A Fully Decoupled Approach for a Class of Reliability-based Optimization Problems in Stochastic Linear Dynamics

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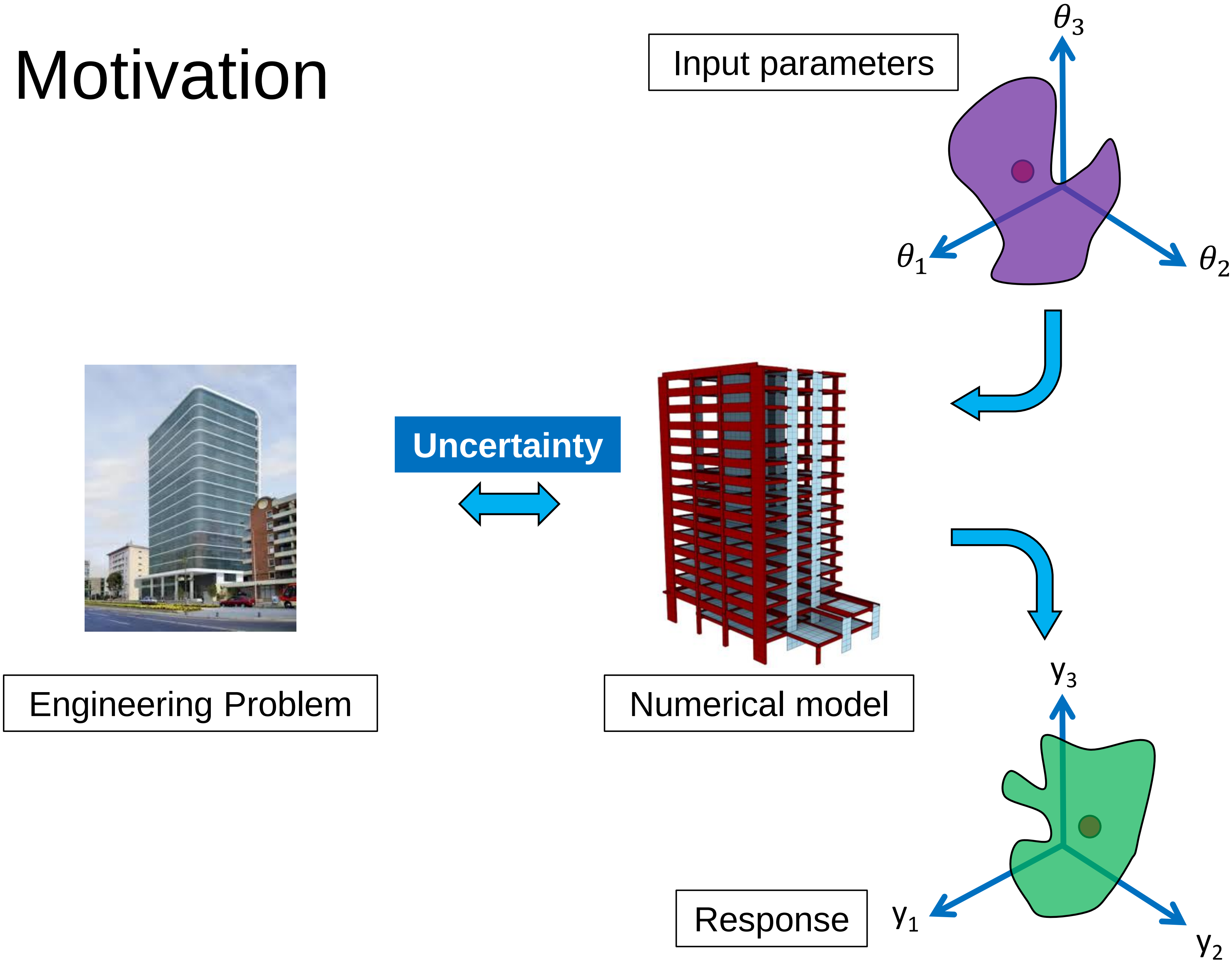
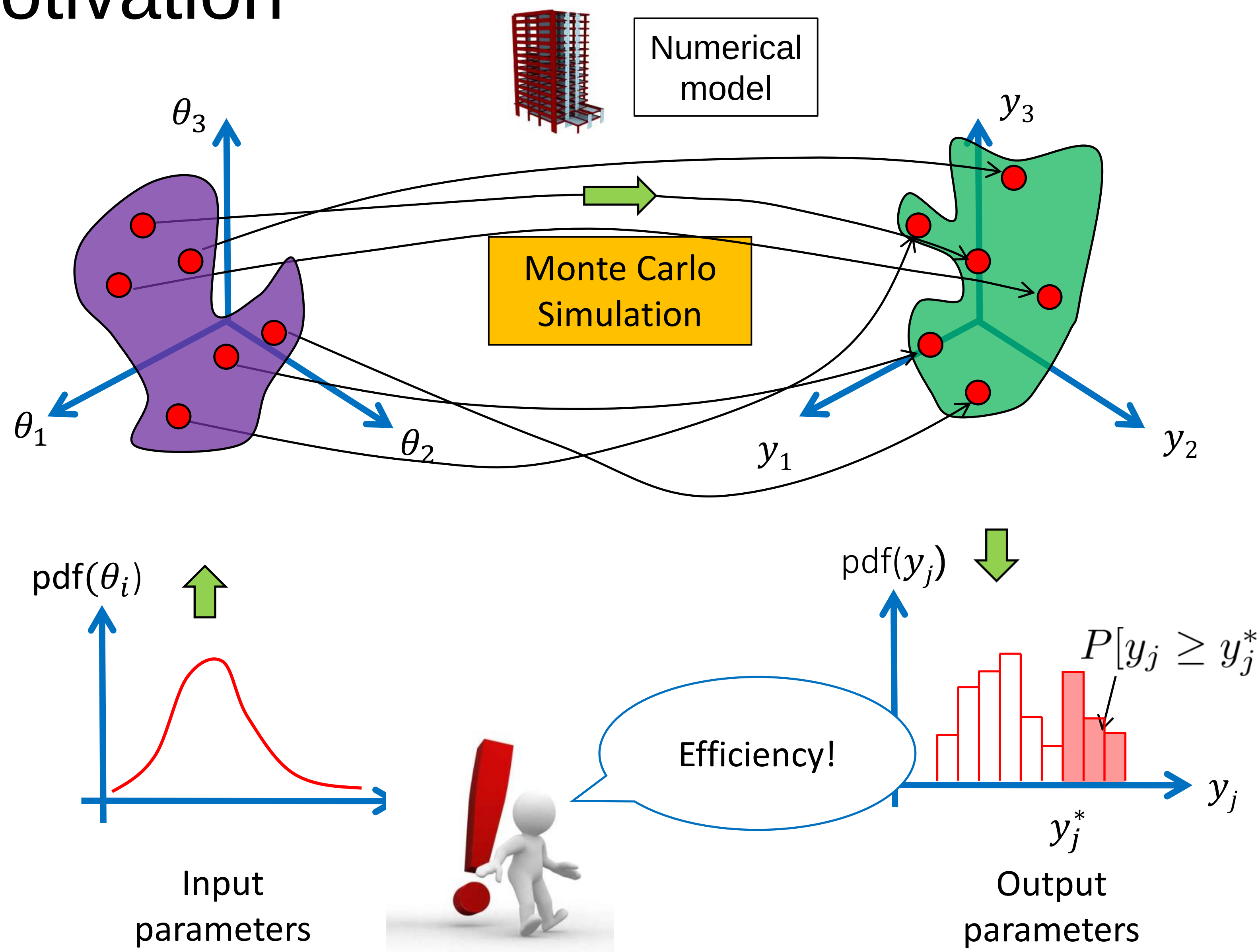
Motivation



