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Interval analysis of the forced vibration of beams with uncertain damage

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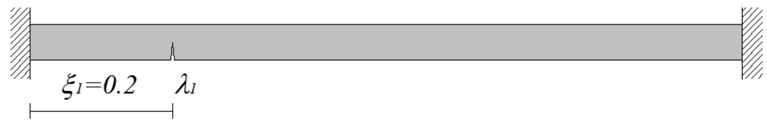
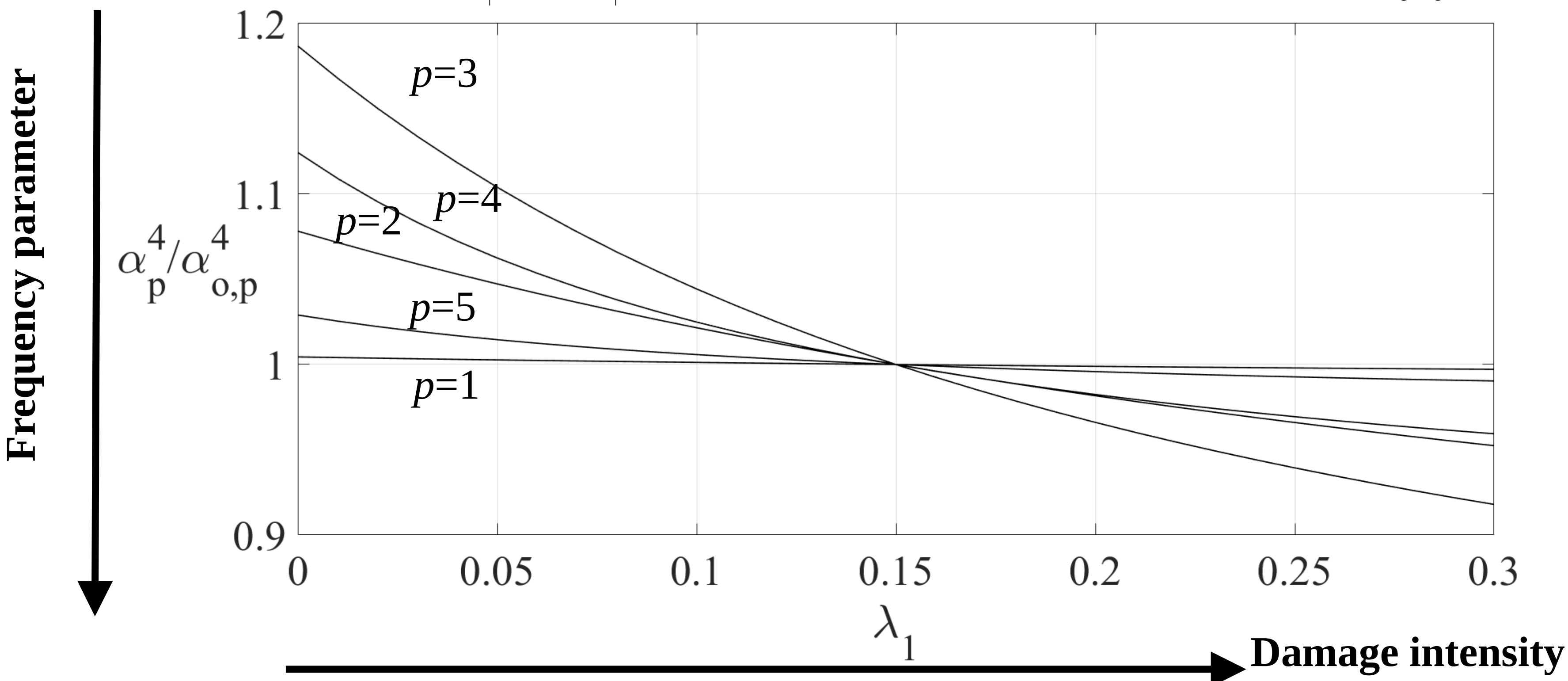
Motivation

The computation of the bounds of the response to the forced vibration of damaged beams in presence of uncertain but bounded damage intensities requires the solution of highly nonlinear equations

The frequency parameters exhibit monotonic trends with respect to the damage intensities

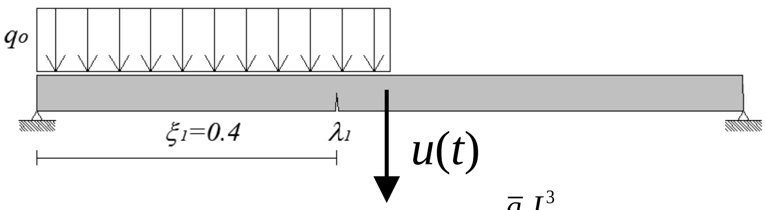


$$\alpha_p^4(\lambda_1) = \frac{mL^4}{E_o I_o}\,\omega_p^2(\lambda_1)$$



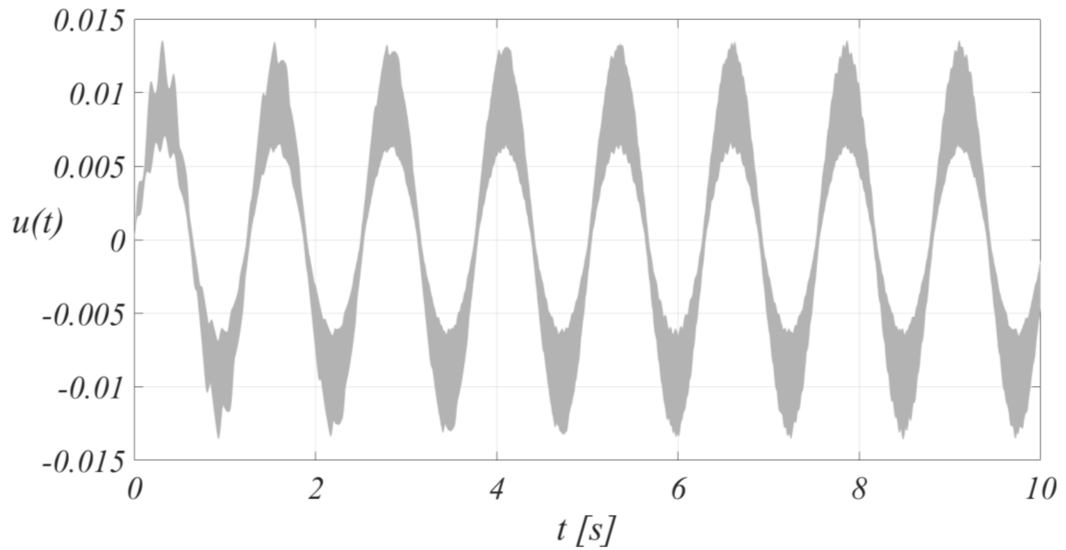
Motivation

The response to the forced vibrations may exhibit non-monotonic trends with respect to the damage intensities in terms of transversal displacement



$$q(t) = q_0 \sin \omega t \qquad \omega = 5\,\text{rad/s} \qquad q_o = \frac{\bar{q}_o L^3}{E_o I_o} = 1$$

$$0 \le \lambda_1 \le 0.4$$



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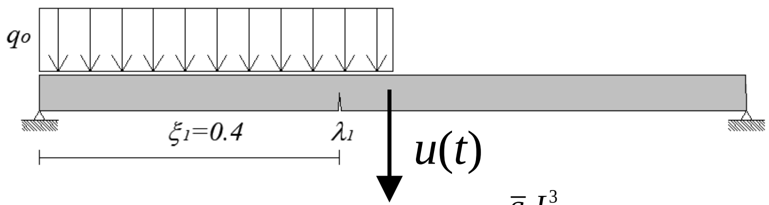

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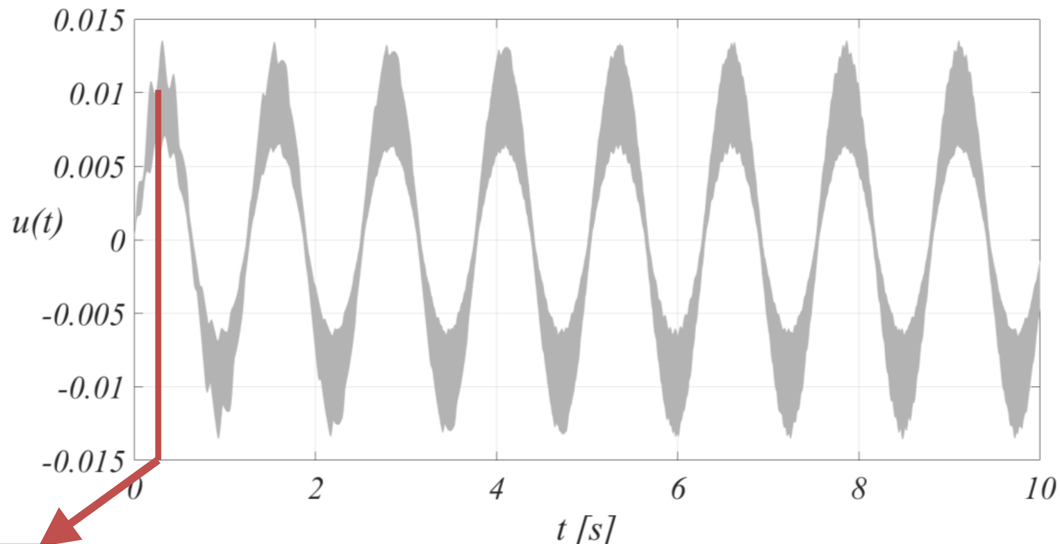
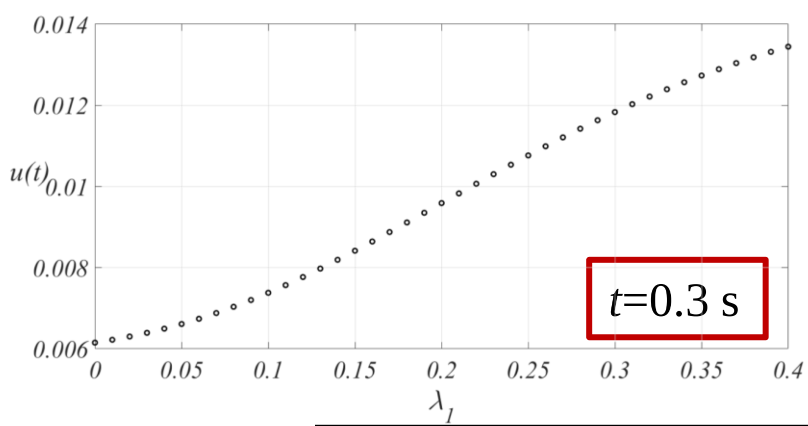
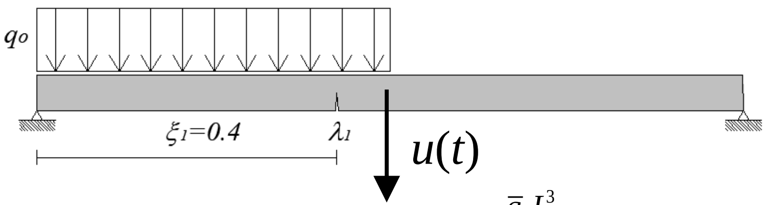




