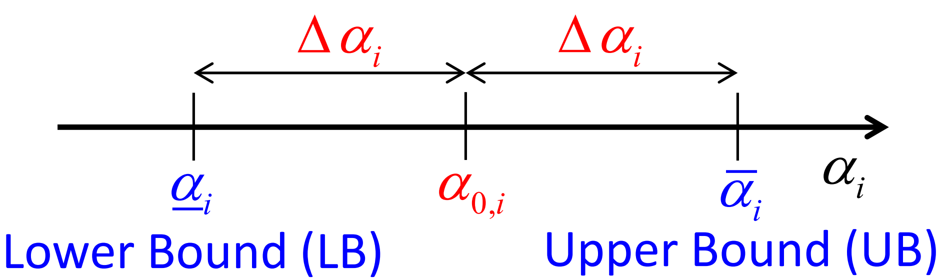
Aims, challenges and tools

- **Aims**:
  - To develop an efficient procedure for computing tight bounds of the interval reliability function of the selected stress process;
  - To identify the most influential uncertain stiffness parameters on structural reliability
- **Challenges**:
  - To reduce the high computational cost required by the propagation of interval uncertainties in the presence of random excitation;
  - To reduce the overestimation due to the *dependency phenomenon* (Moore et al. 2009)
- **Tools**:
  - Sensitivity Analysis
  - *Improved Interval Analysis* (Muscolino and Sofi 2012)

Interval model of uncertainty

- The interval model represents the uncertain parameters as interval variables (Moore 1966)

$$\alpha_i^I = [\underline{\alpha}_i,\ \overline{\alpha}_i] = \{\alpha_i,\ \underline{\alpha}_i \le \alpha_i \le \overline{\alpha}_i,\ \alpha_i \in \mathbb{R}\}$$



$$\alpha_{0,i} = \mathrm{mid}\{\alpha_i^I\} = (\underline{\alpha}_i + \overline{\alpha}_i)/2 \quad \text{Midpoint}$$

$$\Delta\alpha_i = (\overline{\alpha}_i - \underline{\alpha}_i)/2 \quad \text{Deviation amplitude}$$

- In the context of engineering applications, the main drawback of the interval model is the overestimation of the interval solution due to the so-called *dependency phenomenon* (Moore et al. 2009).

• *Example*: $f(x^I) = x^I - x^I;\quad x^I = [1,2] \Rightarrow f(x^I) = [1-2, 2-1] = [-1,1] \neq 0$

$f(x^I) = x^I(1-1) \Rightarrow f(x^I) = 0$

Improved Interval Analysis (IIA), (Muscolino and Sofi 2012)

Improved Interval Analysis

- The Improved Interval Analysis (IIA) (Muscolino, Sofi 2012) is based on the definition of the so-called extra unitary interval (EUI) $\hat{e}_i^I = [-1,+1]$

$$\boxed{\begin{array}{lll} \hat{e}_i^I - \hat{e}_i^I = [0,0]; & \hat{e}_i^I \times \hat{e}_i^I = \left(\hat{e}_i^I\right)^2 = [1,1]; & \hat{e}_i^I \times \hat{e}_j^I = \hat{e}_{ij}^I = [-1,+1], \quad i \neq j; \\ \hat{e}_i^I / \hat{e}_i^I = [1,1]; & x_i \hat{e}_i^I \pm y_i \hat{e}_i^I = \left(x_i \pm y_i\right) \hat{e}_i^I; & x_i \hat{e}_i^I \times y_i \hat{e}_i^I = x_i y_i \left(\hat{e}_i^I\right)^2 = x_i y_i [1,1] \end{array}}$$

- According to the IIA, the generic interval variable $\alpha_i^I = [\underline{\alpha}, \overline{\alpha}] \in \mathbb{IR}$ is expressed in the following *affine form*:

$\hat{e}_i^I = [-1,+1]$ **EUI**

$$\boxed{\alpha_i^I = \underbrace{\alpha_{0,i}}_{\text{Midpoint}} + \underbrace{\Delta\alpha_i}_{\text{Deviation amplitude}} \hat{e}_i^I}$$

- Symmetric interval variable $(\underline{\alpha} = -\overline{\alpha})$

$$\alpha_i^I = \cancel{\alpha_{0,i}} + \Delta\alpha_i \, \hat{e}_i^I$$

Uncertain Young's Moduli

- Consider a *n*-DOFs structure discretized into *N* finite elements.
- Let us assume that r FEs are characterized by uncertain Young's modulus.
- The Young's modulus of the *i*-th FE can be expressed as:

$$E^{(i)}(\alpha_i^I) = E_0^{(i)}\left(1+\alpha_i^I\right) = E_0^{(i)}\left(1+\Delta\alpha_i \hat{e}_i^I\right), \quad i = 1, 2, \ldots, r$$

$$\Downarrow$$

Interval elastic matrix

$$\boxed{\mathbf{E}^{(i)}(\alpha_i^I) = \left(1+\Delta\alpha_i \hat{e}_i^I\right)\mathbf{E}_0^{(i)}}$$

$$\Downarrow$$

Interval stiffness matrix

$$\boxed{\mathbf{K}(\boldsymbol{\alpha}^I) = \mathbf{K}_0 + \sum_{i=1}^{r} \mathbf{K}_i \Delta\alpha_i \hat{e}_i^I}$$

$$\boldsymbol{\alpha}^I = [\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}] = [\alpha_1^I, \quad \alpha_2^I, \quad \ldots, \quad \alpha_r^I]^{\mathrm{T}}$$

Nominal value

i-th deviation stiffness matrix

$$\mathbf{K}_i = \left.\frac{\partial \mathbf{K}(\boldsymbol{\alpha}^I)}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}} = \mathbf{L}^{(i)\mathrm{T}} \mathbf{k}_0^{(i)} \mathbf{L}^{(i)}$$

Equations of motion

- Equations of motion of a quiescent *n*-DOFs classically damped linear structure with *r* interval parameters subjected to a stationary multi-correlated Gaussian stochastic process $\mathbf{F}(t)$:

$$\boxed{\mathbf{M}\ddot{\mathbf{U}}(t,\boldsymbol{\alpha}^I)+\mathbf{C}(\boldsymbol{\alpha}^I)\dot{\mathbf{U}}(t,\boldsymbol{\alpha}^I)+\mathbf{K}(\boldsymbol{\alpha}^I)\mathbf{U}^I(t,\boldsymbol{\alpha}^I)=\mathbf{F}(t)}$$