Input parameters
θ_3
θ_1
θ_2
Engineering Problem
Uncertainty
Numerical model
y_3
Response
y_1
y_2

Motivation


Numerical model
θ_3
θ_1
θ_2
Monte Carlo Simulation
y_3
y_1
y_2
pdf(θ_i)
pdf(y_j)
$P[y_j \geq y_j^*]$
Efficiency!
y_j
y_j^*
Input parameters
Output parameters

Motivation

- **Coping** with optimization and uncertainty: **huge challenge!**

Optimization

Reliability

Deterministic model

- Proposed approach: **decoupling** via operator norm theorem

Optimization

Deterministic model

+

Reliability

Deterministic model

Scope:
- **Gaussian dynamic excitation**
- **Linear structure**
- **Minimization of failure probability**

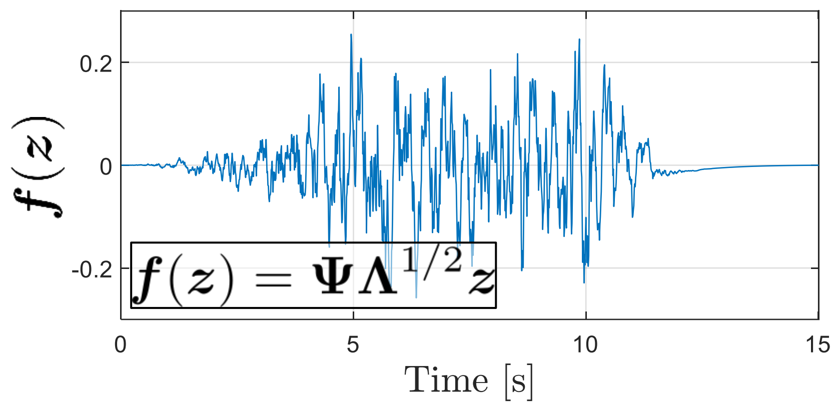
Contents

- Motivation
- **Problem Statement**
- Operator Norm Theorem for RBO
- Example
- Conclusions

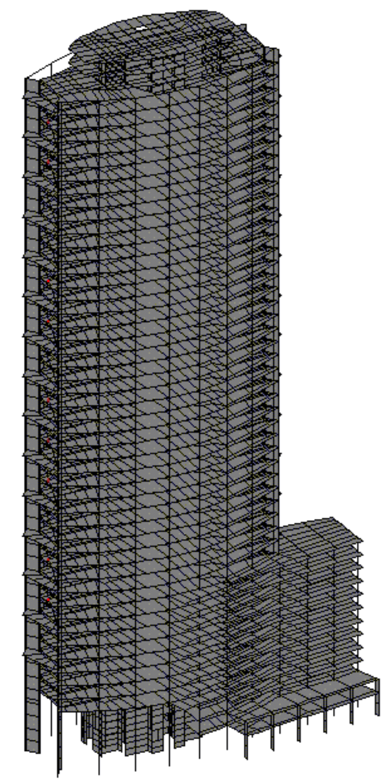
Problem Statement

- **Random Vibration** problem

Stochastic ground acceleration

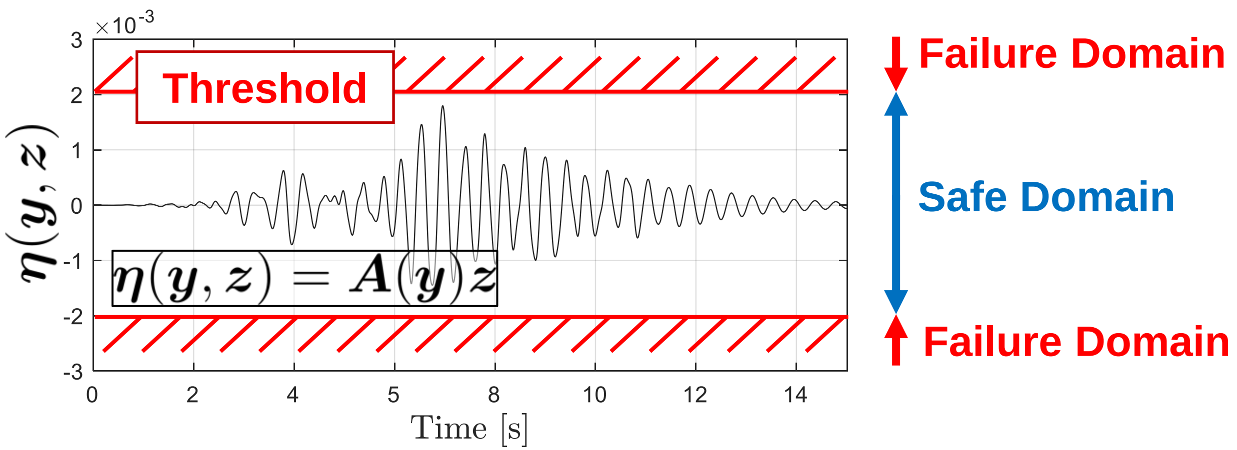


Finite element model



y
(design variables)