Damage intensity

Motivation

The response to the forced vibrations may exhibit non-monotonic trends with respect to the damage intensities in terms of transversal displacement



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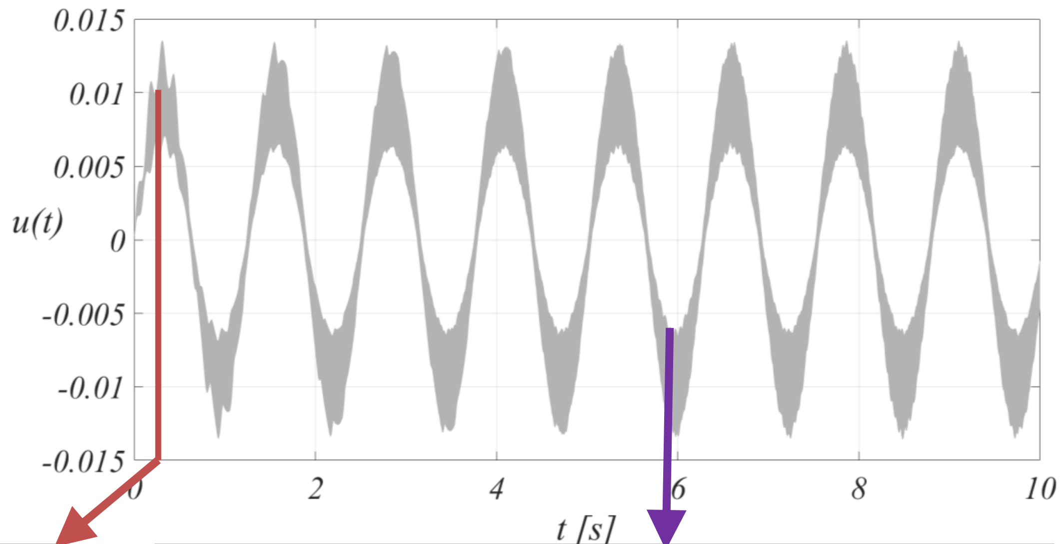
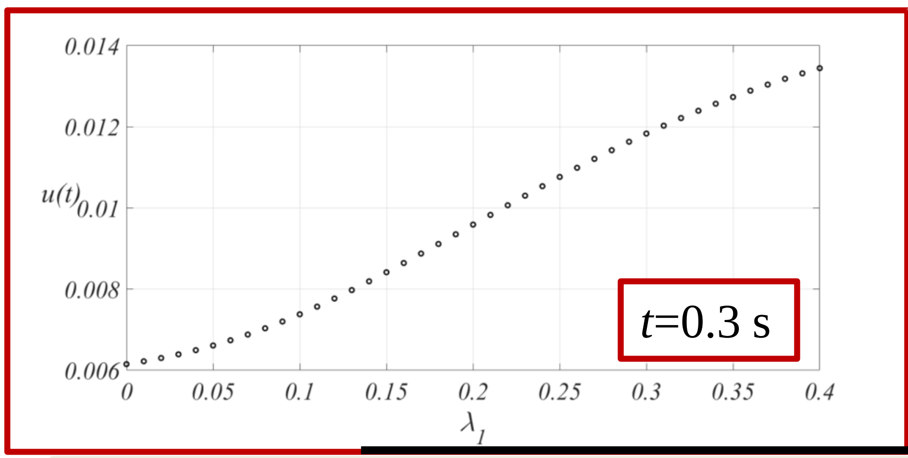
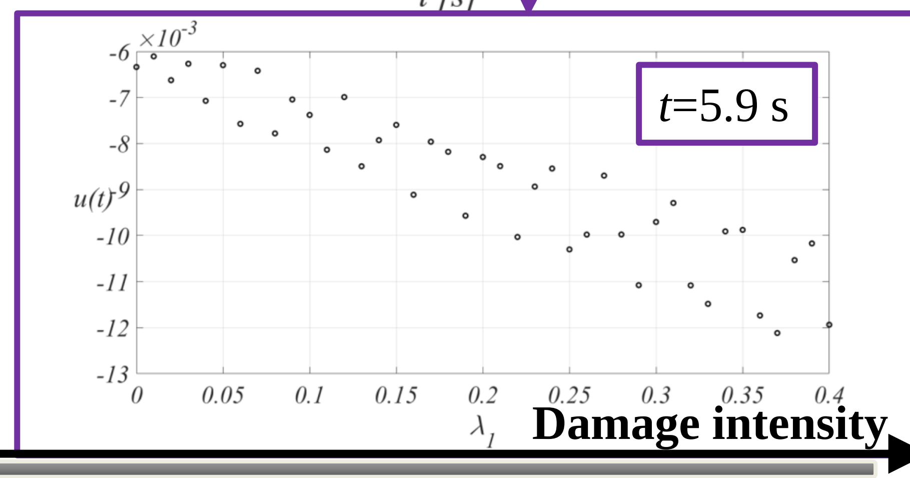
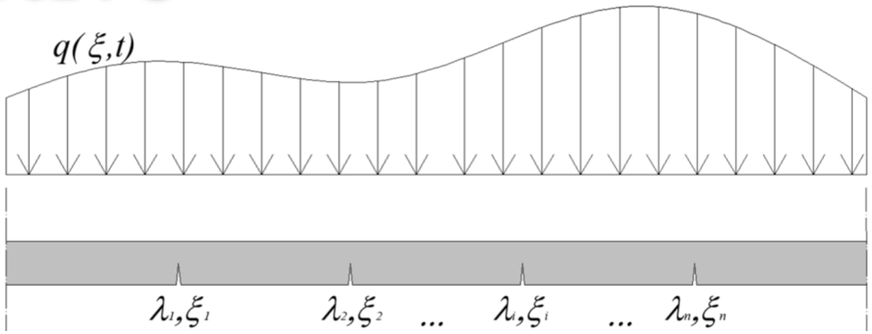






Objective



Problem

Dynamics of Euler-Bernoulli multi-cracked beams

Crack modelling:

- equivalent rotational hinge model (K_i, ξ_i), i=1,…n
- crack intensities λ_i, i=1,…n (here considered as variables of the problem collected in the vector $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots \lambda_i, \ldots \lambda_n]$)

Objective

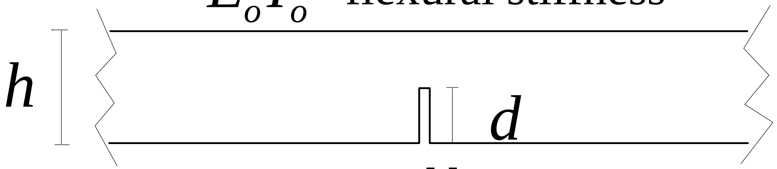
- **Explicit expression** of the eigenproperties of the beam and subsequently of response parameters of the forced vibrations of cracked beams

- Application to direct problems (response variability and identification of the bounds of the response)

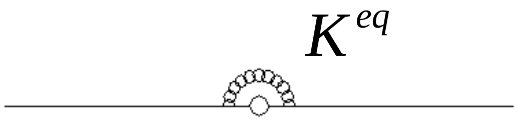
Procedure

- Adoption of the distribution theory (generalised functions)
- Approximated formulas for the computation of the main modal parameters (frequency, generalized modal mass and modal load terms)

The equivalent rotational spring to model concentrated damage



Damaged beam



Equivalent model
(Freund and Herrmann 1976, Gounaris & Dimarogonas 1988)

$$\mu = d/h$$

Stiffness of the equivalent spring

$$K^{eq} = \frac{E_o I_o}{L} \frac{1}{\lambda(\mu)}$$

Dimensionless local compliance

Rizos, Aspragathos & Dimarogonas, 1990

$$\lambda(\mu) = 5.346 \frac{h}{L}(1.86\,\mu^2 - 3.95\,\mu^3 + 16.375\,\mu^4 - 37.226\,\mu^5 + 76.81\,\mu^6 - 126.9\,\mu^7 + 172\,\mu^8 - 143.97\,\mu^9 + 66.56\,\mu^{10})$$

Ostachowicz & Krawczuk, 1991

$$\lambda(\mu) = 3\pi 2\,\mu^2 \frac{h}{L}(0.6384 - 1.035\,\mu + 3.7201\,\mu^2 - 5.1773\,\mu^3 + 7.553\,\mu^4 - 7.332\,\mu^5 + 2.4909\,\mu^6)$$

Bilello, 2001

$$\lambda(\mu) = \frac{h}{L} \frac{\mu(2-\mu)}{0.9(\mu-1)^2}$$

How multiple singularities can be treated without increasing the size of the problem?

The structural response of cracked beams is usually solved by enforcing the continuity conditions at the cracked sections

Transfer matrix method

(*Li 2002*)

$$\begin{bmatrix} y(x_{n,n_n+1,1}) \\ \psi(x_{n,n_n+1,1}) \\ M(x_{n,n_n+1,1}) \\ V(x_{n,n_n+1,1}) \end{bmatrix} = [TT] \begin{bmatrix} y(x_{110}) \\ \psi(x_{110}) \\ M(x_{110}) \\ V(x_{110}) \end{bmatrix} \qquad [TT]=[T_{nn}][T_{(n-1)(n-1)}]\cdots[T_{22}][T_{11}]$$

Theory of distributions

(*Wang and Qiao 2007*)

$$w''''(x,t)+\frac{\rho A}{EI}\ddot{w}(x,t)=\Delta w'''(x_1,t)\delta(x-x_1)+\Delta w''(x_1,t)\delta'(x-x_1)+\Delta w'(x_1,t)\delta''(x-x_1) + \Delta w(x_1,t)\delta'''(x-x_1).$$

Stiffness reduction method

(*Caddemi and Caliò 2008*)

$$E(\xi)I(\xi)=E_oI_o\left[1-\sum_{i=1}^{n}\gamma_i\,\delta(\xi-\xi_{oi})\right]$$

Flexibility modelling

(*Palmeri and Cicirello 2011*)

$$\left[E(\xi)I(\xi)\right]^{-1}=\left[E_oI_o\right]^{-1}\left[1+\sum_{i=1}^{n}\lambda_i\,\delta(\xi-\xi_{oi})\right]$$

Force Green's functions

(*Failla et al. 2008*)

$$\mathbf{R}^{(r)}(x)=[U^{(r)}(x)\Phi^{(r)}(x)M^{(r)}(x)S^{(r)}(x)]^{\mathrm{T}}=\tilde{\omega}_r^2\int_0^L \mathbf{G}^{(P)}(x,y,D)\rho(y)U^{(r)}(y)\delta(x-y)\mathrm{d}y$$

Background

At the University of Catania the theory of distributions has been successfully proposed with reference to:

STATICS *(Biondi and Caddemi 2007, Caddemi et al. 2013)*,

STABILITY *(Caddemi and Caliò 2008, Caddemi et al. 2014)*

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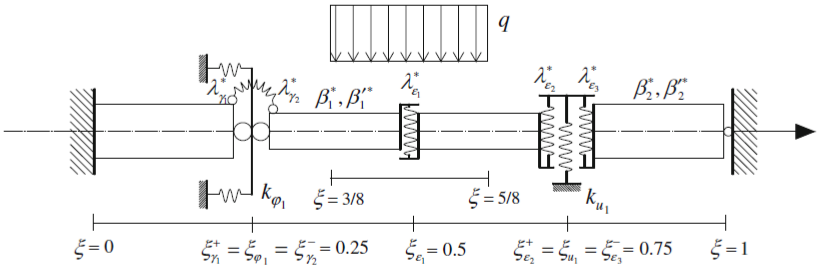

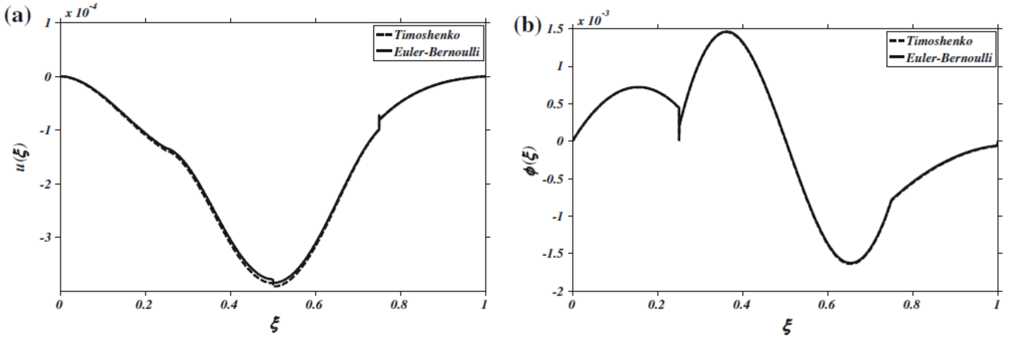

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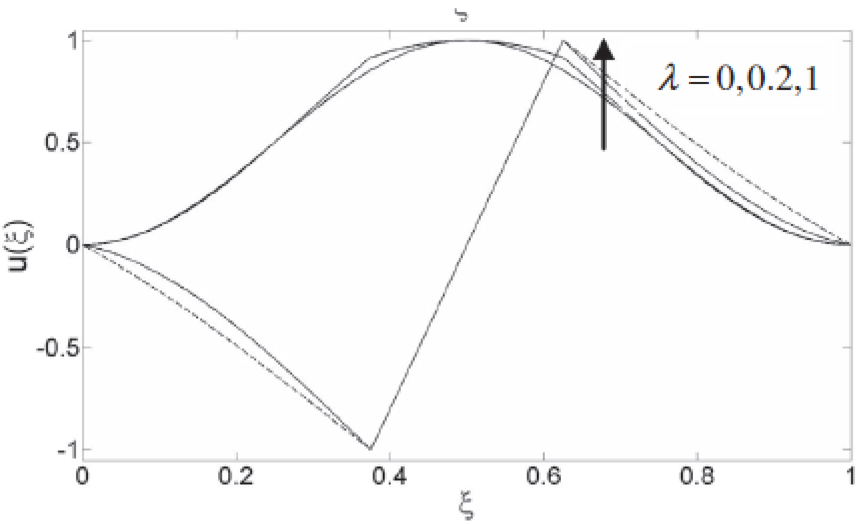

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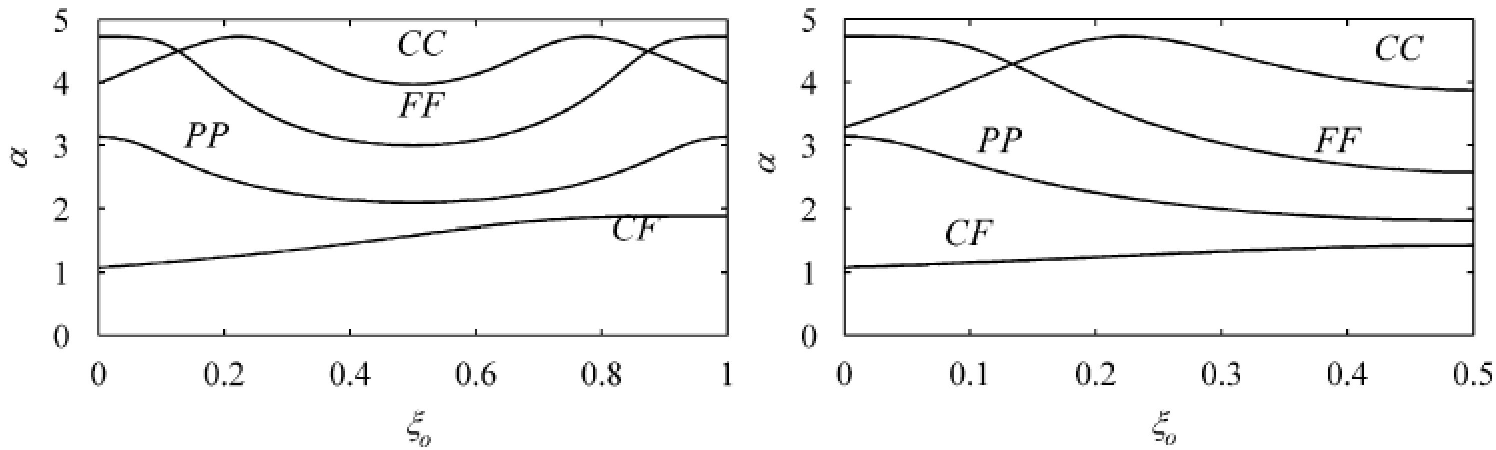
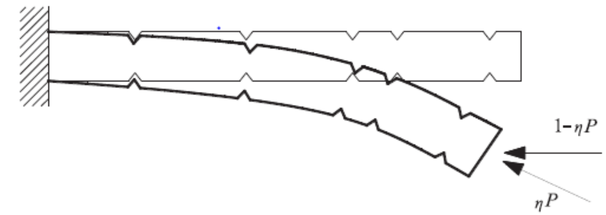


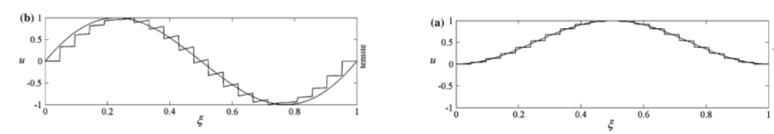
Background

Applications of these basic solutions have been applied to different fields:

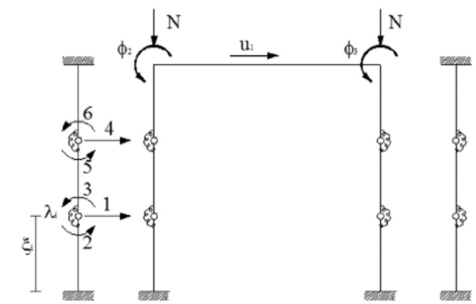
DYNAMIC INSTABILITY of damaged beams (*Caddemi et al. 2014*)



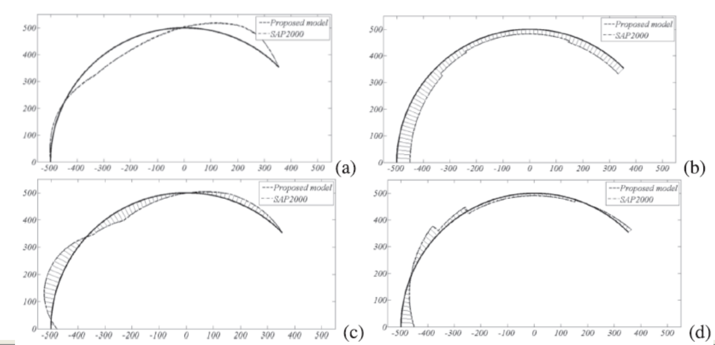
TENSILE BUCKLING (*Caddemi et al. 2015)*



DYNAMICS AND STABILITY of damaged frames (*Caddemi and Caliò 2013, Caddemi et al. 2013, Caddemi et al. 2017*),



STATICS OF MULTI CRACKED ARCHES (*Cannizzaro et al. 2017*)



INVERSE PROBLEMS (*Caddemi and Caliò 2014, Caddemi et al. 2018, Greco et al. 2018*)

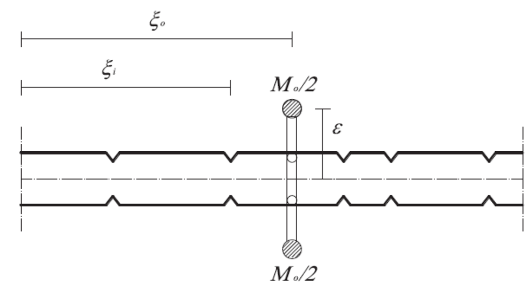
Background

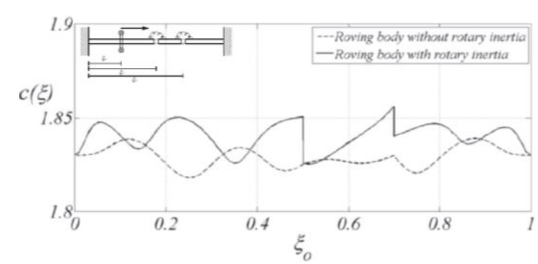
In particular, with reference to inverse problems, a recent work which exploits the properties of a concentrated mass with rotary inertia to locate cracks (*Cannizzaro et al. 2018*) an efficient solution for the dynamics of cracked beams was introduced, able to explicitly infer the mode shapes given the fundamental frequencies.

The computation of the fundamental frequencies, although requiring to handle nonlinear equations, is simplified because the size of the problems is associated to the end conditions only, irrespectively of the number of the along-axis cracks.

$$u^{IV}(\xi,t) - \cancel{\frac{mL^4}{EI}\frac{M_0}{mL}\frac{\varepsilon^2}{L^2}\delta^{I}(\xi-\xi_0)\ddot{u}^{I}(\xi_0,t)} + \frac{mL^4}{EI}\left[1+\cancel{\frac{M_0}{mL}\delta(\xi-\xi_0)}\right]\ddot{u}(\xi,t) =$$

$$= \boxed{\sum_{i=1}^{n} \Delta u^{I}(\xi_i,t)\,\delta^{II}(\xi-\xi_i)}$$





F. Cannizzaro, J. De Los Rios, S. Caddemi, I. Caliò, S. Ilanko, On the use of a roving body with rotary inertia to locate cracks in beams, Journal of Sound and Vibration 425(7) (2018) 275-300.