- $\boldsymbol{\alpha}^I=[\underline{\boldsymbol{\alpha}},\overline{\boldsymbol{\alpha}}]$ $r-$order vector collecting the interval parameters $\alpha_i^I=\Delta\alpha_i\,\hat{e}_i^I$
- $\mathbf{U}^I(t)\equiv\mathbf{U}(\boldsymbol{\alpha}^I,t)$ $n-$order stationary Gaussian vector process of displacements
- $\mathbf{F}(t)=\boldsymbol{\mu}_\mathbf{F}+\tilde{\mathbf{X}}_\mathbf{F}(t)$ stationary multi-correlated Gaussian random process fully characterized by the mean-value vector $\boldsymbol{\mu}_\mathbf{F}=\mathrm{E}\langle\mathbf{F}(t)\rangle$ and the one-sided PSD matrix $\mathbf{G}_{\tilde{\mathbf{X}}_\mathbf{F}\tilde{\mathbf{X}}_\mathbf{F}}(\omega)$

$$\Downarrow$$

Interval stationary Gaussian stress vector process within the *h*-th FE

$$\boxed{\boldsymbol{\sigma}^{(h)}(\boldsymbol{\alpha}^I,\mathbf{x},t)=\left(1+\Delta\alpha_h\hat{e}_h^I\right)\mathbf{E}_0^{(h)}\,\mathbf{B}^{(h)}(\mathbf{x})\,\mathbf{L}^{(h)}\,\mathbf{U}(\boldsymbol{\alpha}^I,t)}$$

Interval stationary Gaussian stress process

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- The j-th component of $\boldsymbol{\sigma}^{(h)}(\boldsymbol{\alpha}^I, \mathbf{x}, t)$ ($\mathbf{x}$ fixed) can be expressed as:

$$\boldsymbol{\alpha}^I = \begin{Bmatrix} \Delta\alpha_1 \hat{e}_1^I \\ \vdots \\ \Delta\alpha_j \hat{e}_j^I \\ \vdots \\ \Delta\alpha_r \hat{e}_r^I \end{Bmatrix}$$

$$\boxed{Y_j^{(h)}(\boldsymbol{\alpha}^I, t) \equiv \sigma_j^{(h)}(\boldsymbol{\alpha}^I, \mathbf{x}, t) = \left(1 + \Delta\alpha_h \hat{e}_h^I\right) \mathbf{r}_j^{(h)\mathrm{T}} \mathbf{U}(\boldsymbol{\alpha}^I, t)}$$

j-th row of the matrix $\mathbf{E}_0^{(h)} \mathbf{B}^{(h)} \mathbf{L}^{(h)}$

Interval mean-value

$$\boxed{\mu_{Y_j^{(h)}}(\boldsymbol{\alpha}^I) = \left(1 + \Delta\alpha_h \hat{e}_h^I\right) \mathbf{r}_j^{(h)\mathrm{T}} \mathbf{K}^{-1}(\boldsymbol{\alpha}^I) \boldsymbol{\mu}_{\mathbf{F}}}$$

Interval one-sided PSD function

$$\boxed{G_{Y_j^{(h)} Y_j^{(h)}}(\boldsymbol{\alpha}^I, \omega) = \left(1 + \Delta\alpha_h \hat{e}_h^I\right)^2 \mathbf{r}_j^{(h)\mathrm{T}} \mathbf{H}^*(\boldsymbol{\alpha}^I, \omega) \mathbf{G}_{\tilde{\mathbf{X}}_\mathbf{F} \tilde{\mathbf{X}}_\mathbf{F}}(\omega) \mathbf{H}^{\mathrm{T}}(\boldsymbol{\alpha}^I, \omega) \mathbf{r}_j^{(h)}}$$

Interval Frequency Response Function (FRF) matrix

$$\boxed{\mathbf{H}(\boldsymbol{\alpha}^I, \omega) = \left[-\omega^2 \mathbf{M} + \mathrm{j}\omega \mathbf{C}(\boldsymbol{\alpha}^I) + \mathbf{K}(\boldsymbol{\alpha}^I)\right]^{-1}, \ \mathrm{j} = \sqrt{-1}}$$

- Multiple occurrences of the same interval variable: overestimation due to the *dependency phenomenon* (Moore et al. 2009).

Extreme value interval random process

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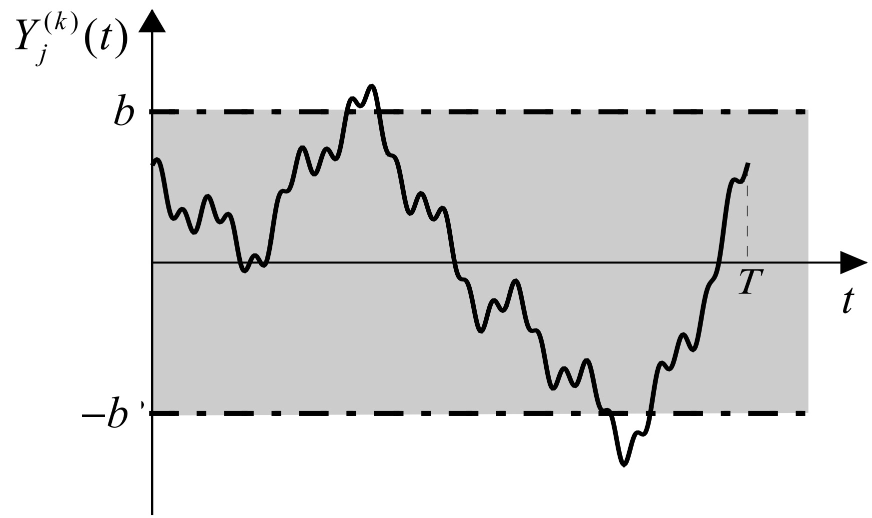
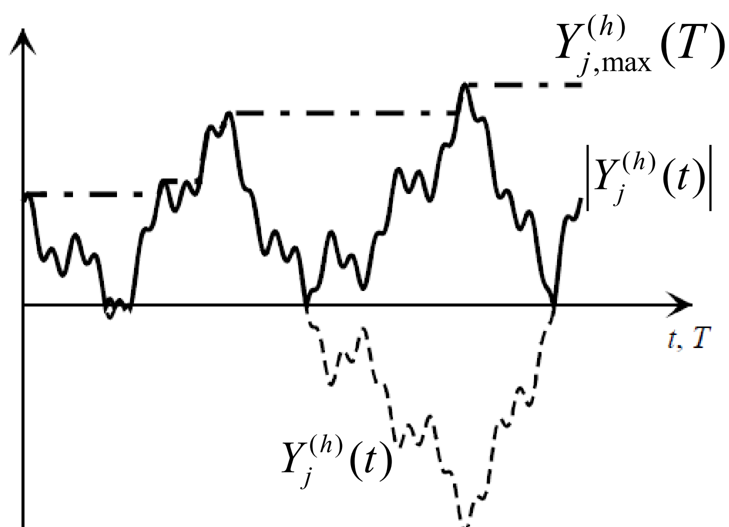
- The *probability of failure* is herein identified with the *first-passage probability*, i.e. failure occurs when the stress or displacement at a critical location firstly exceeds a prescribed safe domain within the time interval [0,*T*].