Stochastic response



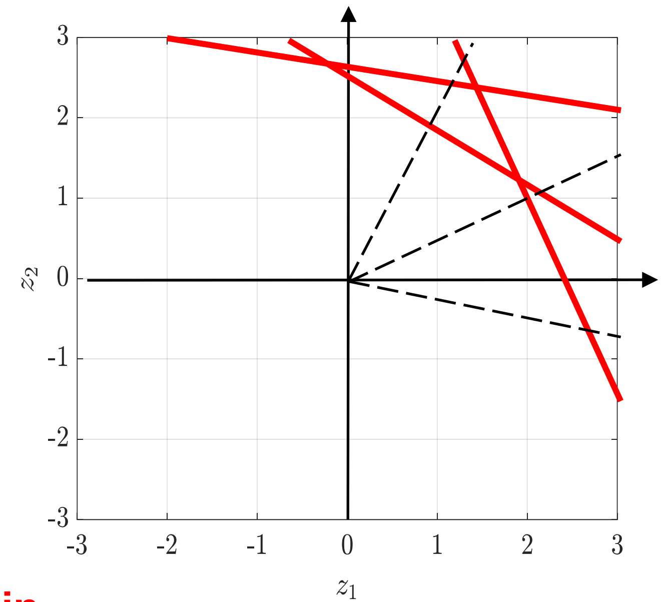
Problem Statement

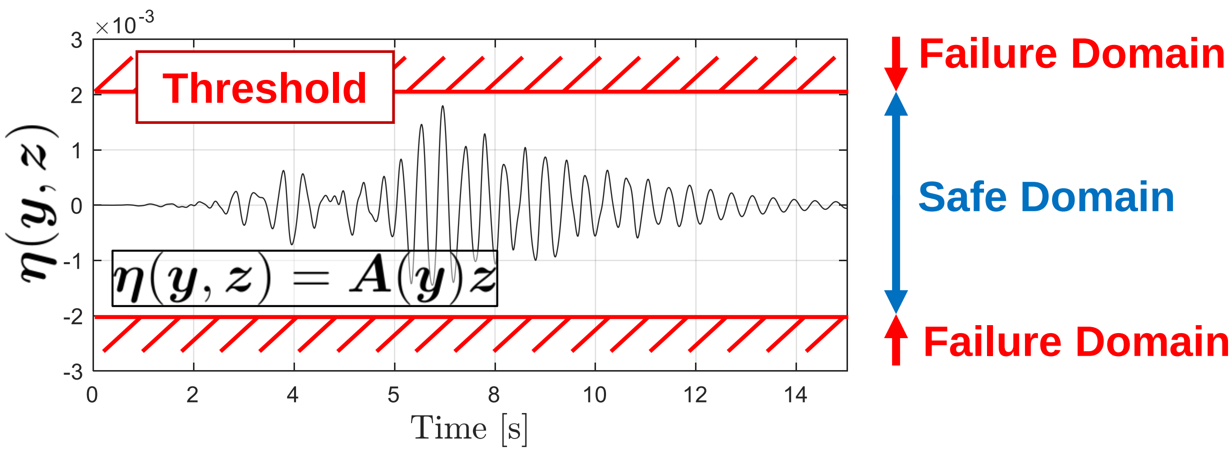
- **<u>First excursion probability</u>**: probability that structural response of interest exceeds acceptable threshold within the duration of the excitation

$$p_F(\boldsymbol{y}) = \int_{\boldsymbol{z}\in\mathbb{R}^{n_D}} I_F\left(\boldsymbol{y},\boldsymbol{z}\right) f_Z\left(\boldsymbol{z}\right) d\boldsymbol{z}$$

Indicator function

PDF, excitation





Der Kiureghian, A. The geometry of random vibrations and solutions by FORM and SORM. Probabilistic Engineering Mechanics, 2000 , 15 , 81-90

Problem Statement

- **<u>Reliability-based optimization (RBO) problem</u>**

$$\boxed{\begin{array}{l} \min_{\boldsymbol{y}} p_F(\boldsymbol{y}) \\ \text{subject to} \\ c_l(\boldsymbol{y}) \leq 0,\ \ l = 1, \ldots, n_C \\ y_j^L \leq y_j \leq y_j^U,\ \ j = 1, \ldots, n_y \end{array}}$$