The governing equation

$$u^{IV}(\xi,t)+\frac{mL^4}{E_oI_o}\ddot{u}(\xi,t)=\sum_{i=1}^{n}\Delta u'(\xi_i,t)\delta''(\xi-\xi_i)+\boxed{q(\xi,t)}$$

Constitutive equation

$$\Delta u'(\xi_i,t)=\lambda_i u''(\xi_i,t) \qquad \lambda_i=\frac{E_oI_o}{K_iL} \qquad \boldsymbol{\lambda}=[\lambda_1,\lambda_2,\ldots\lambda_i,\ldots\lambda_n]$$

Eigenproperties

$$\alpha_r^4=\omega_r^2\frac{mL^4}{EI} \qquad \phi_r(\xi;\boldsymbol{\lambda})=\sum_{j=1}^{4}C_{j,r}(\boldsymbol{\lambda})\,f_{j,r}(\xi;\boldsymbol{\lambda})$$

where

$$f_{j,r}(\xi;\boldsymbol{\lambda})=h_{j,r}(\xi;\boldsymbol{\lambda})+\sum_{k=1}^{n}\bar{h}_k(\xi;\boldsymbol{\lambda})\lambda_k f''_{j,r}(\xi_k;\boldsymbol{\lambda}) \quad , \quad j=1,\ldots,4$$

$$h_{1,r}(\xi;\boldsymbol{\lambda})=\sin\alpha_r\xi; \quad h_{2,r}(\xi;\boldsymbol{\lambda})=\cos\alpha_r\xi; \quad h_{3,r}(\xi;\boldsymbol{\lambda})=\sinh\alpha_r\xi; \quad h_{4,r}(\xi;\boldsymbol{\lambda})=\cosh\alpha_r\xi;$$

$$\bar{h}_{i,r}(\xi;\boldsymbol{\lambda})=\frac{1}{2\alpha_r}\left[\sin\alpha_r(\xi-\xi_i)+\sinh\alpha_r(\xi-\xi_i)\right]U(\xi-\xi_i)$$

The governing equation

	Boundary conditions	**Determinantal equation**
Pinned-pinned	$\phi(0;\boldsymbol{\lambda})=0;\quad \phi''(0;\boldsymbol{\lambda})=0;$ $\phi(1;\boldsymbol{\lambda})=0;\quad \phi''(1;\boldsymbol{\lambda})=0$	$f_{1,r}(1;\boldsymbol{\lambda})\,f''_{3,r}(1;\boldsymbol{\lambda})-f_{3,r}(1;\boldsymbol{\lambda})\,f''_{1,r}(1;\boldsymbol{\lambda})=0$
Clamped-clamped	$\phi(0;\boldsymbol{\lambda})=0;\quad \phi'(0;\boldsymbol{\lambda})=0;$ $\phi(1;\boldsymbol{\lambda})=0;\quad \phi'(1;\boldsymbol{\lambda})=0$	$\left[f_{3,r}(1;\boldsymbol{\lambda})-f_{1,r}(1;\boldsymbol{\lambda})\right]\left[f'_{4,r}(1;\boldsymbol{\lambda})-f'_{2,r}(1;\boldsymbol{\lambda})\right]+$ $-\left[f_{4,r}(1;\boldsymbol{\lambda})-f_{2,r}(1;\boldsymbol{\lambda})\right]\left[f'_{3,r}(1;\boldsymbol{\lambda})-f'_{1,r}(1;\boldsymbol{\lambda})\right]=0$

q_o

Forced vibrations

General solution (free vibrations)

$$z_p^g(t)=e^{-\zeta_p\omega_p(\boldsymbol{\lambda})t}\left[z_p(0)\cos\omega_{p,d}(\boldsymbol{\lambda})t+\frac{\dot{z}_p(0)+\zeta_p\omega_p(\boldsymbol{\lambda})z_p(0)}{\omega_{p,d}(\boldsymbol{\lambda})}\sin\omega_{p,d}(\boldsymbol{\lambda})t\right]$$

ζ_p Modal damping

$\omega_{p,d}(\boldsymbol{\lambda})=\omega_p(\boldsymbol{\lambda})\sqrt{1-\zeta_p^2}$ Damped frequency

Particular solution (steady-state response)

$q(\xi,t)$	$\dfrac{Q_p(t;\boldsymbol{\lambda})}{M_p(\boldsymbol{\lambda})}$	$z_p^p(t)$
$P_o\delta(\xi-\xi_o)\sin\omega t$	$\dfrac{P_o\phi_p(\xi_o;\boldsymbol{\lambda})}{M_p(\boldsymbol{\lambda})}\sin\omega t$	$z_p^p(t)=\dfrac{P_o\phi_p(\xi_o;\boldsymbol{\lambda})}{M_p(\boldsymbol{\lambda})\alpha_p^4(\boldsymbol{\lambda})}\left[C_p(\boldsymbol{\lambda})\sin\omega t+D_p(\boldsymbol{\lambda})\cos\omega t\right]$
$q_o\sin\omega t$	$\dfrac{q_o\int_0^1\phi_p(\xi;\boldsymbol{\lambda})d\xi}{M_p(\boldsymbol{\lambda})}\sin\omega t$	$z_p^p(t)=\dfrac{q_o\int_0^1\phi_p(\xi;\boldsymbol{\lambda})d\xi}{M_p(\boldsymbol{\lambda})\alpha_p^4(\boldsymbol{\lambda})}\left[C_p(\boldsymbol{\lambda})\sin\omega t+D_p(\boldsymbol{\lambda})\cos\omega t\right]$

Constants of integrations

$$C_p(\boldsymbol{\lambda})=\frac{1-\eta_p^2(\boldsymbol{\lambda})}{\left[1-\eta_p^2(\boldsymbol{\lambda})\right]^2+\left[2\zeta_p\eta_p(\boldsymbol{\lambda})\right]^2}$$

$$D_p(\boldsymbol{\lambda})=\frac{-2\zeta_p\eta_p(\boldsymbol{\lambda})}{\left[1-\eta_p^2(\boldsymbol{\lambda})\right]^2+\left[2\zeta_p\eta_p(\boldsymbol{\lambda})\right]^2}$$

Normalized pulsation $\eta_p(\boldsymbol{\lambda})=\alpha_\omega^2/\alpha_p^2(\boldsymbol{\lambda})=\omega/\omega_p(\boldsymbol{\lambda})$

Solution strategy

In the field of continuous systems the dynamic analysis of structures is usually tackled according to the following steps

Governing differential equation

$$u^{IV}(\xi,t)+\frac{mL^4}{E_oI_o}\ddot{u}(\xi,t)=\sum_{i=1}^{n}\Delta u'(\xi_i,t)\delta''(\xi-\xi_i)+q(\xi,t)$$

Solution of a determinantal equation (eigenfrequency $\alpha_r^4=\omega_r^2 mL^4/EI$)

Computation of integration constants $C_{j,r}(\boldsymbol{\lambda})$ and eigenmodes $\phi_r(\xi;\boldsymbol{\lambda})$

Computation of other modal properties:

- Generalized modal mass $M_p(\boldsymbol{\lambda})=\int_0^1\phi_p(\xi;\boldsymbol{\lambda})\phi_p(\xi;\boldsymbol{\lambda})d\xi$
- Modal load term $Q_p(t;\boldsymbol{\lambda})=\int_0^1\phi_p(\xi;\boldsymbol{\lambda})q(\xi,t)d\xi$

Integration of differential equations in the modal space

$$\ddot{z}_p(t)+2\zeta_p\omega_p(\boldsymbol{\lambda})\dot{z}_p(t)+\omega_p^2(\boldsymbol{\lambda})z_p(t)=\frac{E_oI_o}{mL^4}\frac{Q_p(t;\boldsymbol{\lambda})}{M_p(\boldsymbol{\lambda})},\qquad 1,\ldots,n_m$$

Computation of the response via modal superimposition

$$u(\xi,t;\boldsymbol{\lambda})=\sum_{r=1}^{\infty}\phi_r(\xi;\boldsymbol{\lambda})z_r(t)\cong\sum_{r=1}^{n_m}\phi_r(\xi;\boldsymbol{\lambda})z_r(t)$$

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Explicit computation of the frequency parameters $\alpha_r^4(\boldsymbol{\lambda})$

Computation of integration constants $C_{j,r}(\boldsymbol{\lambda})$ and eigenmodes $\phi_r(\xi;\boldsymbol{\lambda})$

Explicit computation of the generalized modal mass $M_p(\boldsymbol{\lambda})$ and of the modal load term $Q_p(t;\boldsymbol{\lambda})$

Integration of differential equations in the modal space

$$\ddot{z}_p(t)+2\zeta_p\omega_p(\boldsymbol{\lambda})\dot{z}_p(t)+\omega_p^2(\boldsymbol{\lambda})z_p(t)=\frac{E_oI_o}{mL^4}\frac{Q_p(t;\boldsymbol{\lambda})}{M_p(\boldsymbol{\lambda})}, \qquad 1,\dots n_m$$

Computation of the response via modal superimposition

$$u(\xi,t;\boldsymbol{\lambda})=\sum_{r=1}^{\infty}\phi_r(\xi;\boldsymbol{\lambda})z_r(t)\cong\sum_{r=1}^{n_m}\phi_r(\xi;\boldsymbol{\lambda})z_r(t)$$

Approximated explicit solutions of the modal properties

The main modal properties, that is eigenfrequencies, generalized modal masses and modal load terms are here explicitly expressed according to the Sherman and Morrison formula (1949), which was already adopted for static problems (*Impollonia* 2006). Damage intensities are considered as variables of the problem.