- Interval *extreme value* random process for the *j*-th stress component over the time interval [0,*T*]

$$\boxed{Y_{j,\max}^{(h)}(\boldsymbol{\alpha}^I, T) = \max_{0 \le t \le T} \left| Y_j^{(h)}(\boldsymbol{\alpha}^I, t) \right|}$$

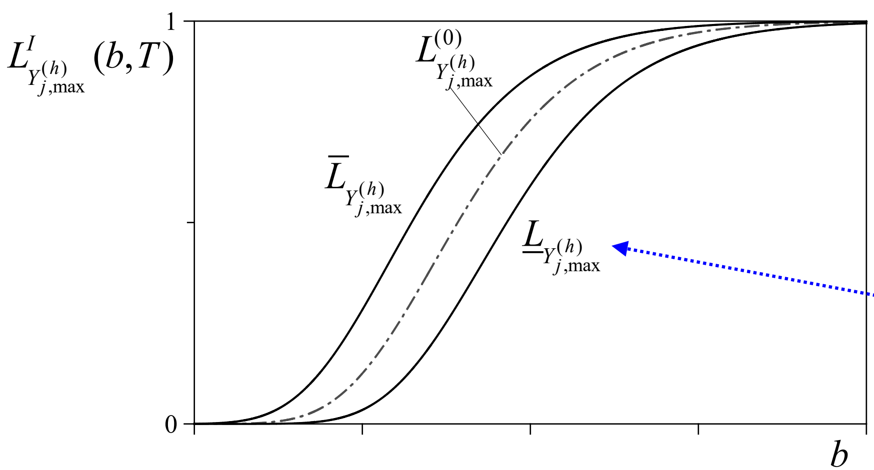


Interval reliability function 1/2

- Interval *cumulative distribution function* (*CDF*) or *interval reliability function*

$$\boxed{L^{I}_{Y^{(h)}_{j,\max}}(b,T) = \Pr\left[Y^{(h)}_{j,\max}(\boldsymbol{\alpha}^{I},T) \le b\right] = \left[\underline{L}_{Y^{(h)}_{j,\max}}(b,T),\quad \overline{L}_{Y^{(h)}_{j,\max}}(b,T)\right]}$$

represents the probability that $Y^{(h)}_{j,\max}(\boldsymbol{\alpha}^{I},T)$ is equal to or less than the critical level (*barrier*) *b>0* within the time interval [0,*T*].



AIM

evaluation of the LB and UB of the interval reliability function

Worst-case scenario ≡ LB

Interval reliability function 2/2

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- If the failure level is high enough, then the interval extension of the classical Rice's formula (1945), based on the Poisson assumption of independent up-crossings of a prescribed threshold, can be applied:

$$\boxed{L^I_{Y^{(h)}_{j,\max}}(\boldsymbol{\alpha}^I, b, T) \approx \exp\left\{-T\,\nu^+_{Y_j^{(h)}}(\boldsymbol{\alpha}^I)\exp\left[-\frac{\left(b-\left|\mu_{Y_j^{(h)}}(\boldsymbol{\alpha}^I)\right|\right)^2}{2\lambda_{0,Y_j^{(h)}}(\boldsymbol{\alpha}^I)}\right]\right\}}$$

$$\Pr\left[Y_{Y^{(h)}_{j,\max}}(\boldsymbol{\alpha}^I, 0) \le b\right] = 1$$ **Unitary initial probability**

$$\boxed{\nu^+_{Y_j^{(h)}}(\boldsymbol{\alpha}^I) = \frac{1}{2\,\pi}\sqrt{\frac{\lambda_{2,Y_j^{(h)}}(\boldsymbol{\alpha}^I)}{\lambda_{0,Y_j^{(h)}}(\boldsymbol{\alpha}^I)}}}$$ **Mean up-crossing rate at level $\left|\mu_{Y_j^{(h)}}(\boldsymbol{\alpha}^I)\right|$**

$$\lambda_{\ell,Y_j^{(h)}}(\boldsymbol{\alpha}^I) = \int_0^\infty \omega^\ell\, G_{Y_j^{(h)}Y_j^{(h)}}(\boldsymbol{\alpha}^I, \omega)\,\mathrm{d}\omega, \quad (\ell = 0, 2)$$ **Interval spectral moments**