- $c_l(\boldsymbol{y})$: deterministic constraints (e.g. physical constraint, budget)
- y_j^L, y_j^U: bounds

<u>Challenges</u>:

- **Double loop**
- **High dimensional probability integral**
- **Probability available through simulation**

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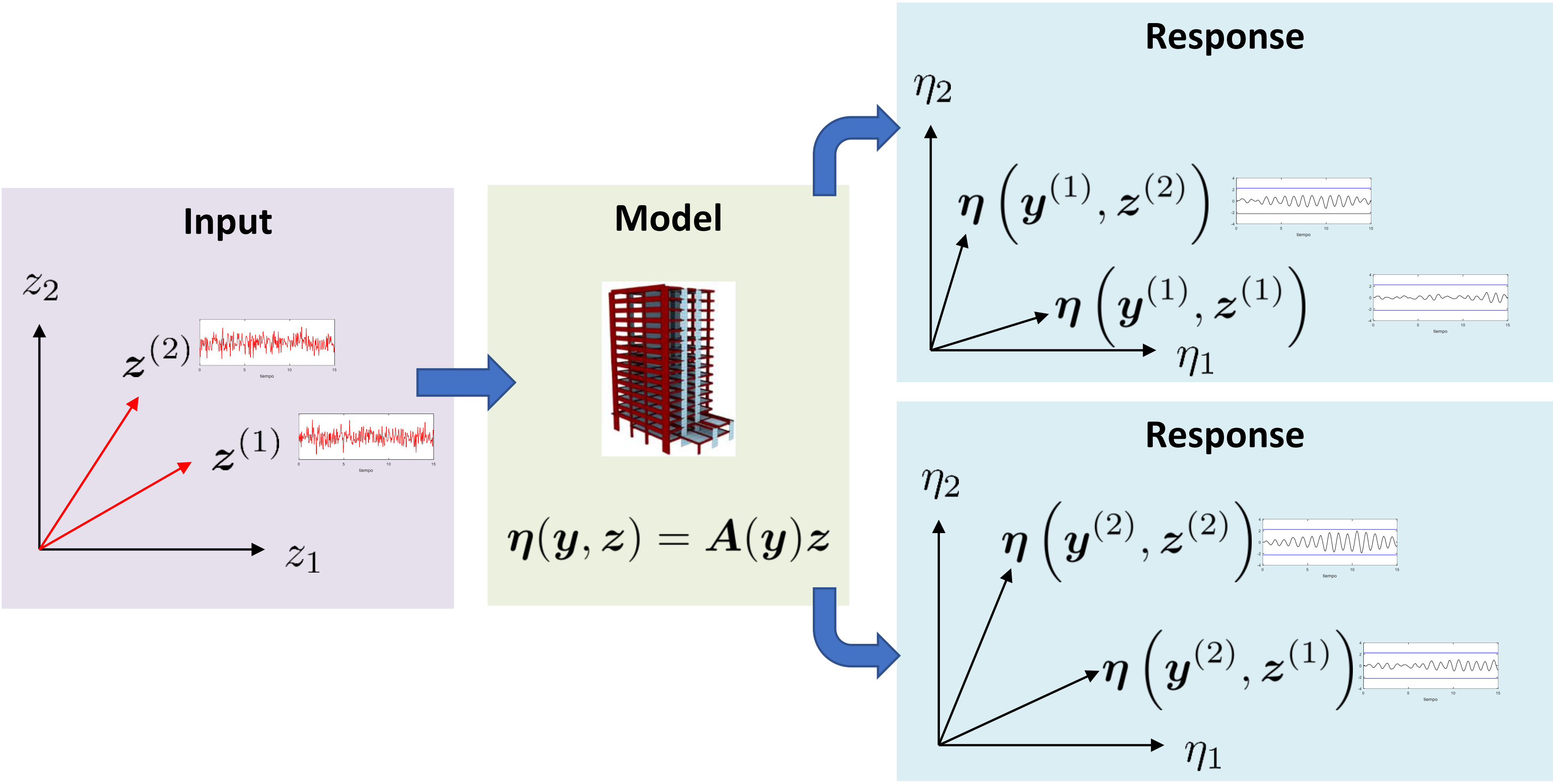
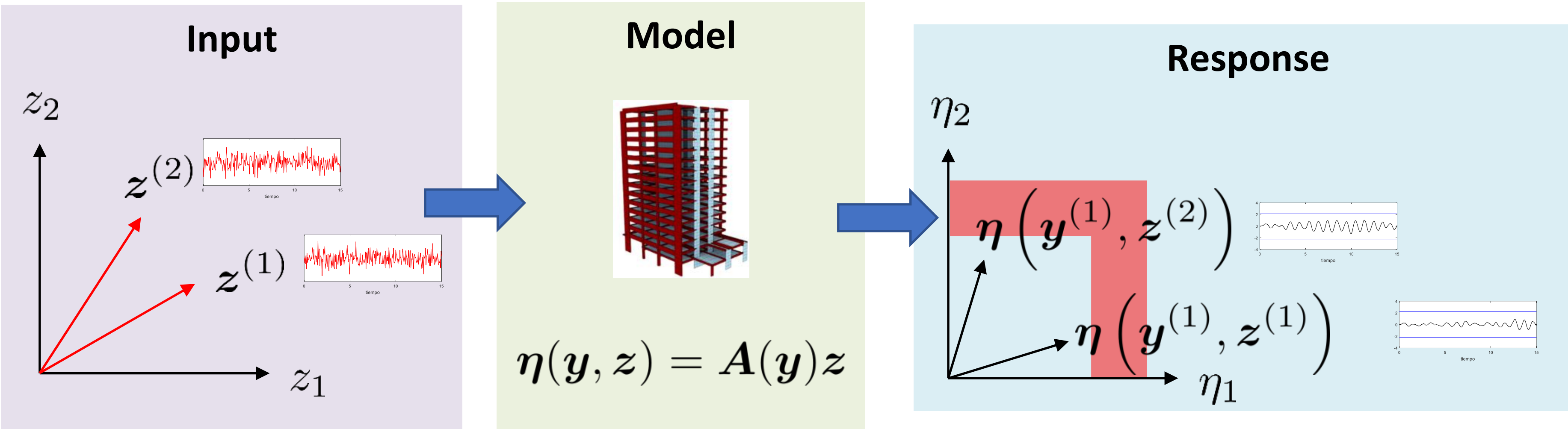
Operator Norm Theorem for RBO