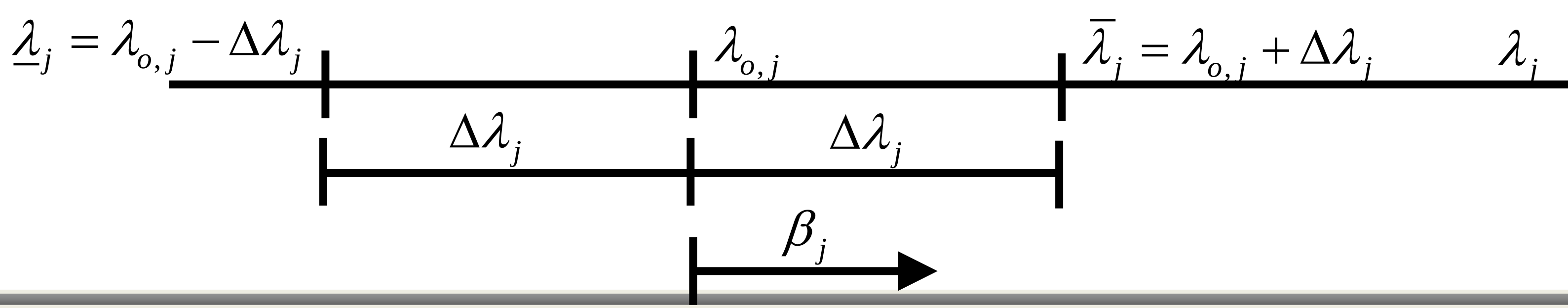
Frequency parameter $$\alpha_p^4(\boldsymbol{\lambda}) \cong \alpha_{o,p}^4 + \sum_{i=1}^{n} \frac{\beta_i a_{p,i}}{1+\beta_i b_{p,i}} \quad , \qquad p=1,\dots,n_m, \qquad \beta_i \in [-\Delta\lambda_i, \Delta\lambda_i]$$

Generalized modal mass $$M_p(\boldsymbol{\lambda}) \cong M_{o,p} + \sum_{i=1}^{n} \frac{\beta_i c_{p,i}}{1+\beta_i d_{p,i}} \quad , \qquad p=1,\dots,n_m, \qquad \beta_i \in [-\Delta\lambda_i, \Delta\lambda_i]$$

Modal load term $$Q_p(t;\boldsymbol{\lambda}) \cong Q_{o,p}(t) + \sum_{i=1}^{n} \frac{\beta_i e_{p,i}(t)}{1+\beta_i f_{p,i}(t)} \quad , \qquad p=1,\dots,n_m, \qquad \beta_i \in [-\Delta\lambda_i, \Delta\lambda_i]$$

Damage intensities

$$\lambda_j = \lambda_{o,j} + \beta_j \qquad j=1,\dots,n \qquad \qquad \beta_j \in [-\Delta\lambda_j, \Delta\lambda_j]$$



Approximated explicit solutions of the modal properties

Frequency parameter $\quad \alpha_p^4(\boldsymbol{\lambda}) \cong \alpha_{o,p}^4 + \sum_{i=1}^{n} \frac{\beta_i a_{p,i}}{1+\beta_i b_{p,i}} \quad , \qquad p=1,\dots,n_m, \quad \beta_i \in [-\Delta\lambda_i, \Delta\lambda_i]$

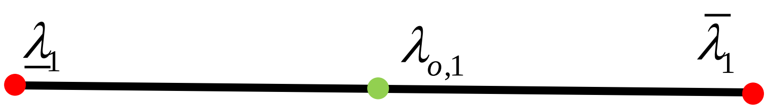
The terms $a_{p,i}, b_{p,i}$ $\; c_{p,i}, d_{p,i}$ $\; e_{p,i}(t), f_{p,i}(t)$ are coefficients to be computed enforcing the correspondence between exact and approximated solutions at given damage configurations.

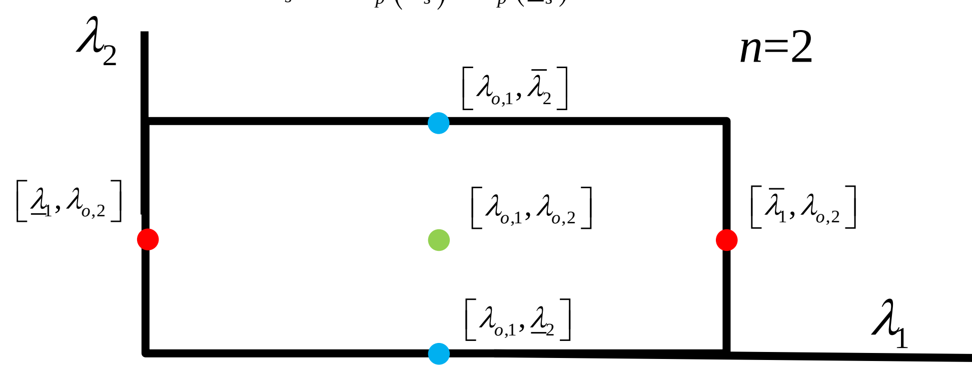
This approach requires computing the exact modal properties at $2n+1$ damage configurations. To uncouple the system of equations the considered damage configurations are:

$$\bar{\boldsymbol{\lambda}}_s = [\lambda_{o,1}, \lambda_{o,2}, \dots, \bar{\lambda}_s, \dots \lambda_{o,n}] \qquad s=1,\dots,n$$
$$\underline{\boldsymbol{\lambda}}_s = [\lambda_{o,1}, \lambda_{o,2}, \dots, \underline{\lambda}_s, \dots \lambda_{o,n}]$$

$$a_{p,s} = \frac{2}{\Delta\lambda_s} \frac{[\alpha_p^4(\bar{\boldsymbol{\lambda}}_s) - \alpha_{o,p}^4][\alpha_{o,p}^4 - \alpha_p^4(\underline{\boldsymbol{\lambda}}_s)]}{\alpha_p^4(\bar{\boldsymbol{\lambda}}_s) - \alpha_p^4(\underline{\boldsymbol{\lambda}}_s)}$$
$$b_{p,s} = \frac{1}{\Delta\lambda_s} \frac{2\alpha_{o,p}^4 - \alpha_p^4(\underline{\boldsymbol{\lambda}}_s) - \alpha_p^4(\bar{\boldsymbol{\lambda}}_s)}{\alpha_p^4(\bar{\boldsymbol{\lambda}}_s) - \alpha_p^4(\underline{\boldsymbol{\lambda}}_s)}$$