Explicit reliability sensitivity

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- The ***reliability sensitivity*** can be derived *analytically* by direct differentiation with respect to $\alpha_i \in \alpha_i^I$

$$S_{L_{Y_{j,\max}^{(h)}},i}(b,T) = \left.\frac{\partial L_{Y_{j,\max}^{(h)}}(\boldsymbol{\alpha},b,T)}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}}$$

$$= C_{Y_j^{(h)}}(b,T)\left\{2\left(b-\left|\mu_{Y_j^{(h)}}^{(0)}\right|\right)\frac{\left|\mu_{Y_j^{(h)}}^{(0)}\right|}{\mu_{Y_j^{(h)}}^{(0)}} S_{\mu_{Y_j^{(h)}},i} + \left[\frac{\left(b-\left|\mu_{Y_j^{(h)}}^{(0)}\right|\right)^2}{\lambda_{0,Y_j^{(h)}}^{(0)}}-1\right] S_{\lambda_{0,Y_j^{(h)}},i} + \frac{\lambda_{0,Y_j^{(h)}}^{(0)}}{\lambda_{2,Y_j^{(h)}}^{(0)}} S_{\lambda_{2,Y_j^{(h)}},i}\right\}$$

$$S_{\mu_{Y_j^{(h)}},i} = \left.\frac{\partial \mu_{Y_j^{(h)}}(\boldsymbol{\alpha})}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}}$$

Sensitivity of the mean-value

$$S_{\lambda_{\ell,Y_j^{(h)}},i} = \left.\frac{\partial \lambda_{\ell,Y_j^{(h)}}(\boldsymbol{\alpha})}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}} = \int_0^\infty \omega^\ell S_{G_{Y_j^{(h)}Y_j^{(h)}},i}(\omega)\,\mathrm{d}\omega$$

Sensitivity of the spectral moments

$$C_{Y_j^{(h)}}(b,T) = -\frac{T}{4\,\pi}\frac{\lambda_{2,Y_j^{(h)}}^{(0)}}{\lambda_{0,Y_j^{(h)}}^{(0)}}\frac{L_{Y_{j,\max}^{(h)}}^{(0)}(b,T)}{\sqrt{\lambda_{0,Y_j^{(h)}}^{(0)}\lambda_{2,Y_j^{(h)}}^{(0)}}}\exp\left[-\frac{\left(b-\left|\mu_{Y_j^{(h)}}^{(0)}\right|\right)^2}{2\lambda_{0,Y_j^{(h)}}^{(0)}}\right]$$

Nominal values in red

Explicit sensitivity of the mean-value

$$\boxed{\mu_{Y_j^{(h)}}(\boldsymbol{\alpha}^I) = \left(1+\Delta\alpha_h \hat{e}_h^I\right)\mathbf{r}_j^{(h)\mathrm{T}}\mathbf{K}^{-1}(\boldsymbol{\alpha}^I)\boldsymbol{\mu}_{\mathbf{F}}}$$

$$\Downarrow$$

$$S_{\mu_{Y_j^{(h)}},i} = \begin{cases} \left.\dfrac{\partial \mu_{Y_j^{(h)}}(\boldsymbol{\alpha})}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}} = \mu_{Y_j^{(h)}}^{(0)} - \mathbf{r}_j^{(h)\mathrm{T}}\,\mathbf{K}_0^{-1}\mathbf{K}_h\,\mathbf{K}_0^{-1}\,\boldsymbol{\mu}_{\mathbf{F}}, & \text{if } i = h \\ \left.\dfrac{\partial \mu_{Y_j^{(h)}}(\boldsymbol{\alpha})}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}} = -\mathbf{r}_j^{(h)\mathrm{T}}\,\mathbf{K}_0^{-1}\mathbf{K}_i\,\mathbf{K}_0^{-1}\,\boldsymbol{\mu}_{\mathbf{F}}, & \text{if } i \neq h \end{cases}$$

Exact sensitivity of the mean-value of the stress process

Explicit sensitivity of the spectral moments

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$$\boxed{S_{\lambda_{\ell,Y_j^{(h)}},i} = \left.\frac{\partial \lambda_{\ell,Y_j^{(h)}}(\boldsymbol{\alpha})}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}} = \int_0^\infty \omega^\ell S_{G_{Y_j^{(h)}Y_j^{(h)}},i}(\omega)\,\mathrm{d}\omega.}$$

$$S_{G_{Y_j^{(h)}Y_j^{(h)}},i}(\omega) = \left.\frac{\partial G_{Y_j^{(h)}Y_j^{(h)}}(\boldsymbol{\alpha},\omega)}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}}$$

$$G_{Y_j^{(h)}Y_j^{(h)}}(\boldsymbol{\alpha},\omega) = \left(\rho_{0,j} / A_{0,j}\right)^2 \left(1+\alpha_j\right)^2 \mathbf{s}_j^{\mathrm{T}} \mathbf{H}^*(\boldsymbol{\alpha},\omega) \mathbf{G}_{\tilde{\mathbf{X}}_\mathbf{F}\tilde{\mathbf{X}}_\mathbf{F}}(\omega) \mathbf{H}^{\mathrm{T}}(\boldsymbol{\alpha},\omega) \mathbf{s}_j, \quad \boldsymbol{\alpha} \in \boldsymbol{\alpha}^I$$

$$\Downarrow$$

Exact sensitivity of the PSD function of the stress process

$$\boxed{S_{G_{Y_j^{(h)}Y_j^{(h)}},i}(\omega) = \begin{cases} \left.\dfrac{\partial G_{Y_j^{(h)}Y_j^{(h)}}(\boldsymbol{\alpha},\omega)}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}} = 2G^{(0)}_{Y_j^{(h)}Y_j^{(h)}}(\omega) + \mathbf{r}_j^{(h)\mathrm{T}} \mathbf{P}_h(\omega) \mathbf{r}_j^{(h)}, & \text{if } i = h; \\ \left.\dfrac{\partial G_{Y_j^{(h)}Y_j^{(h)}}(\boldsymbol{\alpha},\omega)}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}} = \mathbf{r}_j^{(h)\mathrm{T}} \mathbf{P}_i(\omega) \mathbf{r}_j^{(h)}, & \text{if } i \neq h \end{cases}}$$

$$\mathbf{P}_i(\omega) = \mathbf{S}_i^*(\omega) \mathbf{G}_{\tilde{\mathbf{X}}_\mathbf{F}\tilde{\mathbf{X}}_\mathbf{F}}(\omega) \mathbf{H}_0^{\mathrm{T}}(\omega) + \mathbf{H}_0^*(\omega) \mathbf{G}_{\tilde{\mathbf{X}}_\mathbf{F}\tilde{\mathbf{X}}_\mathbf{F}}(\omega) \mathbf{S}_i^{\mathrm{T}}(\omega)$$

$$\mathbf{S}_i(\omega) = \left.\frac{\partial \mathbf{H}(\boldsymbol{\alpha},\omega)}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}} = -(1+\mathrm{j}c_1\,\omega)\,\mathbf{H}_0(\omega)\,\mathbf{K}_i\,\mathbf{H}_0(\omega)$$