- Response $\boldsymbol{\eta}(\boldsymbol{y}, \boldsymbol{z})$ is the result of ***stretching*** loading $\boldsymbol{z}$ by $\boldsymbol{A}(\boldsymbol{y})$
- <u>Less stretching</u> should lead to **smaller** $\boldsymbol{p_F}$

Operator Norm Theorem for RBO



- The amount of ***stretching*** induced by $\boldsymbol{A}(\boldsymbol{y})$ can be bounded by **operator norm theorem**

$$||\boldsymbol{A}(\boldsymbol{y})||_{\infty,2} = \inf\{c(\boldsymbol{y}) \geq 0 : ||\boldsymbol{A}(\boldsymbol{y})\boldsymbol{z}||_{\infty} \leq c(\boldsymbol{y})\,||\boldsymbol{z}||_2\}$$

- <u>Operator norm</u>: find row of $\boldsymbol{A}(\boldsymbol{y})$ with largest Euclidean norm

$$||\boldsymbol{A}(\boldsymbol{y})||_{\infty,2} = \max_{i=1,\ldots,n_R,\ k=1,\ldots,n_T}\left(\sqrt{\boldsymbol{a}_{i_k}\boldsymbol{a}_{i_k}^T}\right)$$

Operator Norm Theorem for RBO

Optimization

Deterministic model

Reliability

Deterministic model

$$\begin{aligned}&\min_{\boldsymbol{y}} \|\boldsymbol{A}(\boldsymbol{y})\|_{\infty,2}\\ &\text{subject to}\\ &c_l(\boldsymbol{y}) \leq 0,\ l = 1,\ldots,n_C\\ &y_j^L \leq y_j \leq y_j^U,\ j = 1,\ldots,n_y\end{aligned}$$

$$p_F(\boldsymbol{y}) = \int_{\boldsymbol{z}\in\mathbb{R}^{n_D}} I_F(\boldsymbol{y},\boldsymbol{z})\, f_Z(\boldsymbol{z}) d\boldsymbol{z}$$