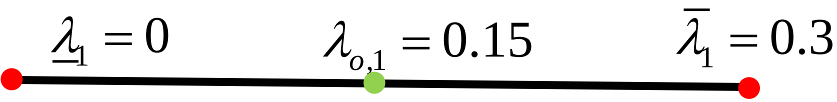
$n=1$

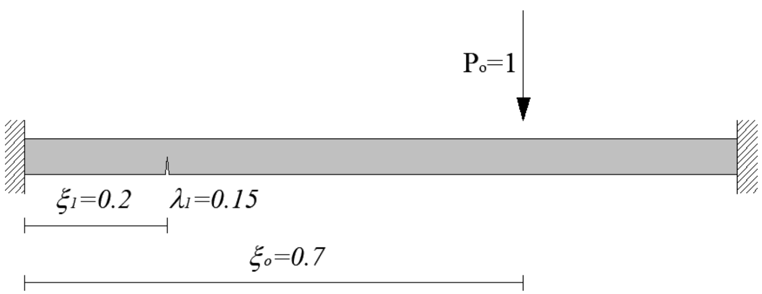


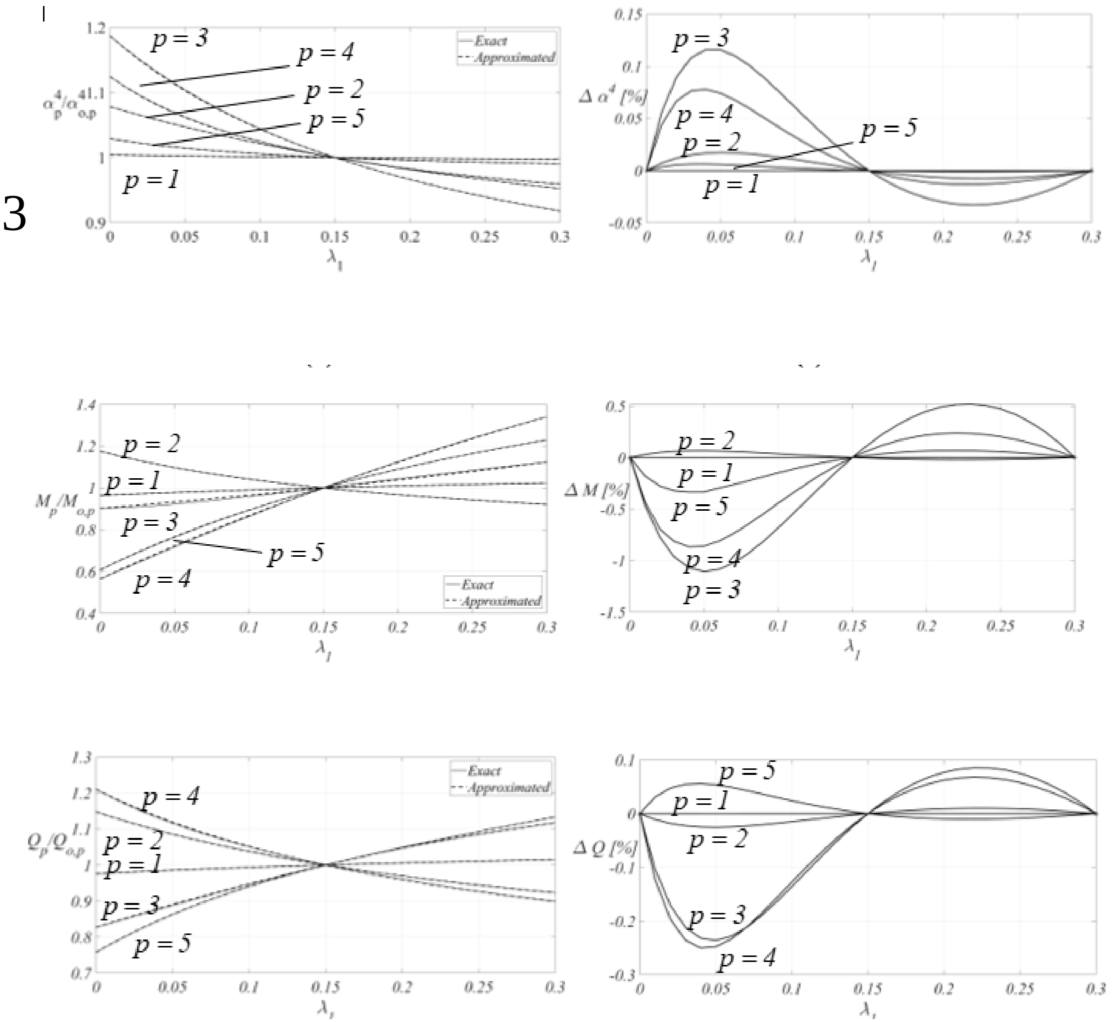


Validation of the proposed approach: single cracked beam

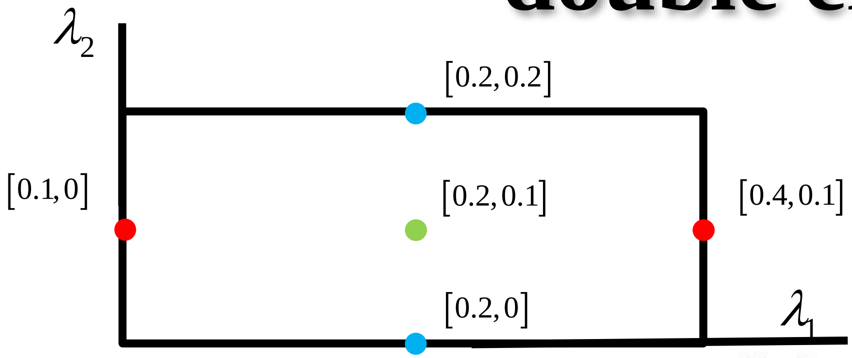
Approximated solution calibrated on

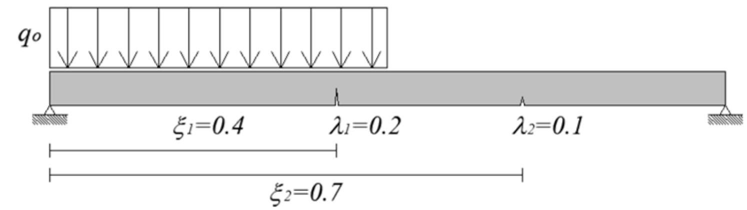


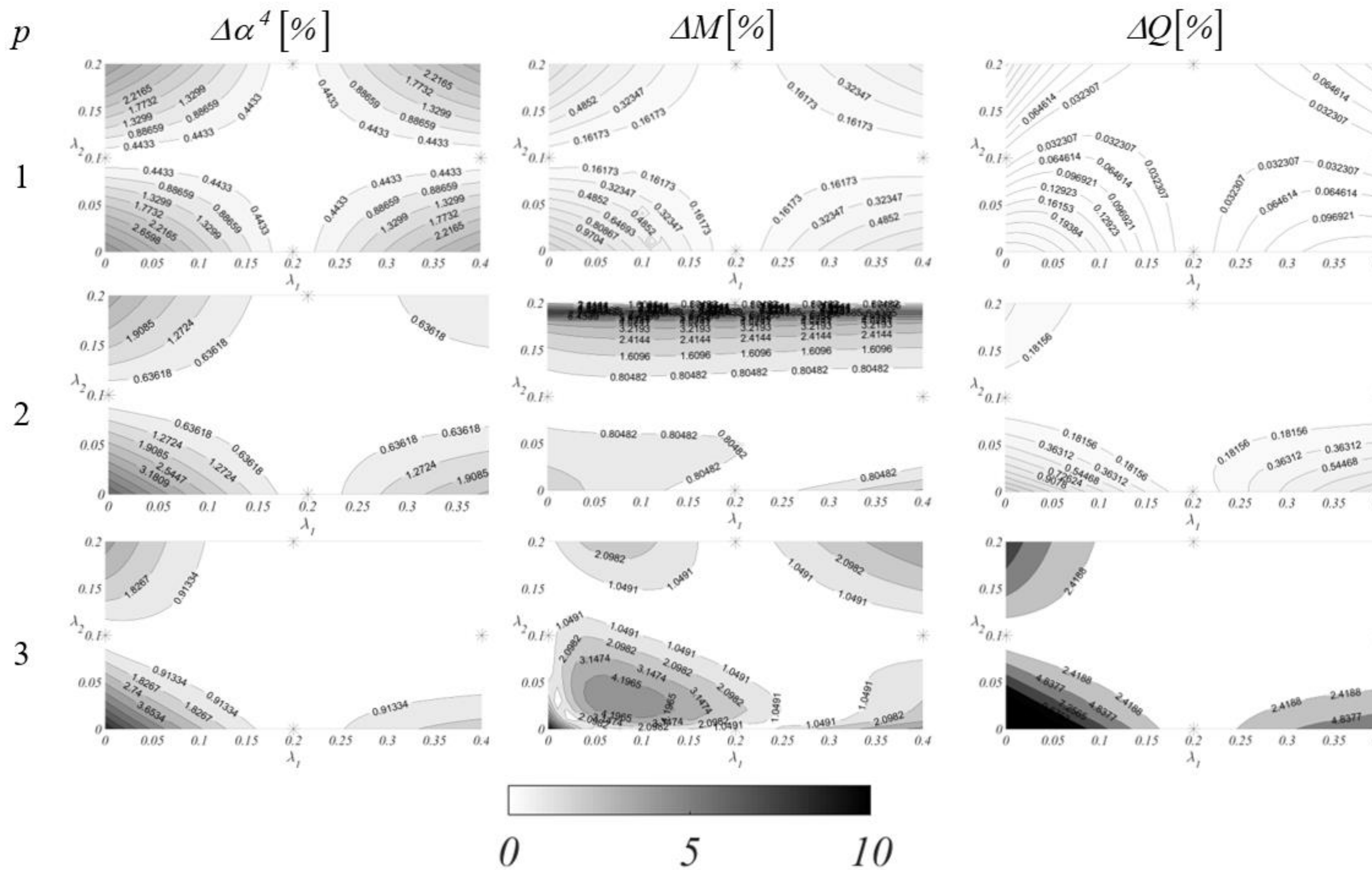




Validation of the proposed approach: double cracked beam





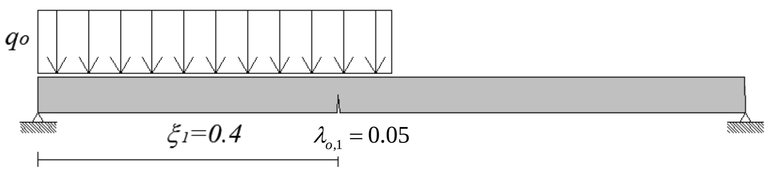


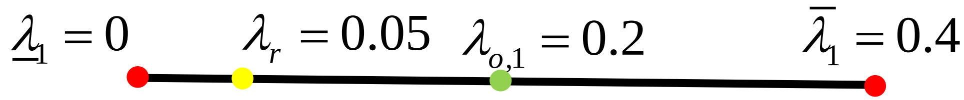
Validation of the proposed approach: time histories

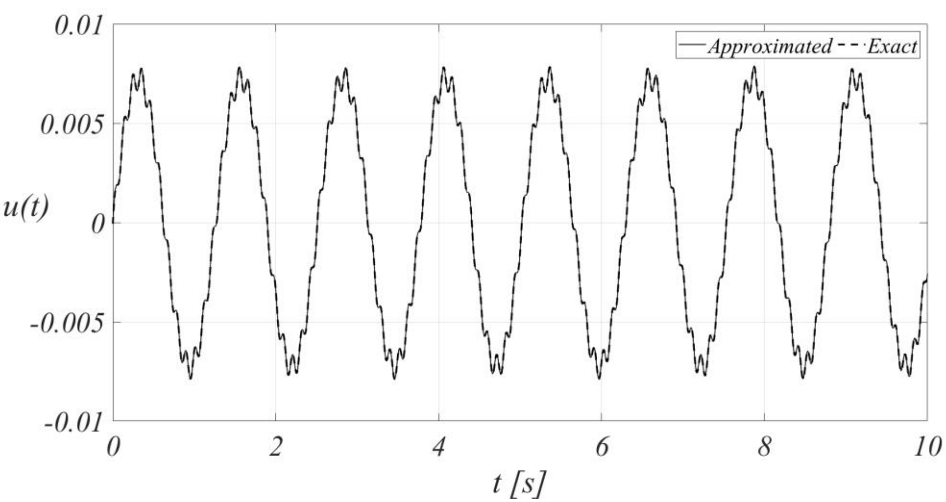
L=5 m, E=2.1*10^{11} N/m^2,
I= 8.33*10^{-6} m^4, m= 75 kg/m

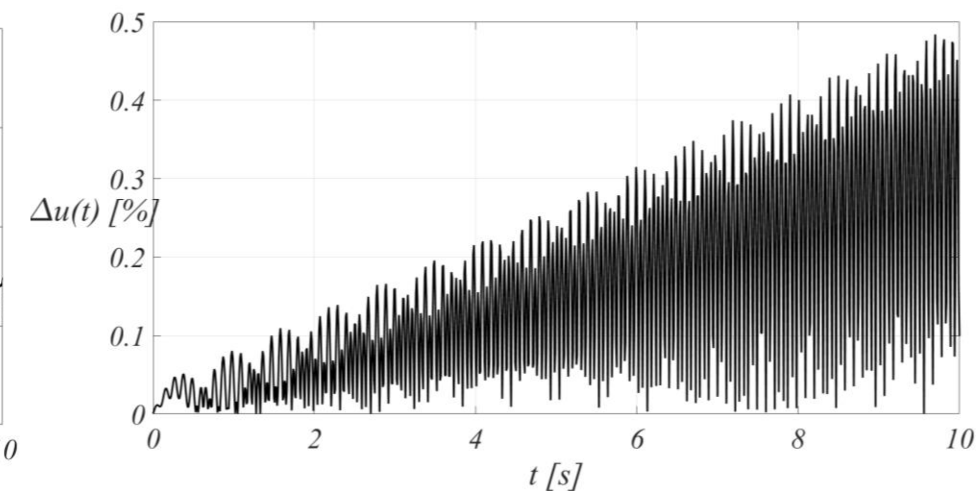
$q_o \sin \omega t \qquad q_o = 1 \qquad \omega = 5\, rad/s \qquad \xi_r = 0.5$

Approximated solution calibrated on









Response bounds

$$r(\xi_r, t; \boldsymbol{\lambda}) = \sum_{p=1}^{n_m} \phi_p^k(\xi_r; \boldsymbol{\lambda}) z_p(t)$$

The damage intensities are considered as uncertain but bounded variables. Possible strategies to compute the response bounds of the dynamic response of excited beams

Here, any response parameter can be provided as an explicit function of the uncertain variables, therefore the research of the bounds is much easier.

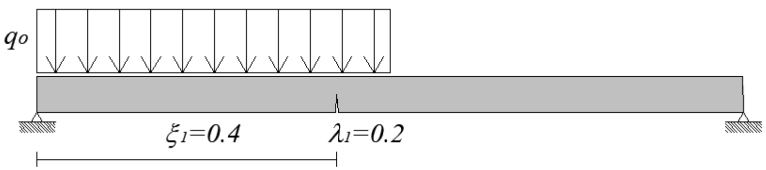
	Eigenproperties computation
Vertex method	2^n
Reanalysis	n_d^n
Proposed strategy	$2n+1$

Due to resonance effects the bounds of the response can be associated to any value of the considered intervals depending on the acting load. Therefore, the vertex method is unsuitable for this kind of problem.