Proposed sensitivity-based procedure

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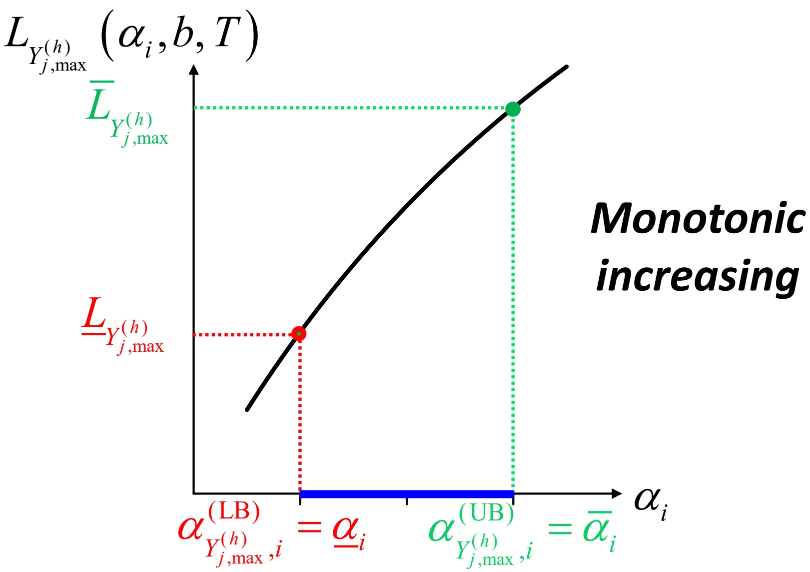
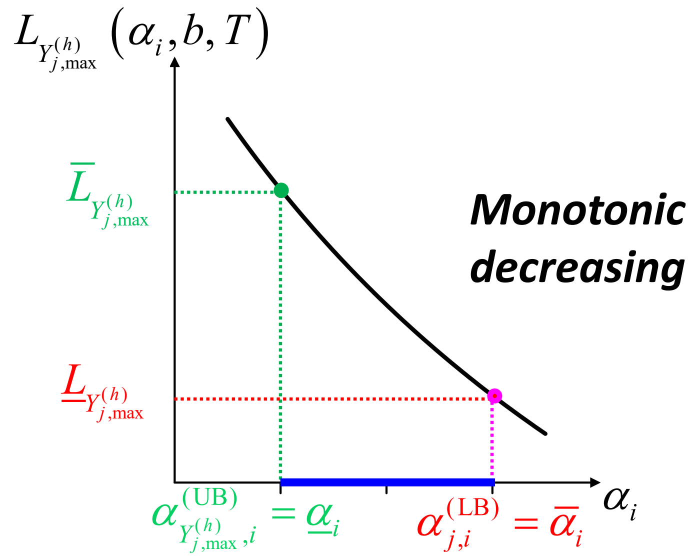
- The combinations of the endpoints of the interval parameters which yield the LB and UB of the interval reliability function can be determined as follows:





$$S_{L_{Y_{j,\max}^{(h)}},i} > 0 \Rightarrow \quad \alpha_{Y_{j,\max}^{(h)},i}^{(\mathrm{UB})} = \overline{\alpha}_i, \ \alpha_{Y_{j,\max}^{(h)},i}^{(\mathrm{LB})} = \underline{\alpha}_i;$$

$$S_{L_{Y_{j,\max}^{(h)}},i} < 0 \Rightarrow \quad \alpha_{Y_{j,\max}^{(h)},i}^{(\mathrm{UB})} = \underline{\alpha}_i, \ \alpha_{Y_{j,\max}^{(h)},i}^{(\mathrm{LB})} = \overline{\alpha}_i$$

$$\Downarrow$$

$$\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{LB})} = \left[\alpha_{Y_{j,\max}^{(h)},1}^{(\mathrm{LB})} \quad \alpha_{Y_{j,\max}^{(h)},2}^{(\mathrm{LB})} \quad \cdots \quad \alpha_{Y_{j,\max}^{(h)},r}^{(\mathrm{LB})} \right]^{\mathrm{T}}; \ \boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{UB})} = \left[\alpha_{Y_{j,\max}^{(h)},1}^{(\mathrm{UB})} \quad \alpha_{Y_{j,\max}^{(h)},2}^{(\mathrm{UB})} \quad \cdots \quad \alpha_{Y_{j,\max}^{(h)},r}^{(\mathrm{UB})} \right]^{\mathrm{T}}.$$

Bounds of the interval reliability function

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$$\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{LB})} = \left[\alpha_{Y_{j,\max}^{(h)},1}^{(\mathrm{LB})} \quad \alpha_{Y_{j,\max}^{(h)},2}^{(\mathrm{LB})} \quad \cdots \quad \alpha_{Y_{j,\max}^{(h)},r}^{(\mathrm{LB})} \right]^{\mathrm{T}}; \quad \boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{UB})} = \left[\alpha_{Y_{j,\max}^{(h)},1}^{(\mathrm{UB})} \quad \alpha_{Y_{j,\max}^{(h)},2}^{(\mathrm{UB})} \quad \cdots \quad \alpha_{Y_{j,\max}^{(h)},r}^{(\mathrm{UB})} \right]^{\mathrm{T}}.$$

$$\underline{L}_{Y_{j,\max}}\left(\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{LB})}, b, T\right) = \exp\left\{ -T\, \nu_{Y_j}^{+}\left(\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{LB})}\right) \exp\left[-\frac{\left(b - \left|\mu_{Y_j}\left(\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{LB})}\right)\right|\right)^2}{2\lambda_{0,Y_j}\left(\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{LB})}\right)} \right] \right\}$$

LB (worst-case scenario)