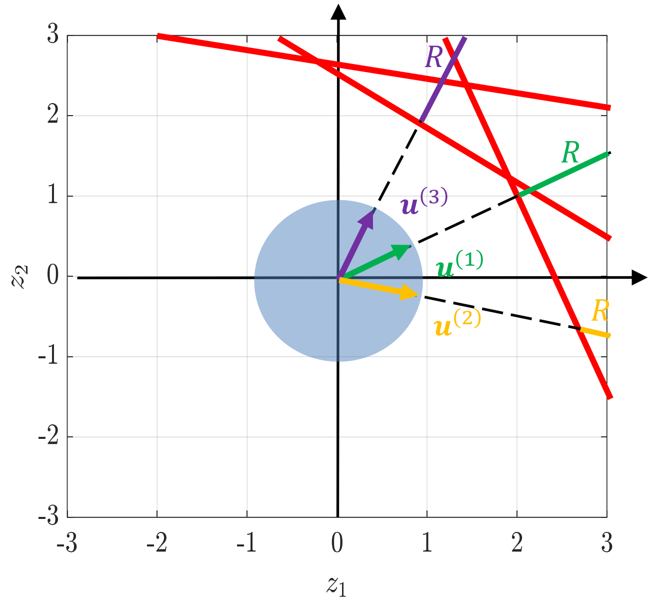
<u>Minimization of operator norm</u>: find $\boldsymbol{y}$ that minimizes amount of stretching induced by $\boldsymbol{A}(\boldsymbol{y})$

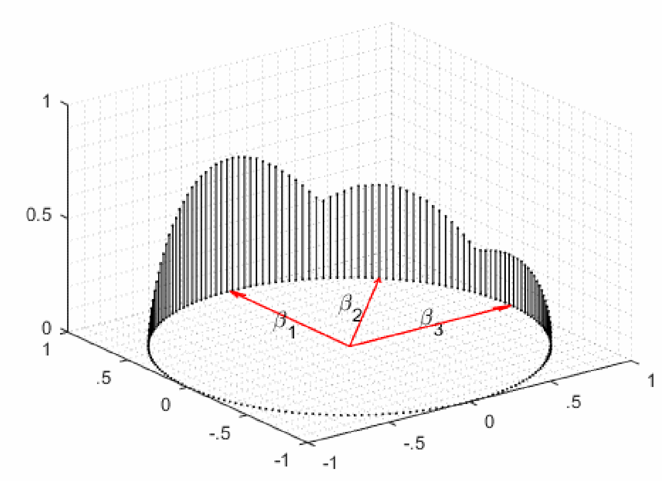
Perform ordinary reliability analysis to determine minimum failure probability

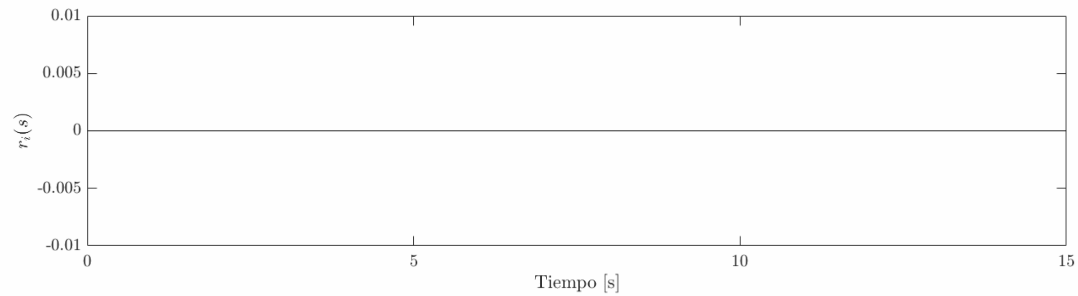
Operator Norm Theorem for RBO

- Probability estimation: **Directional Importance Sampling***

Key concepts for simulation







*Misraji, M.; Valdebenito, M.; Jensen, H. & Mayorga, C. Application of directional importance sampling for estimation of first excursion probabilities of linear structural systems subject to stochastic Gaussian loading. Mechanical Systems and Signal Processing, 2020 , 139 , 106621