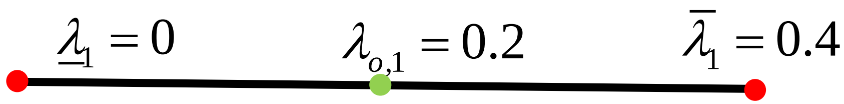
Response bounds - time histories

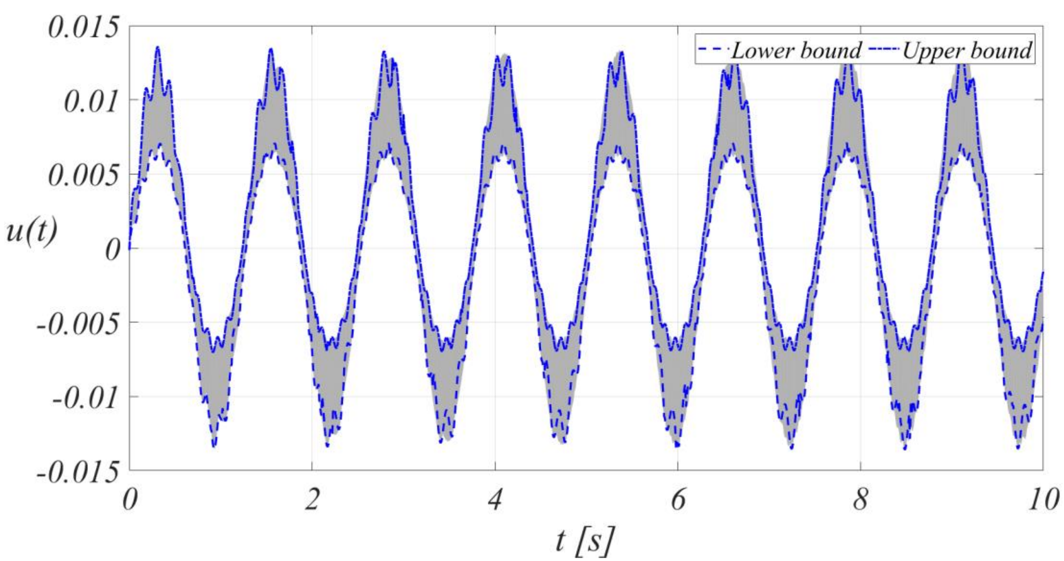
L=5 m, *E*=2.1*10[11] N/m[2],
I= 8.33*10[-6] m[4], *m*= 75 kg/m

$q_o \sin \omega t$ $\quad q_o = 1$ $\quad \omega = 5 rad/s$ $\quad \xi_r = 0.5$ $\quad \zeta_p = 0.05$



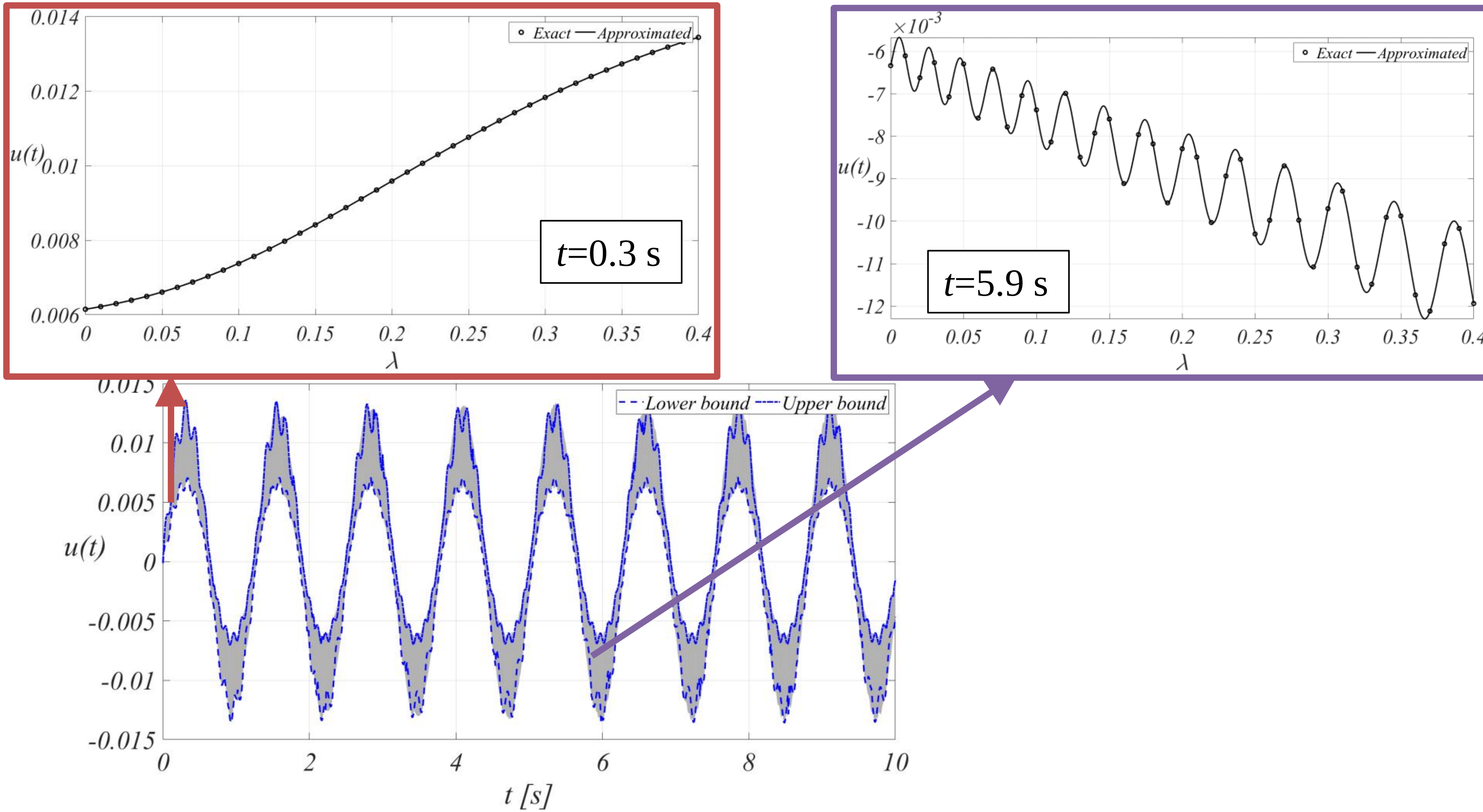
Approximated solution calibrated on





The grey band represents the envelope the envelope of the responses of 41 analyses computed with the reference procedure scanning the damage intensities at evenly spaced steps $\Delta\tilde{\lambda}_1 = 0.01$

Response variability - time history

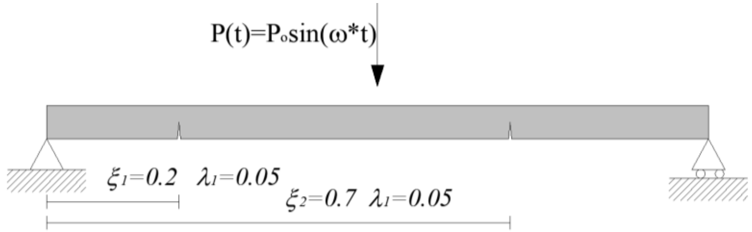


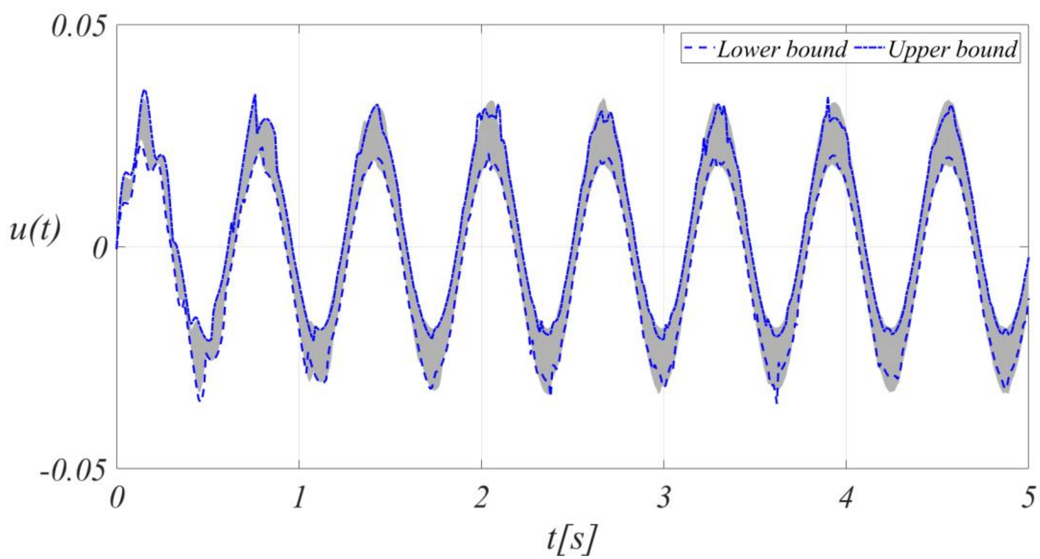
Response bounds - time histories

$$P_o \delta(\xi - \xi_o) \sin \omega t \qquad \omega = 10\, rad/s \quad \xi_L = 0.5$$

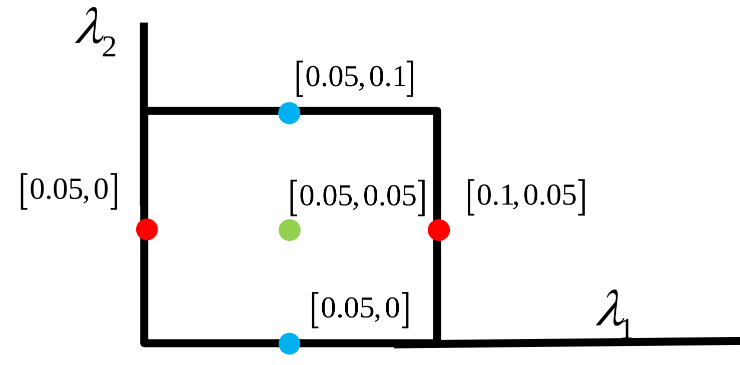
$$\xi_r = 0.5 \qquad \zeta_p = 0.05$$

L=5 m, E=2.1*10^{11} N/m^2,
I= 8.33*10^{-6} m^4, m= 75 kg/m



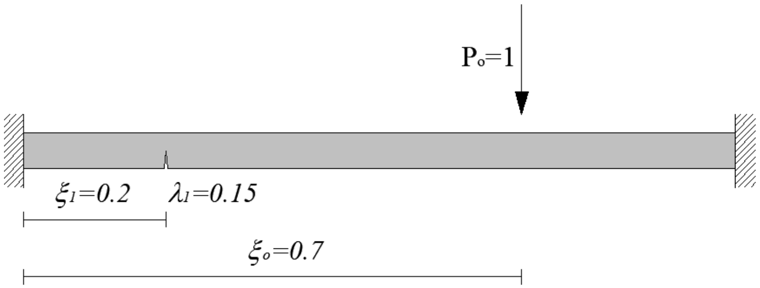


Approximated solution calibrated on



The grey band represents the envelope the envelope of the responses of 121 analyses computed with the reference procedure scanning the damage intensities at evenly spaced steps