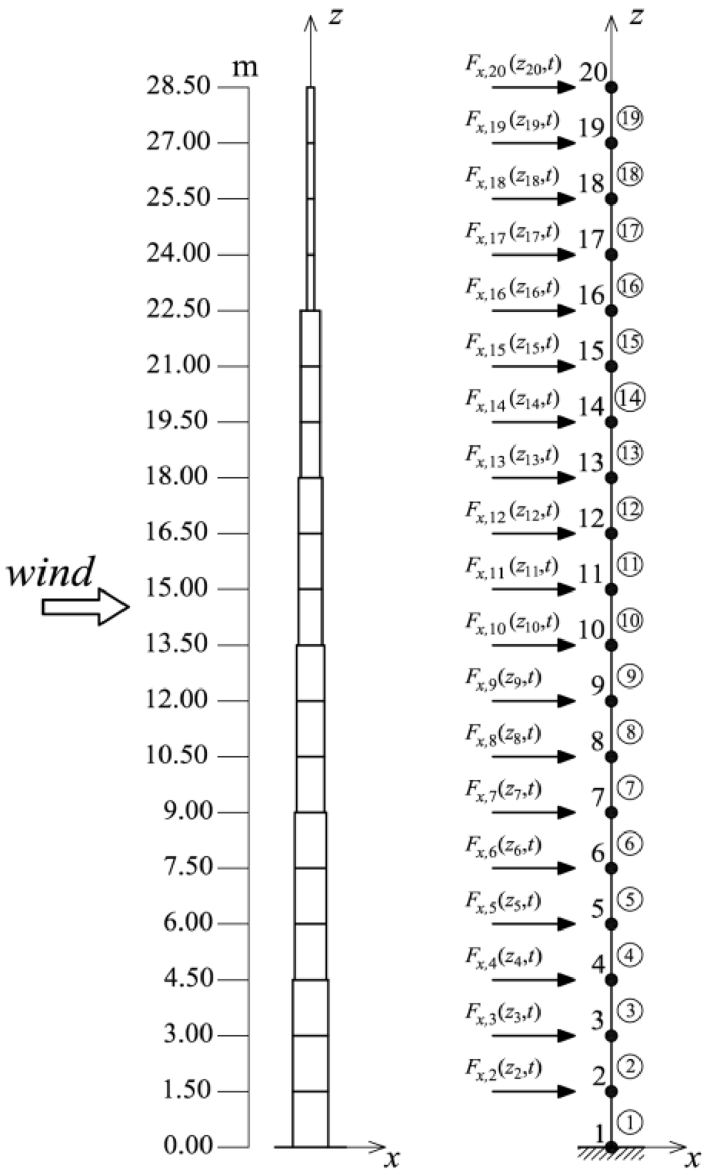
$$\overline{L}_{Y_{j,\max}}\left(\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{UB})}, b, T\right) = \exp\left\{ -T\, \nu_{Y_j}^{+}\left(\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{UB})}\right) \exp\left[-\frac{\left(b - \left|\mu_{Y_j}\left(\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{UB})}\right)\right|\right)^2}{2\lambda_{0,Y_j}\left(\boldsymbol{\alpha}_{Y_{j,\max}^{(h)}}^{(\mathrm{UB})}\right)} \right] \right\}$$

UB

Only 2 stochastic analyses!

Numerical application 1/5

- **Telecommunication antenna under turbulent wind (lumped mass model)**



Node	Height [m]	Outer diameter [m]	Thickness [m]	Lumped mass [t]	Tributary area [m²]
2	1.50	0.9148	0.008	0.3050	1.3722
3	3.00	0.9148	0.008	0.3050	1.3722
4	4.50	0.9148	0.008	0.2754	1.2957
5	6.00	0.8128	0.0071	0.2458	1.2192
6	7.50	0.8128	0.0071	0.2458	1.2192
7	9.00	0.8128	0.0071	0.2211	1.143
8	10.50	0.7112	0.0063	0.1964	1.0668
9	12.00	0.7112	0.0063	0.1964	1.066
10	13.50	0.7112	0.0063	0.1760	0.9906
11	15.00	0.6096	0.0056	0.1530	0.9144
12	16.50	0.6096	0.0056	0.1505	0.9144
13	18.00	0.6096	0.0056	0.1288	0.8382
14	19.50	0.508	0.005	0.2671	0.762
15	21.00	0.508	0.005	0.1071	0.762
16	22.50	0.508	0.005	0.1650	0.526275
17	24.00	0.1937	0.0045	0.0429	0.29055
18	25.50	0.1937	0.0045	0.1329	0.29055
19	27.00	0.1937	0.0045	0.0429	0.29055
20	28.50	0.1937	0.0045	0.0214	0.145275

$$E_{0,i} = E_0 = 210\,\text{GPa};$$

$$\zeta_0 = 0.01$$

$$T_0 = 0.723\text{s}$$

- 12 uncertain Young's moduli:

$$E_i^I = E_0(1+\Delta\alpha\,\hat{e}_i^I),(i=1,2,..,12)$$

- Attention is herein focused on the evaluation of the range of the interval reliability function for the axial stress random process $Y_1^{(1)}(\boldsymbol{\alpha}^I,t) \equiv \sigma_1^{(1)}(\boldsymbol{\alpha}^I,t)$ of FE 1.

Numerical application 2/5

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- **Wind velocity field**

$$W(z,t)=w_s(z)+\tilde{W}(z,t)$$

$$w_s(z)=w_{s,10}\left(\frac{z}{10}\right)^{\beta}$$

mean wind velocity

zero-mean stationary Gaussian random process

$$G_{\tilde{W}\tilde{W}}(\omega)=4K_0 w_{s,10}^2\frac{\chi^2}{\omega\left(1+\chi^2\right)^{4/3}}$$

(Davenport, 1961)

- **Cross-Power Spectral Density**

$$S_{\tilde{W}_i\tilde{W}_j}(z_i,z_j;\omega)=S_{\tilde{W}\tilde{W}}(\omega)f_{ij}(\omega)$$

$$f_{ij}(\omega)=\exp\left\{-\frac{|\omega|\sqrt{C_z^2(z_i-z_j)^2}}{\pi\left[w_s(z_i)+w_s(z_j)\right]}\right\}$$

(Coherence function)