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Example

- SDOF system

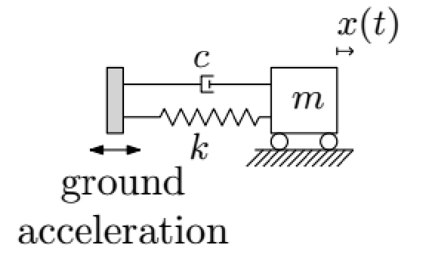
- Stochastic ground acceleration (Clough-Penzien + Shinozuka-Sato), **more than 1000 rv's**

- <u>Design variable</u>: stiffness k

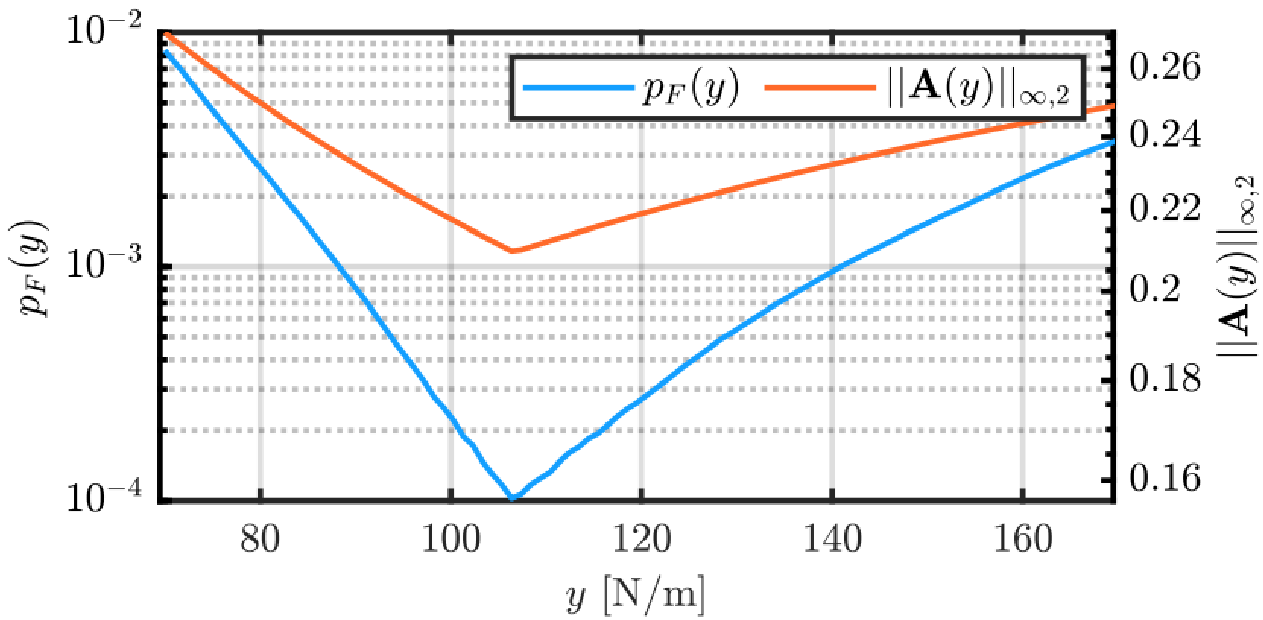
- <u>Failure criteria</u>

  - Maximum displacement below a threshold

  - Maximum acceleration below a threshold



Example



- Operator norm serves as suitable proxy for failure probability
- Speed up of about two orders of magnitude

*Faes, M. G. & Valdebenito, M. A. Fully decoupled reliability-based design optimization of structural systems subject to uncertain loads. Computer Methods in Applied Mechanics and Engineering, 2020 , 371 , 113313
*Faes, M. & Valdebenito, M. Fully decoupled reliability-based optimization of linear structures subject to Gaussian dynamic loading considering discrete design variables. Mechanical Systems and Signal Processing, 2021 , 156 , 107616

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Conclusions

- Operator norm theorem offers a framework for decoupling optimization and reliability

- Limited to a specific class of problems

- References

- Faes, M. G. & Valdebenito, M. A. Fully decoupled reliability-based design optimization of structural systems subject to uncertain loads. Computer Methods in Applied Mechanics and Engineering, 2020 , 371 , 113313
- Faes, M. & Valdebenito, M. Fully decoupled reliability-based optimization of linear structures subject to Gaussian dynamic loading considering discrete design variables. Mechanical Systems and Signal Processing, 2021 , 156 , 107616