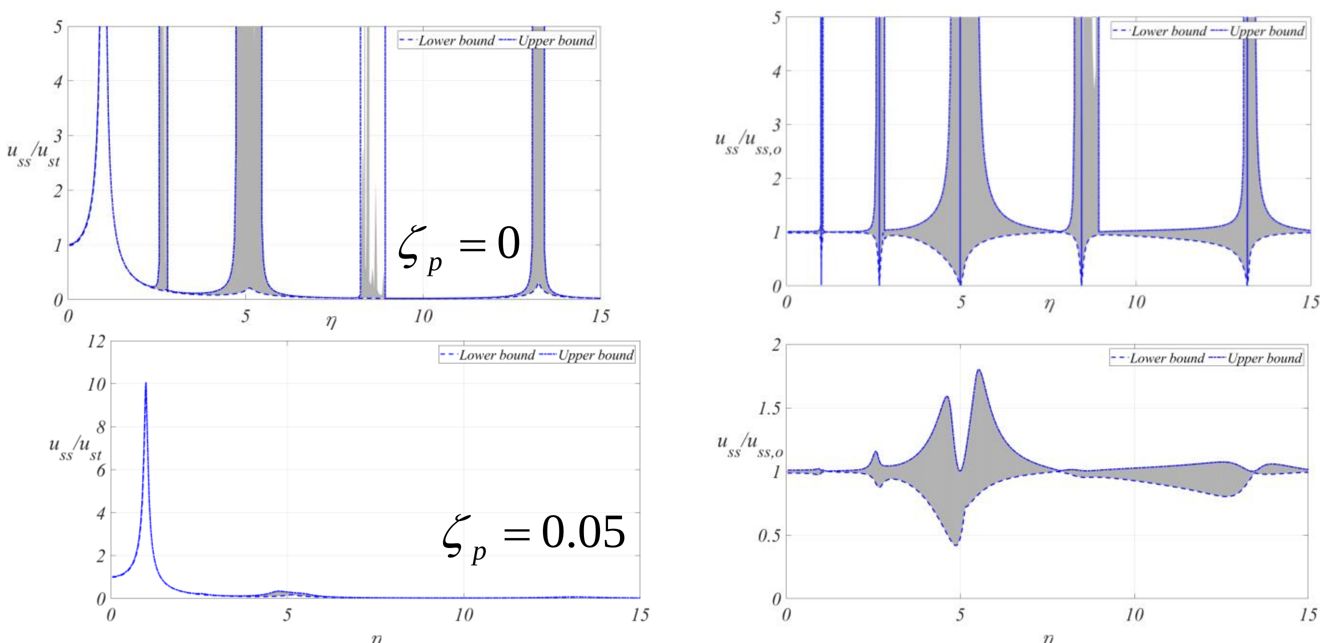
Response bounds –spectra



$$P_o\delta(\xi-\xi_o)\sin\omega t \quad \xi_r = 0.5 \quad \xi_L = 0.7$$

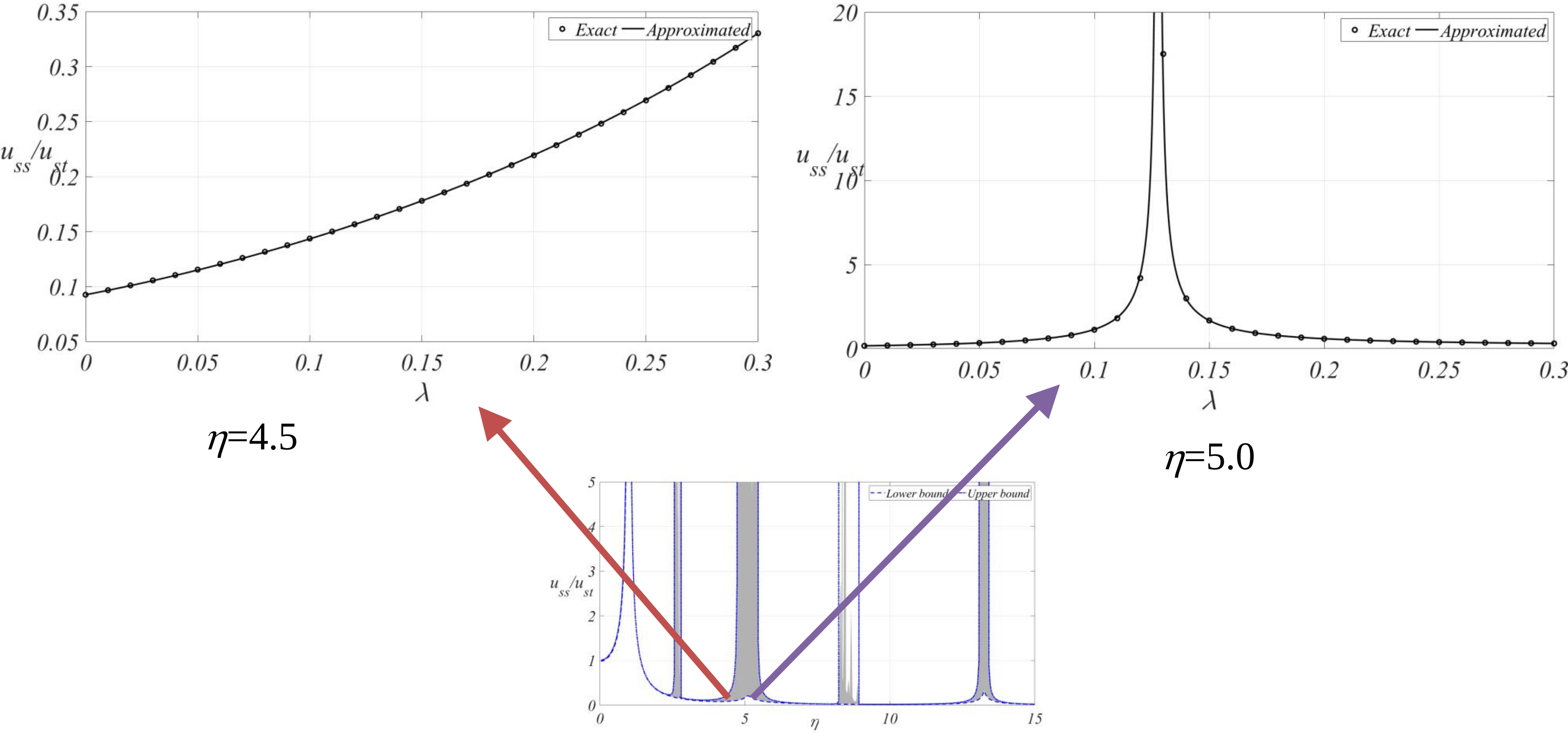
Approximated solution calibrated on

$$\underline{\lambda}_1 = 0 \qquad \lambda_{o,1} = 0.15 \qquad \overline{\lambda}_1 = 0.3$$



The grey band represents the envelope the envelope of the responses of 31 analyses computed with the reference procedure scanning the damage intensities at evenly spaced steps

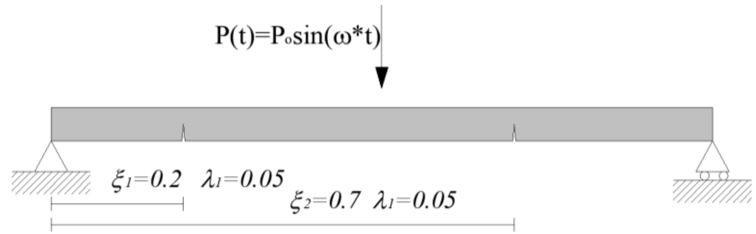
Response variability - spectra

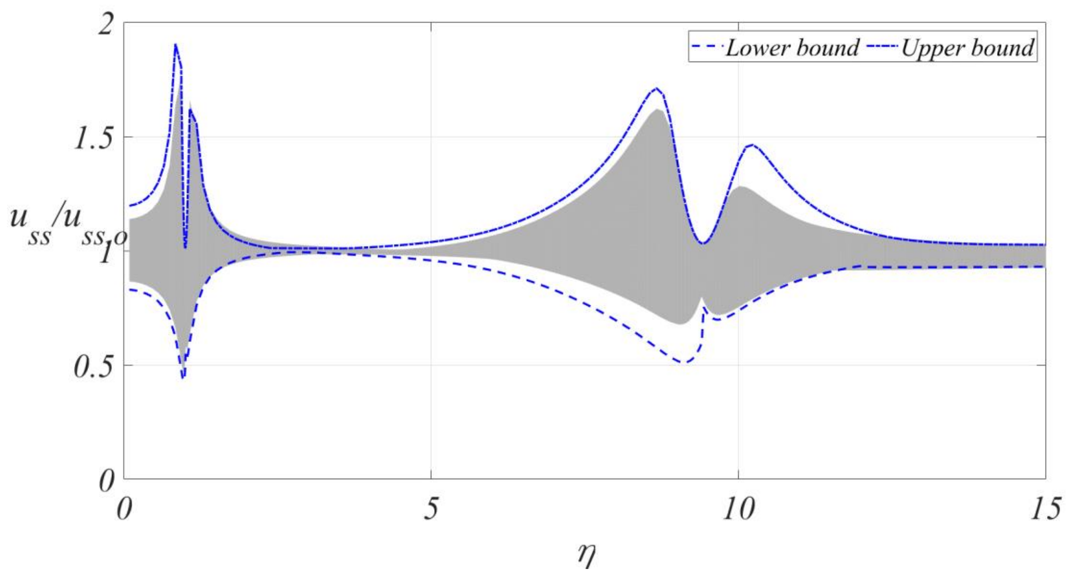


Response bounds - spectra

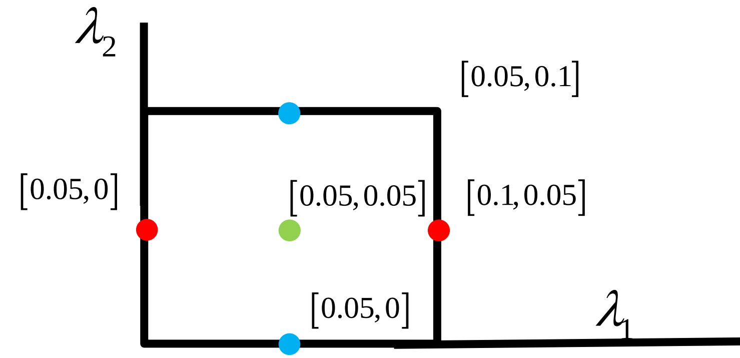
L=5 m, E=2.1*10^{11} N/m^2, I= 8.33*10^{-6} m^4, m= 75 kg/m

$q_o \sin \omega t \quad q_o = 1 \quad \omega = 5\,rad/s \quad \xi_r = 0.5 \quad \zeta_p = 0.05$





Approximated solution calibrated on



The grey band represents the envelope the envelope of the responses of 121 analyses computed with the reference procedure scanning the damage intensities at evenly spaced steps

Conclusions and future developments

✓The forced vibrations of the Euler-Bernoulli multi-cracked beam have been treated by means of the theory of distributions

✓An explicit expression of the main modal parameters as a function of the crack intensities has been provided and validated

✓The computational efficiency of the proposed approach has been shown when multiple analyses are needed

✓The explicit expression of the response parameters has been employed to investigate the response variability of the forced vibration of damaged beams

✓Applications to identify the response bounds when the crack intensities are considered as uncertain but bounded variables are provided both for the case of time histories and response spectra

✓Although falling outside the scope of this study, the explicit expressions here provided were also employed for inverse problems identifying crack severities from frequency measurements

➢ Extension of the proposed approach to damaged frame structures might be the object of future studies