- **Forces on the wind-exposed nodes**

$$F_{x,i}(z_i,t)=\frac{1}{2}\rho C_D A_i W^2(z_i,t)=F_{x,i}^{(s)}+\tilde{F}_{x,i}(z_i,t)\approx\frac{1}{2}\rho C_D A_i w_s^2+\rho C_D A_i\tilde{W}(z_i,t)w_s,$$

$$(i=1,2,\ldots,19)$$

$\rho=1.25\ \mathrm{kg/m^3};\ \ C_D=1.2.$

Numerical application 3/5

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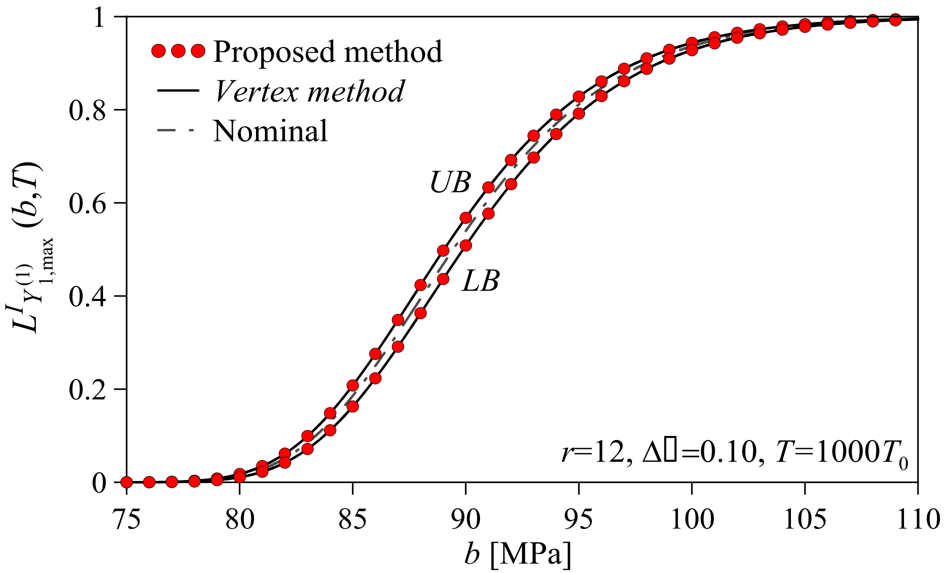
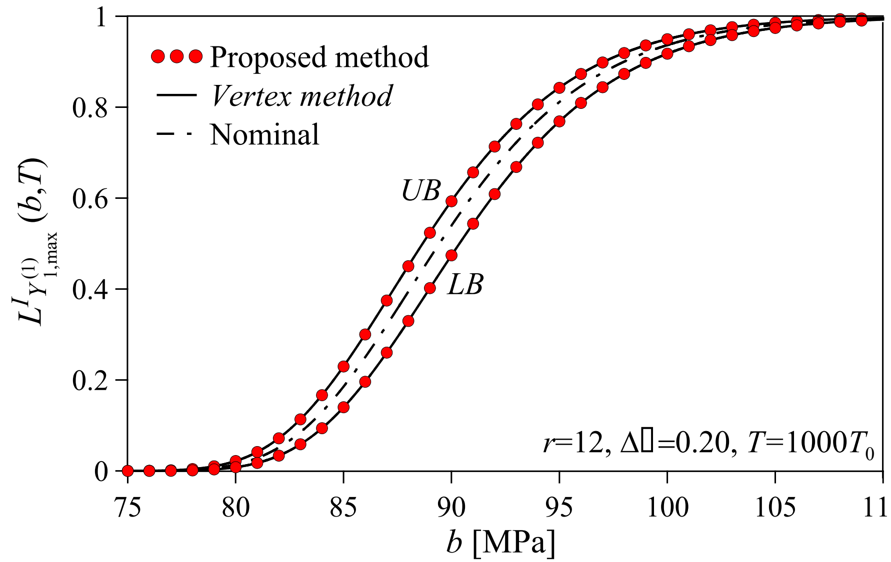
- **LB and UB of the interval reliability function of $Y_1^{(1)}(\boldsymbol{\alpha}^I, t)$: comparison between the proposed method and the *vertex method***





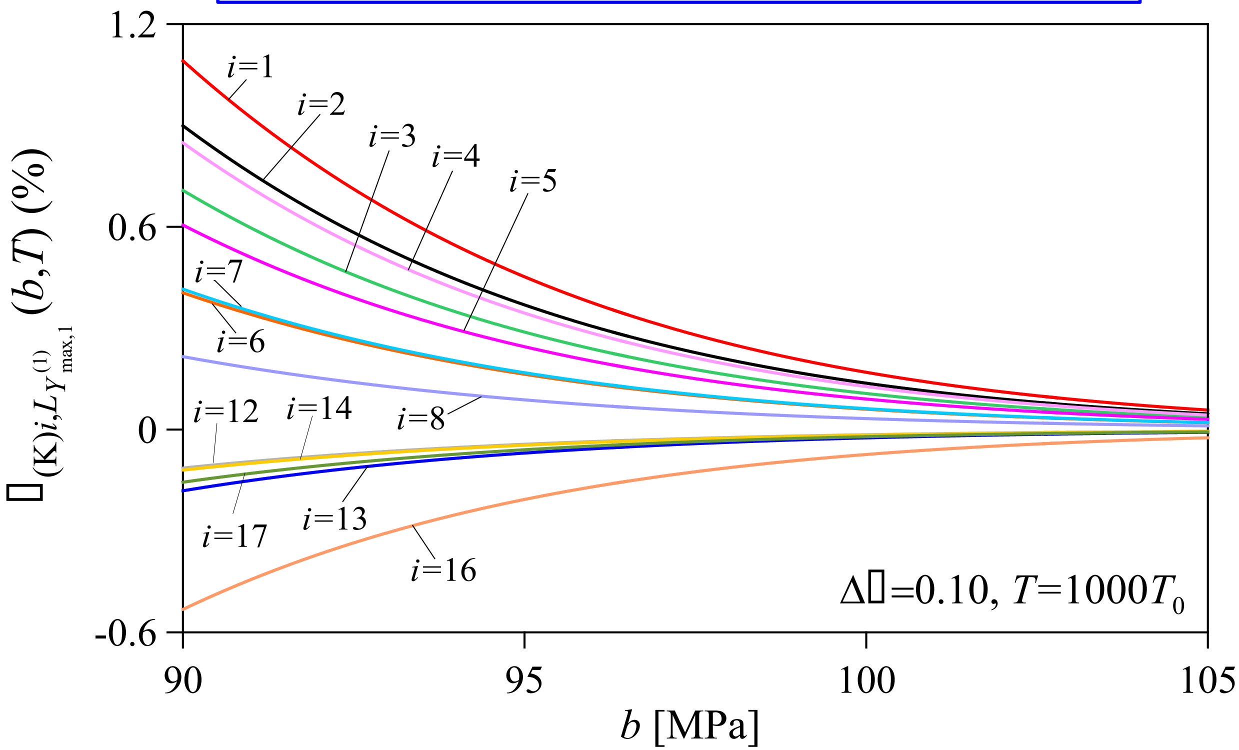
Method	Number of stochastic analyses
Vertex method	2^{12}
Proposed method	2

Numerical application 4/5

REC2021

- **Function of sensitivity of the interval reliability function of** $Y_1^{(1)}(\boldsymbol{\alpha}^I, t)$

$$\varphi_{i,L_{Y_{j,\max}^{(h)}}}(b,T)(\%) = \frac{S_{L_{Y_{j,\max}^{(h)}},i}(b,T)}{L^{(0)}_{Y_{j,\max}^{(h)}}(b,T)}\Delta\alpha_i \times 100$$

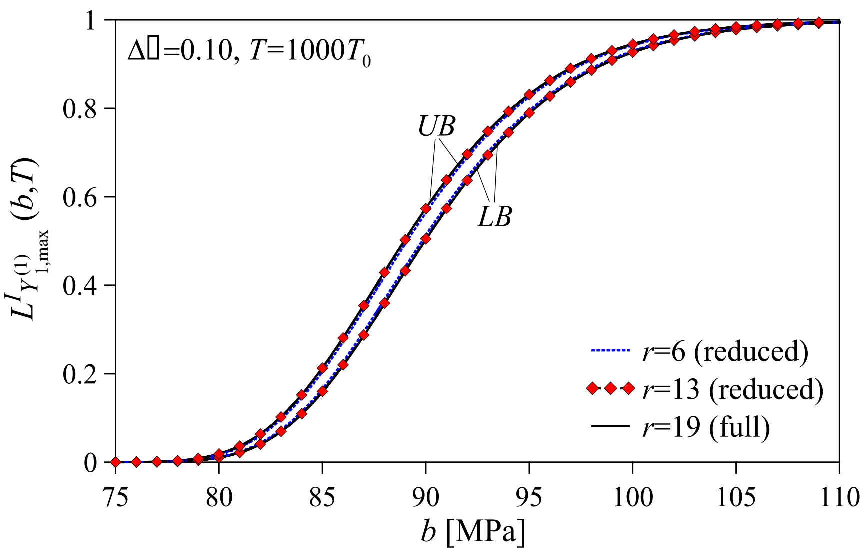


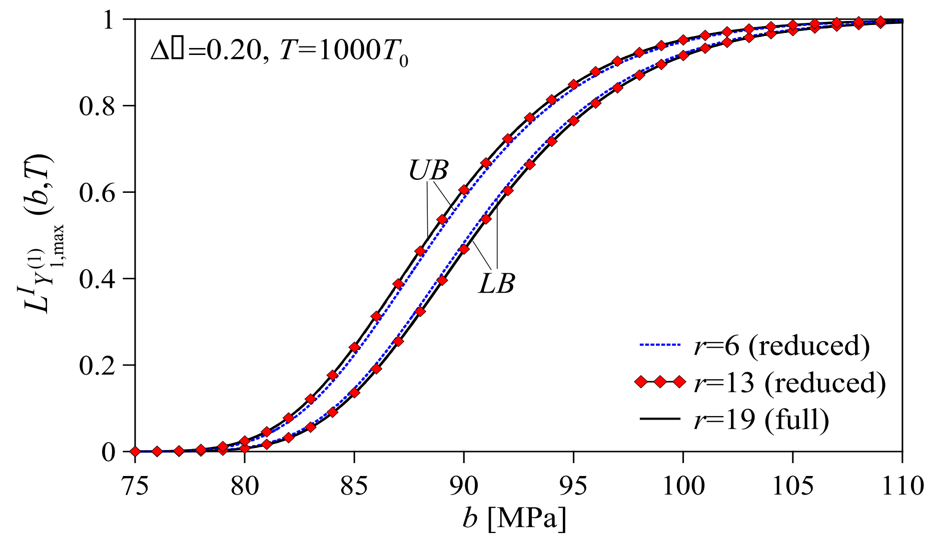
b=90 Mpa corresponds to a probability p=0.5 of not being excedeed within the observation time *T* when all the uncertain parameters are set equal to their nominal value

Numerical application 5/5

REC2021

- **Proposed LB and UB of the interval reliability function of $Y_1^{(1)}(\boldsymbol{\alpha}^I, t)$ obtained considering full and reduced uncertain input parameters**





$$r = 6\,(\text{reduced}) \quad E^{(i)I} = E_0\left(1 + \Delta\alpha \hat{e}_i^I\right), (i = 1, 2, 3, 4, 5, 16)$$

$$r = 13\,(\text{reduced}) \quad E^{(i)I} = E_0\left(1 + \Delta\alpha \hat{e}_i^I\right), (i = 1, \ldots, 16, 7, 8, 12, 13, 14, 16, 17)$$

$$r = 19\,(\text{full}) \quad E^{(i)I} = E_0\left(1 + \Delta\alpha \hat{e}_i^I\right), (i = 1, 2, \ldots, 19)$$

Conclusions

- Reliability analysis of linear structures with interval stiffness parameters subjected to stationary multi-correlated Gaussian random processes has been addressed.
- A novel sensitivity-based approach for the evaluation of the bounds of the interval reliability function of a selected stress process has been proposed.
- **Main achievements**:
  - very accurate estimates of the bounds of the interval reliability function of a selected stress process by performing only two stochastic analyses of the structure
  - reduced computational effort compared to combinatorial procedures
  - Insights into the impact of stiffness fluctuations on structural performance allowing the identification of the parameters which may be set equal to the nominal values → Enhancement of the computational efficiency

Thanks for your attention!